A

FAMILIAR EXPLANATION

OF

ARITHMETIC;

PART I.

CONTAINING

SIMPLE AND COMPOUND RULES, REDUCTION, AND THE ELEMENTARY RULES APPLIED TO DECIMALS.

BY THE

REV. FREDERICK CALDER, M.A.

HEAD MASTER OF THE GRAMMAR SCHOOL, CHESTERFIELD.

NEW EDITION.

LONDON:
LONGMAN, GREEN, LONGMAN, AND ROBERTS.
1861.

The Arithmetical Questions in Part II. of this Arithmetic are sold separately, 55 pp. *Price* 1*s*.

Also, The Answers to both Parts, 1*s*.

By the same Author,

KEY to Lund's Mensuration. *Price* 3*s*.

ADVERTISEMENT TO NEW EDITION OF PART I.

THIS edition differs from the preceding ones, simply in containing the application of the four elementary rules to decimals; the object being to teach even young pupils the advantages of decimals over all other calculations, and so prepare them to appreciate the benefits of a decimalized system of money, weights, and measures.

ARITHMETIC.

NOTATION AND NUMERATION.

1. When several things of the same kind are placed before the eye, even of a child, and a person either takes some of them away, or puts some more to them, the child perceives that there has taken place an alteration,—but not in the *kind* of the articles. Hence he first gets his idea of *number*, and asks the question, How many things?

Or, if he divides a heap of marbles among his companions, giving two to each till the heap is all gone, the question comes into his mind—How many times round can I go?

Now the science which will enable him to answer the questions of *How many things?* and *How many times?* is Arithmetic.

His ideas of numbers are as yet expressed only in words; but he afterwards learns that these numbers can be written shorter in *figures*. Now the art of changing these words into figures which have the same meaning is called Notation, and that of reading figures in their proper words is called Numeration.

The following are the figures or digits used in Arithmetic, and by means of which all numbers whatever may be expressed.

	0,	and it is thus read in words,		*Nought* or *cipher*,
	1,	"	"	*One*,
	2,	"	"	*Two*,
	3,	"	"	*Three*,
	4,	"	"	*Four*,
	5,	"	"	*Five*,
(A)	6,	"	"	*Six*,
	7,	"	"	*Seven*,
	8,	"	"	*Eight*,
	9,	"	"	*Nine*,

being altogether TEN in number. *

2. The reason why there should be used only ten different figures is, because there are ten fingers on the hand; and these have been early used for counting: thus, beginning with the thumb of one hand, and going through both hands, a boy finds that he can count up to *ten;* but when he comes to the last or tenth finger, and he wishes to count any more, he must make some mark or note to shew that he has been *once* round. This is done by taking the figure 1, and placing a 0 at its right hand, writing it thus, 10, where the figure 1 which formerly stood for one only, now stands for one round of the fingers, or *one ten,* which we call *Ten.*

If now I begin to go over the hands again, and thus make *two* rounds, or two tens, which are called *Twenty,* I ought to write a figure 2, where I before had 1, and write **20**: If I take 3 rounds, or 3 tens, I must write 3 instead of 2, and say **30**, or Thirty: So also

4 rounds give 4 tens, or **40**, called *Forty.*
5 rounds give 5 tens, or **50**, called *Fifty.*

* The numbers 1, 3, 5, 7, &c., are called *odd*, and 2, 4, 6, 8, &c., are called *even* numbers.

(B) 6 rounds give 6 tens, or 60, called *Sixty*.
7 rounds give 7 tens, or 70, called *Seventy*.
8 rounds give 8 tens, or 80, called *Eighty*.
9 rounds give 9 tens, or 90, called *Ninety*.
10 rounds give 10 tens, or 100, which has now a new name, called a hundred, or ONE *Hundred*.

Exs. 1. Write down in figures, Forty, Seventy, Sixty, Thirty, Ninety, Twenty, Fifty, Ten, Eighty, One hundred.

Write down in words, 20, 60, 90, 100, 50, 30, 80, 70, 40.

OBS. The pupil should be perfectly ready at reading and writing the above examples, before he advances a step further.

We now find that when two figures are put at the end of the 1, as in **100**, this figure is called ONE *hundred*; so, if we write 2 instead of 1, and have

	200,	we shall call it	*Two hundred,*
	300,	„	*Three hundred,*
	400,	„	*Four hundred,*
	500,	„	*Five hundred,*
(C)	600,	„	*Six hundred,*
	700,	„	*Seven hundred,*
	800,	„	*Eight hundred,*
	900,	„	*Nine hundred,*
	1000,	„	*Ten hundred,*

which is called ONE *Thousand*.

Exs. 2. Write down in figures, Three Hundred, Eight Hundred, Seven Hundred, One Hundred, Ten Hundred, or One Thousand, Four Hundred, Two Hundred, Five Hundred.

Write down in words, 600, 800, 300, 100, 1000, 900, 400, 700, 200, 500.

3. We now learn that when *three* figures are placed after the 1, this figure 1 is called *One Thousand*; and we shall find it convenient to put a comma before these three figures, and write 1,000. When, therefore, we see three figures at the end of the 1, and a comma placed, we shall easily remember to read in words, ONE *Thousand*.

So, if there are three figures after the figure 2, we shall write

	2,000,	which is read	*Two thousand,*
and in like manner,	3,000,	„	*Three thousand,*
	4,000,	„	*Four thousand,*
	5,000,	„	*Five thousand,*
(D)	6,000,	„	*Six thousand,*
	7,000,	„	*Seven thousand,*
	8,000,	„	*Eight thousand,*
	9,000,	„	*Nine thousand,*
	10,000,	„	*Ten thousand,*

and in like manner, changing 10 into 20, 30, &c., up to 100, we have

	20,000,	which is read	*Twenty thousand,*
	30,000,	„	*Thirty thousand,*
	&c.		&c.
(E)	60,000,	„	*Sixty thousand,*
	&c.		&c.
	90,000,	„	*Ninety thousand,*
	100,000,	„	*One hundred thousand.*

And still further, changing 100 to 200, 300, &c., up to 1,000, we have

	200,000,	which is read	*Two hundred thousand,*
	300,000,	„	*Three hundred thousand,*
	&c.		&c.
(F)	600,000,	„	*Six hundred thousand,*
	700,000,	„	*Seven hundred thousand,*
	&c.		&c.
	1,000,000,	„	*One thousand thousand,*

and this is called by a new name, ONE *Million.*

Exs. 3. Write down in figures, Five Thousand, Nine Thousand, Thirty Thousand, Three Hundred Thousand, Eighty Thousand, Seventy Thousand, Seven Hundred Thousand, One Million.

Write in words, 4,000, 80,000, 600,000, 800,000, 1,000, 70,000, 1,000,000.

4. Having now *six* figures after the 1, and a comma placed after every three figures, (counting from the right

hand to the left,) we must remember that all figures to the left of these six right-hand figures are MILLIONS; we shall thus have

	2,000,000,	which is read	*Two millions,*
	&c.		&c.
	5,000,000,	"	*Five millions,*
(G)	6,000,000,	"	*Six millions,*
	&c.		&c.
	10,000,000,	"	*Ten millions,*
	&c.		&c.
(H)	60,000,000,	"	*Sixty millions,*
	100,000,000,	"	*One hundred millions,*
	200,000,000,	"	*Two hundred millions,*
	&c.		&c.
(I)	600,000,000,	"	*Six hundred millions,*
	&c.		&c.
	1,000,000,000,	"	*One thousand millions,*
(K)	6,000,000,000,	"	*Six thousand millions.* *

Exs. 4. Write down in figures, Four Millions, Nine Millions, Seventy Millions, One Hundred Millions, Three Hundred Millions, Eight Hundred Millions.

Write down in words, 40,000,000, 8,000,000, 60,000,000, 2,000,000, 50,000,000, 80,000,000, 600,000,000, 1,000,000,000.

5. We have now shown how to read in words any single figure, either by itself, or when followed by any number of figures; and we will now write out in the form of a table the results which we have just been proving.

* We do not often require numbers higher than these. We can, however, read as follows.

	10,000,000,000,	that is	*Ten thousand millions,*
	&c.		&c.
(L)	60,000,000,000,	"	*Sixty thousand millions,*
	&c.		&c.
	100,000,000,000,	"	*One hundred thousand millions,*
	&c.		&c.
(M)	600,000,000,000,	"	*Six hundred thousand millions,*
	&c.		&c.
	1,000,000,000,000,	"	*One thousand thousand millions,*

or, *One million millions,* which is called ONE BILLION. If there were six more figures to the right of the 1, it would become One *Trillion;* if 6 more, a *Quadrillion;* and so on, the highest separate name being a *Nonillion.*

Collecting a single line from each of the preceding groups of figures, viz. those marked A, B, C, D, E, F, G, H, I, K, L, M, and placing them so that the last figures in each row will stand under each other, we have

A.	 6	which is	*Six.*
B.	.... 6 0	„	*Sixty.*
C.	.... 6 0 0	„	*Six Hundred.*
D.	.... 6, 0 0 0	„	*Six Thousand.*
E.	.... 6 0, 0 0 0	„	*Sixty Thousand.*
F.	.... 6 0 0, 0 0 0	„	*Six Hundred Thousand.*
G.	.... 6, 0 0 0, 0 0 0	„	*Six Millions.*
H.	.... 6 0, 0 0 0, 0 0 0	„	*Sixty Millions.*
I.	...... 6 0 0, 0 0 0, 0 0 0	„	*Six Hundred Millions.*
K.	.. 6, 0 0 0, 0 0 0, 0 0 0	„	*Six Thousand Millions.*
L.	6 0, 0 0 0, 0 0 0, 0 0 0	„	*Sixty Thousand Millions.*
M.	6 0 0, 0 0 0, 0 0 0, 0 0 0	„	*Six Hundred Thousand Millions.*

Units.
Tens.
Hundreds.
Thousands.
Tens of Thousands.
Hundreds of Thousands.
Millions.
Tens of Millions.
Hundreds of Millions.
Thousands of Millions.
Tens of Thousands of Millions.
Hundreds of Thousands of Millions.
&c.

6. Underneath the last line are written the names of all the values which the 6 has in all its different positions; and if these are learnt by the pupil, the value of any one figure which he may desire to know will be found immediately. Thus, if I have the number 500000, without the commas to help me to see *at once* what the 5 stands for, I shall begin with the right-hand figure, and proceed to the left, pointing to the figures one after another with the finger or pencil, and shall say, ***units, tens, hundreds, thousands***, &c. till I come to the 5, and I shall then find that I

have come to *hundreds of thousands;* hence the 5 in the above number stands for *five hundred thousands.*

7. We have now to show how all the intermediate numbers, between those which we have explained, are to be written both in figures and words.

In (1) we went up as far as Ten or 10; and then went to Twenty or 20; the intermediate numbers are

In words.	In figures.
Eleven	11
Twelve	12
Thirteen	13
Fourteen	14
Fifteen	15
Sixteen	16
Seventeen	17
Eighteen	18
Nineteen	19
Twenty	20

Take now any one of these numbers, as 16, and it will be observed that it is composed of ten and six; that is, the 1 stands for *ten,* as in the figures 10, and the 6 is added to it; so 17 means ten and seven together; 19 means ten and nine together; and so on.

In like manner we can fill up all the places between 20 and 30;

Thus,	24	stands for	20 and 4,	or *Twenty-four.*
	27	"	20 and 7,	or *Twenty-seven.*
so also,	32	"	30 and 2,	or *Thirty-two.*
	45	"	40 and 5,	or *Forty-five.*
	49	"	40 and 9,	or *Forty-nine.*

And the same method is used for expressing the numbers between 50 and 60, between 60 and 70, &c.

Exs. 5. Write in figures, Fourteen, Twenty-two, Sixty-one, Seventy-Seven, Eighty-five, Ninety-three, Forty-eight, Fifty-one, Thirty-four, Ninety-one, Nineteen.

Write in words, 28, 37, 45, 89, 73, 64, 58, 17, 21, 88, 33, 99.

8. Proceeding beyond 100, we fill up all the numbers between 100 and 200, by writing all the numbers from 1 up to 99, instead of one or both of the ciphers in 100.

Thus, if I have to write *one hundred* and *fifty-seven*, I first put down 1 for the *one hundred*, and then, instead of the two ciphers, I write *fifty-seven* in figures, that is, 57; so that it becomes 157. Similarly, *one hundred* and *seventy-nine* is written 179; and *one hundred* and *eight* is 108, where I see that the number eight, or 8, takes the place of only the *latter* of the two ciphers; but in one hundred and fifty, or 150, the 50, being 5 tens, stands as a 5 in the tens' place, and the latter cipher is unaltered.

The numbers between 200 and 300, between 300 and 400, &c., are filled up in exactly the same way; so that if a pupil has learnt thoroughly what has been written above, he will see at once that

375	stands for	*Three hundred and seventy-five.*
401	„	*Four hundred and one.*
517	„	*Five hundred and seventeen.*
833	„	*Eight hundred and thirty-three.*
760	„	*Seven hundred and sixty.*
999	„	*Nine hundred and ninety-nine.*

Having now learnt how to write in words and in figures all numbers from 1 to 1,000, the pupil will easily see that all the numbers which he has just been learning will fill up the space from 1,000 to 2,000, from 2,000 to 3,000, &c.

so that	1324	stands for	*One thousand three hundred and twenty-four.*
	2075	„	*Two thousand no hundreds and seventy-five,* or *Two thousand and seventy-five.*
	3104	„	*Three thousand one hundred and four.*

Exs. 6. Write in figures, Five Hundred and Seven, Eighteen Hundred and Nineteen, Four Thousand and Seventeen, Nine Hundred and Twenty, Eight Hundred and Eleven, Nine Thousand Four Hundred and Sixteen, Two Thousand and Four.

Write in words, 302, 427, 109, 704, 1845, 1900, 7324, 8907 1401, 3870, 1044, 9009, 10,000.

9. I need not describe at length the mode of writing all the numbers from 10,000 to 20,000, 30,000, &c.; or from 100,000 to 1,000,000, &c. If the pupil will bear in mind what was said in (3) and (4), that all millions have *six* figures after them, and all thousands *three* figures after, he will find little difficulty in writing down any proposed number. Also, when learning to write down long numbers, he should place nine dots, as in the margin, to guide him where to put the three different terms *millions*, *thousands*, and numbers below 1000.

Mill^ns	Thous^s	
...	...	...
345,	126,	327.

Thus, if I have to write down 345 millions, 126 thousands, and 327, I must place the 345 under the first three dots, the 126 under the next three, and the 327 under the last.

I see here that all the 9 places are filled up by the figures which I had to put down; but if the numbers given had been such as not to occupy all the places, then the empty places between the first and last figures must be filled with ciphers. For instance, if a beginner has to write 25 millions, 7 thousand, and 29, he should ... ...
first write as follows: . . . . . 25, 7, 29.
and then filling up the empty places, he
will have . . . . 25, 007, 029.
And he will now see the use of the ciphers, namely, to keep all the other figures in the places which we have shewn to be the proper ones.

Obs.—I do not place a 0 under the last dot to the left, because there are no figures to the left of it, to be kept in their proper places.

Exs. 7. Write down in figures, Fifty Thousand and Six, One Hundred and Seventy Thousand and Eighteen, Four Hundred and Five Millions Thirty-nine Thousand and One, Three Thousand and Five Millions Six Hundred and Nine, One Million and Forty-three.

Write down in words, 700,401, 1,400,906, 40,010, 80,040,017, 400,401,090, 36,011, 3,245,068,018. *

ADDITION.

10. To ADD is to collect together two or more numbers into one sum.

In order to add together any numbers, we must collect, separately, all those numbers which are of the same kind; thus, units must be added to units, tens to tens, hundreds to hundreds, and so on.

When, therefore, we have to find the sum of any numbers, we must so place them under one another, that all the figures in the units' places may be in an upright row; then the tens, hundreds, thousands, &c., if there be any, will also be arranged in upright rows.

11. To help the learner to add together any two numbers, each less than 10, I insert the following ADDITION TABLE.

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

* The figures which we have been describing are *Arabic* Numerals.

The Roman Numerals were I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, M for 1000.

Any one of these placed to the *right* of a larger numeral was to be *added* to it; thus VI represented V and I or 6; XII stood for 12; and CLXX for 170.

But a numeral placed to the *left* of a larger numeral was to be *subtracted* from it; thus IV stood for 4; XL for 40.

Sometimes, also, IↃ was written for 500, and CIↃ for 1000.

It may be learnt or read thus. Place a finger of the left-hand on any one of the figures in the left-hand row; and as you move your finger to the right, add to this figure the numbers in the highest row one after another. The amounts will be under the finger at every step. Thus, pointing to 4 with the left-hand, and to the 1, 2, 3, &c., of the top line with the right-hand, and then moving both hands to the right, we read 4 and 1 make 5, 4 and 2 make 6, 4 and 3 make 7; and so on, to the end of that line; and I see that the amounts 5, 6, 7, &c., are exactly under the 1, 2, 3, &c., of the top line, and in the same row as the left-hand 4.

If now I wish to find from this table the sum of any two numbers, each less than 10, as 7 and 8, I point with the left-hand to the 7, in the left-hand row, and with the right-hand to the 8 in the top row; then moving the right-hand straight down till it comes opposite the left-hand, I find I come to the number 15. This 15 is the *sum* of the numbers 7 and 8.

12. We have seen in NUMERATION, that the figure 1 in the second place from the right-hand counts as 10; therefore, to add 10 to any number under 10, we have only to place the figure 1 to the left of that number; thus, if I have to add 10 to 7, I merely write 17. So also, to add 10 to a number greater than 10, as to 46, I add 1 to the figure in the *tens'* place, so that 46 and 10 make 56. By the same method 235 and 10 make 245; 1427 and 10 make 1437. In like manner, to add 100, I increase the figure in the *hundreds'* place by 1, so that 245 and 100 make 345; 3156 and 100 make 3256.

We shall now have no difficulty in adding together two numbers, one of which is more than 10, and the other less

than 10, as 16 and 3. For 16 is the same as 10 and 6. I therefore add the 3 to the 6, making 9; and keeping the 1 in the units' place, make 19 in all.

$$\begin{array}{r}16\\ \underline{3}\\ 19\end{array}$$

So also, to add 16 and 7, I first add the 7 to the 6, making 13, and write down the 3; and for the 10 in this number 13, I put 1 more in the tens' place, and make in all 23.

$$\begin{array}{r}16\\ \underline{7}\\ 23\end{array}$$

In like manner, if I have to add 28 and 7, I shall first add the 7 to the 8, making 15, and put the 5 in the units' place; and for the 10 in this 15, I add 1 to the 2 in the tens' place, making 35.

$$\begin{array}{r}28\\ \underline{7}\\ 35\end{array}$$

And so I might add any two such numbers, as 36 and 7, making 43; 46 and 7, making 53; 146 and 7, making 153; 248 and 8, making 256.

Exs. 8. Write down the sums of 16 and 10, 26 and 10, 13 and 5, 18 and 9, 26 and 7, 45 and 8, 78 and 4, 89 and 1, 101 and 8, 104 and 7, 156 and 8, 345 and 9, 426 and 7.

13. Now let it be required to find the sum of 2548, 8027, 9, and 765.

Arranging the numbers according to the directions given above, so that all the figures in the units' places may be in an upright row, they stand thus:

$$\begin{array}{r}2548\\ 8027\\ 9\\ \underline{765}\\ 11349\end{array}$$

Beginning at the right-hand or units' row, and adding, I say;—5 and 9 are 14, 14 and 7 are 21, 21 and 8 are 29, that is, 2 tens and 9 units; I place the 9 units under the row of units, and carry the 2 tens to the next row, which consists of tens. I now add this second row, just as I did the first; it amounts to 12 tens, and with the two tens carried, it is 14 tens, (or 140,) that is, 1 hundred and 4 tens; I place the 4 under the row of tens, and carry the 1 hundred to the next row, which

consists of hundreds. The third row, with the 1 carried, amounts to 13 hundreds, (or (1,300), that is, 1 thousand and 3 hundreds; I put down the 3 under the row of hundreds, and carry the 1 thousand to the fourth row: this amounts to 11 thousands; and as there are no more rows to add, I put down the whole of the 11. The complete answer is, Eleven thousand three hundred and forty-nine (11349). And so we might have added any number of rows. Hence we have the following Rule for SIMPLE ADDITION:

RULE. Arrange the numbers to be added under one another, so that the figures in the units' places may be in an upright row, and draw a line under the whole.

Add up the first row to the right; and of the amount just found place the right-hand or units' figure under this row, and carry the other figure, or figures, if there be any, to the second row. Find the sum of this second row, adding in the figure carried from the first row; put down the right-hand figure, and carry the remaining ones as before. Proceed in like manner to the last row; and when its sum is found, place the whole of it to the left of the figures already set down.

OBS. The operation of Addition is sometimes denoted by the sign $(+)$ *plus*, which, placed between two numbers, shows that they are to be added together. Also, the sign $(=)$ *equal to*, placed betweed any two quantities, signifies that they are equal. Thus $4 + 5 = 9$, is read 4 *plus* 5 *equals* 9, or 4 added to 5 makes 9. So also, $4 + 5 + 3 + 7 = 19$; or the *sum* of 4, 5, 3, and 7, is 19.

EXS. 9. Find the value of

1. $643 + 879 + 245 + 134 + 851 + 405 + 137$.
2. $145 + 672 + 834 + 1000 + 987 + 451 + 1687$.

3. 1144 + 328 + 456 + 987 + 101 + 376 + 451.
4. 4176 + 3459 + 1111 + 4321 + 6897 + 14562.
5. 8976 + 3178 + 9017 + 5632 + 14587 + 8765.
6. 4301 + 9872 + 4632 + 1829 + 5437 + 684 + 9187.
7. 8743 + 1986 + 4530 + 12875 + 1493 + 6421.
8. 1879 + 8431 + 9645 + 4287 + 1673 + 7482.
9. 8735 + 6419 + 4327 + 9180 + 14327 + 6875.
10. 3456 + 875 + 10010 + 7435 + 8962 + 67.
11. 8432 + 98765 + 20101 + 3476 + 823 + 10479.
12. 658 + 1034 + 98710 + 3279 + 14826 + 4113.
13. 84291 + 103456 + 8732 + 91011 + 178452 + 74379.
14. 110456 + 83019 + 75146 + 238108 + 14679 + 8001.
15. 148765 + 75832 + 64101 + 75 + 80015 + 9873.
16. 143856 + 28739 + 41032 + 999 + 91458 + 100000 + 87532.
17. 457890 + 32576 + 987542 + 17639 + 4010 + 101010 + 67348.
18. 111111 + 43210 + 87641 + 12479 + 71 + 375 + 118934.
19. 843217 + 641839 + 4813756 + 5204187 + 28756 + 4315821 + 867509.
20. 43201 + 384917 + 1007 + 138589 + 710234 + 86549 + 684305.
21. 148302 + 7649 + 45831 + 776789 + 145384 + 784591 + 638414.
22. 487643 + 1897421 + 684579 + 4458321 + 9237548 + 8459320.
23. 876410 + 1984321 + 765899 + 4101873 + 600750 + 9999.
24. 4865320 + 6421987 + 468359 + 146721 + 1098638 + 1016.
25. 687532 + 4591408 + 7432017 + 5689432 + 874310 + 189 + 6345681.
26. 875439 + 8674392 + 5108469 + 83167 + 410509 + 1673245 + 587410.
27. 5681094 + 6138765 + 494587 + 238654 + 1647829 + 30178 + 472.
28. 784586 + 87532 + 1049761 + 853742 + 945681 + 410392 + 7684910.
29. 487653 + 768923 + 9898989 + 445671 + 830187 + 941245 + 7631408.
30. 456786 + 2389124 + 837540 + 975218 + 111456 + 832075 + 1410987.
31. 8740192 + 1432786 + 910437 + 64985 + 87563 + 11468492 + 753864.
32. 4132875 + 6894360 + 5119387 + 7256918 + 14071728 + 1271449.
33. 489735 + 2010387 + 5897654 + 319771 + 864987 + 410684 + 34586.
34. 8764591 + 143286 + 375 + 914268 + 4372145 + 896451 + 98817532.
35. 348765 + 1487659 + 2018437 + 498765 + 14329 + 87642 + 1073496.
36. 4368291 + 78543 + 689201 + 45763 + 75 + 101789 + 2345682.

SIMPLE SUBTRACTION.

14. To Subtract a less number from a greater is to find how many more units there are in the greater number than in the less; or, to find what number added to the less will make the greater. This number is called the *difference.*

First, I must learn to find the difference between two numbers, whereof one is less than 10, and the other does not exceed it by 10; as for instance between the numbers 9 and 15. For this purpose the Addition Table may be used as a Subtraction Table; thus:—I find the smaller of the two numbers in the left-hand row, and move my finger to the right, till I come to the larger one; I then look to the top of the upright row in which my finger is, and there I find the difference. For instance, taking the numbers 9 and 15, I look for 9 in the left-hand row, and move to the right till I come to 15: at the top of the upright row that my finger now touches, I find 6: this is the difference between the 9 and 15.

15. I now can find the difference between any two numbers. Thus, let it be required to find the difference between 43854 and 2628.

Placing the numbers so that the units' figures shall be nnder each other, I commence as follows; 8 from 4,—it cannot be taken; I therefore observe that the 54 in the top line is the same as 40 + 14, or 4 tens and 14; and I subtract the 8 from this 14, finding a remainder 6,

$$\begin{array}{r} 43854 \\ 2628 \\ \hline 41226 \\ \hline \end{array}$$

which I place underneath. I now remember that I have no longer 50 in the upper line, but 40; that is, the figure in the tens' place must not be counted as 5, but as 4. I therefore take the 2 tens in the lower line from the 4 tens, and have 2 tens as the difference, which I put down. So, also, 6 hundreds from 8 hundreds gives 2 hundreds; 2 thousands from 3 thousands gives 1 thousand: and since there are no tens of thousands in the lower line to be taken from the 4 tens of thousands in the upper line, the 4 must be brought down as it is. The whole remainder is 4 tens of thousands, 1 thousand, 2 hundreds, 2 tens, and 6, or forty-one thousand two hundred and twenty-six. (41226.)

16. In performing the first subtraction of the above example, because I could not take the 8 from the 4 in the units' place, I added 10 to the 4, or, as it is often called, I *borrowed* 10, and counted the figure 5 in the tens' place, as 4, and then subtracted the 2 below it from the 4; but the common way, after having added this 10 in any subtraction, is to leave the figure in the top line of the next place unaltered, but add 1 to the figure under it, and then subtract. Thus, I add 1 to the 2 in the lower line, and then say, 3 from 5, which gives me the difference 2, just as before.

Hence, when it is required to find the difference of two numbers, we have this

RULE. Place the less number under the greater, so that the figures in the units' places may be under one another. Begin at the right-hand; and, if possible, subtract the lower figure from the upper, placing the difference underneath; but if the lower figure is greater than the upper, add 10 to the upper, and then subtract.

Proceed in the same manner with each pair of figures;

but remember that when in any subtraction you have added 10 to the upper figure, you must, in performing the *next* subtraction, add 1 to the figure in the *lower* line. If there remain any figures in the upper line, from which none are to be subtracted, bring them down in their proper order.

Obs. The operation of Subtraction may also be expressed by the sign (–) *minus*, which, placed between two quantities, shows that the latter is to be taken from the former. Thus, when I wish to say that if 2 be taken from 7, the remainder is 5, I write $7 - 2 = 5$, which is read, 7 minus 2 equals 5; or, the *difference* between 7 and 2 is 5.

Exs. 10. Find the value of

1. 45863 – 21051.
2. 68745 – 61032.
3. 58900 – 48799.
4. 456789 – 123456.
5. 785432 – 694310.
6. 689321 – 278543.
7. 5894101 – 785326.
8. 8497103 – 1456897.
9. 4132987 – 875619.
10. 8976410 – 2301019.
11. 9832145 – 4567891.
12. 7640189 – 178923.
13. 1432897 – 567898.
14. 3456890 – 126901.
15. 2117584 – 456897.
16. 18432745 – 1857106.
17. 68935842 – 1065719.
18. 78568932 – 6458109.
19. 385879276 – 7750987.
20. 107964201 – 6423189.
21. 351068743 – 49969836.
22. 784500000 – 6891019.
23. 89760001 – 4321890.
24. 321047607 – 76498321.
25. 987654321 – 123456789.
26. 64208882 – 7134284.
27. 1111111111 – 123456789.
28. 9684766554 – 807060504.
29. 483718088 – 403090871.
30. 9308615607 – 930830730.
31. 7386543210 – 4567898765.
32. 8762748898 – 796534289.

QUESTIONS INVOLVING ADDITION AND SUBTRACTION.

Exs. 11.

1. From the sum of 3276 and 189, take the sum of 375 and 456.
2. From the sum of 14095 and 8376, take the difference between 427 and 999.
3. From the sum of 98763, 3275, and 144, take the difference between 10000 and 1001.
4. To the number 10001 add 8399; from the sum subtract 4376; then add 1879; then subtract both 327 and 185; what will remain?

5. Out of 200 marbles, a boy gives away 20, 30, 40, and 50 to four other boys; how many has he left?
6. If the upper line in a subtraction sum be 3456, and the remainder be 749, what is the lower line?
7. If the lower line be 4896, and the remainder be 960, what is the upper line?
8. From one million I take ninety-nine thousand and nine; to the remainder I add seven thousand and fifteen; what will be the amount?
9. Write down in figures and signs; three hundred and seven, added to one thousand and one, is equal to the difference between ninety-two, and one thousand four hundred.
10. Write down in figures and signs; seventy, plus 18, added to five thousand and four, diminished by seven hundred and nine, amounts to four thousand three hundred and eighty-three.
11. Write in words, $1000 - 457 + 193 - 75 = 661$.
12. Write in words, $3045 + 6208 = 10001 - 748$.
13. The flood took place 2348 years B.C.; how many years from that date to the year 1851 A.D.?
14. How many years from the first year of the fifth century to 1850?

SIMPLE MULTIPLICATION.

17. To Multiply one number by another is to see what the first number amounts to, when repeated as many times as there are units in the second number.

Thus, to multiply 7 by 5 is to repeat the number 7 *five* times, that is, it is $7 + 7 + 7 + 7 + 7$, which by addition we find to be 35; and therefore we say that 5 times 7, or 7 multiplied by 5, equals 35. This result, 35, is called the *product* of 7 and 5; also 7 is called the *multiplicand*, and 5 the *multiplier*.

In like manner, by performing successive additions, we might find the product of any two numbers; but it is not convenient to obtain these results by addition, any further than to find the sum of 12 repeated 12 times, or the pro-

duct of 12 by 12. A table which contains in order the products of any two numbers not greater than 12, is called the MULTIPLICATION TABLE, and is as follows:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The above is thus formed: the first row to the left contains the numbers 1 to 12. Taking the second row with 2 at the top, and going downwards, we find that the row is formed by adding 2 at every step; thus, 2 and 2, or twice 2, are 4; 4 and 2, or 3 times 2, are 6; 6 and 2, or 4 times 2, are 8; and so on, till we come to 12 times 2, or 24. In like manner the 7th row is found by beginning with 7, and adding 7 successively, which gives the row 7, 14, 21, 28, &c. . . . . . . 84. And so for all the other rows.

And if I wish to find the product of any two numbers, each not exceeding 12, as 7 and 8, I point with the left-

hand to the one number, 7, in the left-hand row, and with the right-hand to the other, 8, in the top row, as in the Addition Table; then moving the right-hand straight down till it comes opposite to the left-hand, I find that I come to the number 56; this is the *product* of the numbers 7 and 8.

18. Now let it be required to multiply any number, as 527, by any number not greater than 12, for instance, by 9.

Since $527 = 500 + 20 + 7$, therefore, if I multiply each of these three quantities by 9, the sum of all the products will be the whole product of 527 by 9. I have, then, as the first product, 9 times 7, or 63; as the second, 9 times 20, or 9 times 2 tens, which is 18 tens, or 180; as the third, 9 times 5 hundreds, which gives 45 hundreds or 4500: and, as shown in the margin, the sum of these three products is 4743.

$$\begin{array}{rl} 527 & \\ 9 & \\ \hline 63 & = 9 \text{ times } 7 \\ 180 & = 9 \quad ,, \quad 20 \\ 4500 & = 9 \quad ,, \quad 500 \\ \hline 4743 & = 9 \text{ times } 527 \end{array}$$

19. But in practice we perform this work in one line, and add the products as we go on, thus; 9 times $7 = 63$; put down the 3 units, and carry 6 tens; 9 times 2 tens = 18 tens, and with the 6 carried, is 24 tens, or 2 hundreds and 4 tens; put down the 4 tens, and carry the 2 hundreds; 9 times 5 hundreds = 45 hundreds, and with the 2 hundreds carried = 47 hundreds, or 4 thousands 7 hundreds; put down both these figures, and the result is 4743, as before.

$$\begin{array}{r} 527 \\ 9 \\ \hline 4743 \end{array} \qquad (A)$$

Exs. 12. Form the following products.

I. 3456789 by 2, 3, 4, 5, 6, 7, successively.
II. 6410954 by 3, 4, 5, 6, 7, 8.
III. 1875697 by 4, 5, 6, 7, 8, 9.
IV. 6183249 by 7, 8, 9.

We have already seen in Numeration, that if a cipher be placed at the right-hand of a number, every figure in that number has ten times the value that it had before; that is, to multiply a number by 10, I need only place a cipher after it; so, also, to multiply by 100, I place two ciphers; by 1000, three ciphers; and so on.

20. Again, since to multiply the above number, 527, by 10, I merely add a cipher, making 5270; therefore, to multiply it by 20, which is 2 tens, I multiply by 10 and by 2, that is I add one cipher, and then multiply by 2, making 10540. In like manner to multiply by 40, I add a cipher, and multiply by 4; by 60, I add a cipher, and multiply by 6.

So, also, to multiply by 200, I add two ciphers, and then multiply by 2; by 700, add two ciphers, and multiply by 7; by 5000, add three ciphers, and multiply by 5.

The following are examples of the manner in which such multiplications are performed.

$$\begin{array}{r} 3247 \\ 60 \\ \hline 194820 \\ \hline\hline \end{array} \qquad \begin{array}{r} 8295 \\ 700 \\ \hline 5806500 \\ \hline\hline \end{array} \quad \text{(B)}$$

Exs. 13.

I. Multiply 3456789 by 10, 100, 10000.
II. „ 785632 by 20, 30, 600, 8000.
III. „ 4568301 by 9000, 80, 600, 70000.

21. Let us now see the effect of a cipher in the middle of a multiplier. For example, if I have to multiply 48295 by 1703.

Here, multiplying by the 3, according to (A), I obtain the first line of the product; next I multiply by the 700,

according to (B); next by 1000, by merely adding 3 ciphers to the multiplicand, 48295. By adding these three products, I obtain the whole product: but the work is

	48295			48295	
	1703			1703	
	144885	= prod^t by	3	144885	
(C)	33806500	= ,,	700	3380650	(D)
	48295000	= ,,	1000	48295	
	82246385	= ,,	1703	82246385	

usually written as in (D), where all the ciphers are omitted in writing out the products, except the single one in the second line, which came from our having a cipher in the second place of the multiplier.

In working any sum, then, it will be seen that it is not necessary to write down all the ciphers as in (C), but only to place each succeeding row one place farther to the left than the preceding one; but if there be a cipher in the multiplier, it is to be written down before multiplying by the next figure, and the row next below will be thereby thrown one place more to the left.

Hence, when two numbers are given to be multiplied together, we have this

Rule. Place the smaller number beneath the larger, so that the figures in the units' places may be under one another.

Begin with the right-hand figure in the multiplier, and multiply the units in the multiplicand; put down the right-hand figure of this product, and carry the tens, if there be any. Proceed in like manner through the first row, up to the last figure in the multiplicand, when the whole product must be put down.

Multiply in the same way by each figure in the multiplier, placing the first or right-hand figure of each row under the

second figure of the former row; but if there be a cipher in the multiplier, place it under the second figure of the row last formed, and multiply by the next figure in the multiplier, as usual; and in the succeeding multiplication, if there be one, place the row *two* places to the left. Add together all the rows; the result will be the product of the two numbers.

NOTE. The operation of Multiplication may also be expressed by the sign ($\times$), which, placed between two numbers, shews that they are to be multiplied together; thus, $7 \times 5 = 35$, which is read, 7 multiplied by 5 equals 35; or, 7 *into* 5 equals 35.

Exs. 14. Express the following words in signs and figures.

1. Seven multiplied by four, plus three, minus eight, is equal to twenty-three.
2. Nineteen added to three hundred and twenty-five, is equal to forty-three, multiplied by eight.
3. The sum of forty-five and one hundred and nineteen, is equal to the product of four and forty-one.
4. The difference between one thousand, and three hundred and fifty-five, is equal to the product of forty-three and fifteen.

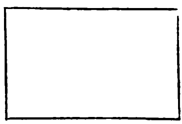
Exs. 15. Form the following products.

I. 6183249 by 11, 12, 13, 14, 15, 16.
II. 8375426 by 124, 347, 645, 809.
III. 45789213 by 653, 842, 976, 1048.
IV. 387940128 by 3456, 7894, 6781, 8592.

1. 41[illegible]952 $\times$ 4568.
2. 168942189 $\times$ 13076.
3. 87401329 $\times$ 16849.
4. 53298476 $\times$ 14370.
5. 764109836 $\times$ 4756.
6. 514276439 $\times$ 28309.
7. 8750146081 $\times$ 76112.
8. 458964107 $\times$ 13847.
9. 743286491 $\times$ 68940.
10. 784923607 $\times$ 42816.
11. 814906735 $\times$ 87492.
12. 436892198 $\times$ 57194.
13. 764381479 $\times$ 60879.
14. 9850124657 $\times$ 134806.
15. 56401389617 $\times$ 48907.
16. 8976432518 $\times$ 23874.
17. 1468529645 $\times$ 46879.
18. 3874598460 $\times$ 85423.

22. If I now turn to the Multiplication Table, I find that the number 144 at the right-hand lower corner tells me the number of small squares in the whole square, that is, in a square which has 12 equal parts in each side; so that, if each of these parts measures one inch, the whole square will contain 144 squares, each one inch both in length and breadth, or 144 square inches. I thus learn that the number of square inches in the whole square is found by multiplying the number of inches in the length by the number in the breadth.

And if I try this rule on any portion of this square, so that it be enclosed by 4 lines and shaped as in the figure, called an oblong, I shall find it true. For if I take a piece 7 inches long, and 5 broad, I shall find that there are in it 7 times 5 squares, or 35 square inches; and this number 35 is at the right-hand lower corner of the oblong. If I take a piece 9 inches long, and 5 broad, I shall find 45 squares; so that I can now tell at once how many square inches or feet are contained in any square or oblong, if I know how many inches, or feet, it is in length and breadth; for I have only to multiply the length by the breadth.



If also in each of these small squares a tree were placed, I could tell the number of trees in an oblong clump, by counting the number in length, and the number in breadth, and finding their product.

Exs. 16.

1. A board is 17 inches long, and 10 inches broad, how many squares, an inch each way, are there in its surface?
2. A box is 9 inches long, and 7 inches broad, how many square inches are there in the top and bottom?

3. If the above box be 6 in. high, how many square inches in the sides?
4. In a surface, divided like a chess-board, there are 15 divisions on each side, how many squares in the board?
5. In an oblong plantation, where the trees are planted in regular rows, there are 175 in one side, and 150 in the next side; how many trees are there in all?

SIMPLE DIVISION.

23. To divide one number by another, is to find how often the second nnmber may be subtracted from the first; or, what number multiplied by the second will produce the first. Thus, if it be required to divide 35 by 5, I subtract 5 from 35, till there be either nothing left, or till the number left be less than 5. I find I can subtract the 5 just 7 times, and therefore I say that 35 contains 7 fives, or that, when divided by 5, it gives the answer 7. Here 35 is called the *dividend*, 5 the *divisor*, and 7 the *quotient*.

Also, the operation of Division is represented by the sign ($\div$). Thus, 35, $\div$ 7 $=$ 5, is read, 35 divided by 7 equals 5; or, the quotient of 35 when divided by 7 is 5.

24. When the number to be divided is not greater than 144, and the divisor not greater than 12, the Multiplication Table may be used as a Division Table. Thus, since I know that if 5 and 8 were to be multiplied together, the result would be the number in the table where the two rows from 5 and 8 meet, viz. 40; therefore, if I make 5 the divisor, and 40 the dividend, then the number 8 standing over the 40 will be the quotient. So, also, if 8 were the divisor, and 72 the dividend, the number 9 at the top of the row over 72 would be the quotient.

But we shall not find all the numbers between 1 and 144 in this Multiplication Table; for instance, if I wish to

divide 43 by 5, and I look in the row beginning with 5, I run my finger along the row till I come to the *next number below* 43, that is, 40; and since I find that 8 is at the top of the row, therefore 8 is the quotient; but there are 3 out of the 43 which are not divided; and this work of division which I have just been performing would be thus expressed,—5 in 43 goes 8 times, and 3 over.*

25. But if the dividend consist of four or five places of figures, as 3276, and the divisor be still under 12, as 9, we cannot see at once how many times 9 will produce 3276, but must take several steps to find the result.

Now the 32 in 3276 means 32 *hundreds;* since, then, we know how often 9 is contained in 32, viz. 3 times and 5 over, we know how many times it is contained in 32 *hundreds,* viz. 3 *hundred* times, and 5 *hundred* over. Take 300 times 9, or 2700 from 3276, and we have 576 over, which has not yet been divided. Again, since 570 is 57 *tens,* and 9 in 57 goes 6 times and 3 over, therefore 9 in 57 tens goes 6 tens, or 60 times, and 3 tens, or 30, over. Subtract 60 times 9 from 576, and we have 36 over; and we see that 9 in 36 goes 4 times exactly: therefore the number 3276 has been entirely divided by 9; and the quotient is 300, and 60, and 4, or 364.

$$\begin{array}{rrl}
 & 9)\ 3276 & \\
 & \underline{2700} & = 300 \text{ times } 9 \\
 & 576 & \\
 & \underline{540} & = \ \ 60 \text{ times } 9 \\
 & 36 & \\
 & \underline{36} & = \ \ \ \ 4 \text{ times } 9 \\
\text{or,} & 3276 & = \underline{364 \text{ times } 9}
\end{array} \qquad \text{(E)}$$

The above may be so arranged as to form a very simple sum, thus:

$$\begin{array}{r}
9)\ \underline{3276} \\
\underline{364}
\end{array}$$

And the manner of per-

* The pupil will afterwards be shown that if 43 be divided by 5 it is correct to say, that the quotient is eight and *three-fifths,* or, as it is written $8\frac{3}{5}$. See Art *Fractional Quotient,* in Appendix to Part II. of the Arithmetic.

forming the operation is to say, 9 in 32 goes 3 times and 5 over; put down the 3 under the 2, and take the 5 to the 7, the next figure in the dividend, calling it 57; 9 in 57 goes 6 times and 3 over; put down the 6 under the 7, and take the remaining 3 to the 6, calling it 36; 9 in 36 gives a quotient 4, which put under the 6. Hence, for dividing by a number not greater than 12, we have this

Rule. Place the divisor to the left of the dividend, separating them by a curved line. Take as many figures of the dividend as are necessary to make the number taken at least equal to the divisor: see how often the divisor is contained in this number, and place the quotient under the last of the figures so taken: if there be any remainder, annex it to the next figure of the dividend, and divide as before; but if the remainder and the figure so taken be less than the divisor, place a cipher as quotient, and take another figure and divide. But if there be no remainder, divide the next figure alone, if possible; and if not, put a cipher under it, and take two or more figures if necessary. Proceed in this manner till all the figures in the dividend are taken. If there be any final remainder, place it a little to the right of the quotient.

This method, as shown in the second form of the above example, is called Short Division. But as the sum was worked at first, where the subtractions after every division were performed on the paper, instead of in the mind, the process is called Long Division; and we shall have thus to work almost all examples, where the divisor is greater than 12; the only difference being, that the ciphers might have been omitted in the lines which are subtracted, as was shown in (D) in the multiplication sum in page 22.

Exs. 17. Divide

I. 3849628 by 2, 3, 4, 5, successively.
II. 89764320 by 6, 7, 8, 9.
III. 148763592 by 5, 7, 9, 8.
IV. 4587692015 by 8, 9, 11, 12.

26. The above Rule may also be used for dividing by numbers greater than 12, if they can be exactly split up into two other numbers, each less than 12; as for instance, by 21, which is 7 times 3; by 24, which is 6 times 4. Thus, if we have the number 2476 to be divided by 21, we shall divide, first by one of the numbers, as 7, and then by the other, 3. And to show that this process will bring a correct result, let us suppose we had 2476 apples to be divided into heaps of 21: if we divide 2476 by 7, we shall have the quotient 353, which gives the number of heaps of 7, and 5 single apples over; and since 3 heaps of 7 will make one heap of 21, therefore, if we now divide these 353 heaps of 7 by 3, we shall have a quotient consisting of heaps of 21; this is 117, and the rem^{r} 2 represents 2 heaps of 7, which is the same as 14 single apples, and with the former remainder, 5, gives 19 apples rem^{s} from the whole division.

$$21\left\{\begin{array}{l} 7)\ \underline{2476} \\ \ \ 3)\ \underline{353}\ -\ 5 \\ \qquad \underline{\underline{117}}\ -\ 2 \end{array}\right\}19\ \text{rem}^{r}$$

Here we see, that the number remaining from the second division, namely, 2, required to be multiplied by the first divisor, viz. 7, to give the *real* second rem^{r}, and then it was added to the former remainder. So that when I wish to obtain the complete remainder, 19, from the two partial remainders, 5 and 2, I shall say, $2 \times 7 + 5 = 19$, the true remainder. Therefore, in dividing as in Short Division, by a number which is formed by the multiplication of two numbers under 12, we have, for finding the remainder, this

RULE. Multiply the second remainder by the first divisor, and add in the first remainder. If there be no second remainder, the first remainder will be the true one.

Exs. 18. Divide

I. 37643291 by 14, 16, 18, 21.
II. 874013695 by 25, 36, 56, 81.
III. 172345698 by 72, 49, 121.

Exs. 19. Perform the following operations.

1. 289764235 ÷ 27.
2. 4568972501 ÷ 33.
3. 46895017846 ÷ 55.
4. 7592041863 ÷ 81.
5. 5697421078 ÷ 121.
6. 6498721417 ÷ 120.
7. 5689743209 ÷ 132.
8. 78964012385 ÷ 144.
9. 3456785231 ÷ 84.
10. 4680397642 ÷ 96.
11. 58742136847 ÷ 99.
12. 84397652108 ÷ 108.

27. But since most of the numbers that we have to use as divisors cannot be broken up exactly into two numbers each under 12, we must use the method of (E), or of Long Division, when dividing by any ordinary divisor larger than 12. I will give another Example.

Let it be required to divide 3487906 by 754.

Now as in Art. (25) we showed how to divide 3276 by 9 at several steps, so we must work in this example. Take the 3487 as being the smallest number of the dividend that can be divided by 754; and since there are 3 places after these four figures, we know that this 3487 signifies 3487 *thousands*, and therefore when divided by 754, the quotient will be so many *thousands*.

We have first to see how often 754 goes in 3487; this is nearly the same as seeing how often 7 hundred goes in 34 hundred, or 7 in 34; that is, we neglect the two right-hand figures and then divide. Here 7 in 34 goes 4 times and something over, therefore 700 in 3400 goes 4 times,

and something over: but we said that 3487 in this place meant 3487 thousands, therefore 754 in 3487 thousands goes 4 thousands and something over; put this 4000 in the quotient; and that we may see how much is over, subtract from the whole dividend, 4000 times 754, or 3016 thousands, and we have a rem^{r} 471906. Taking this rem^{r}

```
754) 3487906
     3016000 = 4000 times the divisor.
     -------
      471906
      452400 =  600      "
      ------
       19506
       15080 =   20      "
       -----
        4426
        3770 =    5      "
        ----
         656
        ====
that is, 3487906 - 656 = 4625 times the divisor.
```

as a new dividend, and beginning, as before, by dividing into the first four figures 4719, which are so many *hundreds*, we find that 754 in 4719 goes 600 times, and something over: as before, we subtract 600 times 754 from the new dividend, to see how much is over, and we find 19506. Again, dividing by 754, we find that it goes 20 times and something over, and our rem^{r} after subtracting is 4426: here 754 goes 5 times, and something over; and by subtracting again, we have the last rem^{r} 656; and our whole quotient is 4000 + 600 + 20 + 5, or 4625.

We have here performed the process at full length; but if we put down only those figures which are necessary in the operation, the sum will appear thus.

```
754) 3487906 (4625
     3016
     ----
      4719
      4524
      ----
       1950
       1508
       ----
        4426
        3770
        ----
         656
        ====
```

We here see that after each

subtraction it is not necessary to bring down to the rem^{r} more than one figure of the dividend, though it sometimes happens, as in Short Division, that the rem^{r} is so small, that even when one figure is brought down, the rem^{r} still is less than the divisor; in this case a cipher must be placed in the quotient, shewing that there has been no division performed, and another figure must be brought down, and the work proceeded with as before.

I insert one more Example, which will show that care must be taken when one figure is brought down, and yet no division performed.

Divide 1746549138 by 4587.

```
4587) 1746549138 (380760
      13761
      -----
       37044
       36696
       -----
         34891
         32109
         -----
          27823
          27522
          -----
            3018
            ====
```

Here, when, after the second subtraction, the 9 was brought down, the dividend 3489 was smaller than the divisor, and no division could be performed; I therefore placed a 0 in the quotient, and brought down another figure, and the divisor then could go into the dividend, 7 times. So, also, after the last subtraction, when 8 was brought down, the number 3018 was too small to be divided, and a 0 was placed in the quotient, and 3018 was left as a remainder.

It seems unnecessary, after going so fully through the above example, to state a separate rule for Long Division, since the only difference in working Short and Long Division is, that the subtractions necessary to find the rem^{rs}, after every division, are in *Long* Division performed on the paper, whereas in *Short* Division they are performed by the memory, and only the quotient is put down.

OBS. It is worth remarking, that in finding how often the divisor will go into the dividend or any of the rem[rs], we cannot always get a correct result by merely dividing the first figure of the divisor into the first, or first two figures of the dividend. Thus, in the first division of the example given above, though 4 in 17 goes 4 times, yet 4587 in 17465 will not go 4 times. Nothing but practice will enable a pupil to hit upon the correct number at once.

Also, after each subtraction it must especially be noticed whether the rem[r] is larger than the divisor; if so, the last figure in the quotient was too small, and a larger figure must be tried: or else either the last multiplication or subtraction was incorrect.

EXS. 20.

Find the required quotients in the following Examples.

1.	$35689742 \div 23$.	13.	$98754426817 \div 563$.
2.	$1468920357 \div 29$.	14.	$14576148903 \div 684$.
3.	$3689740125 \div 37$.	15.	$3875421986 \div 796$.
4.	$41562389075 \div 39$.	16.	$45862175647 \div 891$.
5.	$3875401267 \div 41$.	17.	$16894321678 \div 1531$.
6.	$7649801325 \div 43$.	18.	$3459806546 \div 4270$.
7.	$2607458913 \div 47$.	19.	$98423614578 \div 1094$.
8.	$146892750438 \div 53$.	20.	$57302648192 \div 4326$.
9.	$689432167 \div 153$.	21.	$451728954320 \div 5783$.
10.	$4987531681 \div 257$.	22.	$52198736929 \div 68754$.
11.	$1728956043 \div 345$.	23.	$10943268759 \div 43281$.
12.	$8653219547 \div 436$.	24.	$94544671038 \div 96329$.

28. There is one kind of divisors with which, though greater than 12, we can yet divide in one line, as 10, 20, 30, 40, 500, 800, 1000, &c.; that is, the numbers 1 to 12 followed by one or more ciphers.

For instance, to divide 37458 by 20. Performing the operation by Long Division, we have the work as (F): or

if we cut off the cipher in the 20, and the last figure in the dividend, namely 8, the work would stand as in (G).

```
20) 37458 (1872          2,0) 3745,8 (1872
    20                        2
    ---                       --
    174                       17
    160                       16
    ---                       --
     145      (F)              14      (G)
     140                       14
     ---                       --
       58                        5
       40                        4
       --                        -
       18                        1
       ==                        =
```

And if the 8 which was cut off be now brought down to the last rem^{r} 1, we shall have the same quotient and rem^{r} as in the former operation; hence this second operation is correct, and we may thus put the work of (G) in the form of Short Division,

```
2,0) 3745,8
     1872 „ 18 remʳ.
```

where, in performing this division, we use the last rem^{r} 1 as though it were 10, and adding the 8 which was cut off, make the whole rem^{r} 18. And this appears to be reasonable; for since we have used the divisor 20 as though it were 2, therefore a rem^{r} 1 is to be counted as 10.

In like manner, if I divided by 200, or 300, I should cut off *two* figures from both divisor and dividend, and divide by 2 or 3; if by 2000, or 3000, I should cut off *three* figures, and divide by 2 or 3. If by 1000, I have only to cut off three figures from the dividend for a remainder, and keep the figures not cut off as a quotient.

```
2,00) 387,59                1,000) 986,421
      193 „ 159 remʳ               986 „ 421 remʳ

            7,000) 5435,189
                   776 „ 3189 remʳ
```

Exs. 21. Divide

I. 38749102 by 10, 100, 1000.
II. 258749157 by 20, 30, 40, 500, 6000.
III. 78542963 by 900, 8000, 500,000.

It was shown in (22) that the product of the numbers representing the length and breadth of an oblong or square, in feet or inches, gave us the number of square feet or inches in the surface. So, also, if the number of square feet or inches in a surface, and the length of it, be known, the breadth will be found by dividing the surface by the length. Of course, if the breadth be given, the length can be found by dividing the surface by the breadth.

Hence we know the following facts.

Length $\times$ breadth = oblong surface.
Surface $\div$ length = breadth.
Surface $\div$ breadth = length.

Exs. 22.

1. If there are 120 square inches in a surface, and it be 12 inches long, how broad is it?
2. There are 3625 square inches in the surface of an oblong slab, and it is 25 inches broad, how long is it?
3. A box measures 76 inches round, and its sides contain 2812 square inches, what is its depth?
4. The same box is 20 inches long, how many square inches in the top and bottom?
5. A board is 15 inches broad, how long must it be that there may be 450 square inches in *both* its surfaces?
6. An oblong plantation contains 175 trees planted regularly one foot apart; if it contain 25 in length, how many in breadth? If 35 in length, how many in breadth?
7. How many feet round will the above two oblongs be?

Also, since

$$\text{Divisor} \times \text{quotient} + \text{remainder} = \text{dividend},$$

therefore, when any three of these quantities are known, the fourth one can be found.

Exs. of this kind will be found below.

Exs. 23.

MISCELLANEOUS QUESTIONS INVOLVING THE SIMPLE RULES.

1. What number subtracted from 35890101 will leave 67842?
2. Divide $175 + 368 + 459$ by $3685 - 3174$.
3. The product of two numbers is 387659; one of them is 6432; what is the other?
4. How much does 384501 amount to, when repeated 999 times?
5. There are two numbers, the greater of which is three times 3728, and the less is twice 1479, what is the difference?
6. The divisor is 35, the quotient 38975, and remainder 17, what is the dividend?
7. Twelve hundred and ten workmen receive among them in 3 months £18150; how much is that for each workman per month?
8. How many fifties are there in five hundred and ten millions?
9. The sum of two numbers is 38976, and one of them is 3459, what is the other?
10. The sum of two numbers is 45873, and the greater of them is 31267, what is their product?
11. What is the difference between the 11th and 12th parts of 42768?
12. If light travels at the rate of 192,000 miles per second; how far must the sun be from us, if his light is 490 seconds reaching us?
13. In a crew of 847 men, each man receives £3 a month; how much is paid to the whole crew in 12 months?
14. If 7848 marbles are divided equally among 72 boys, how many will each have?
15. A field in the form of a double square is 75 yards broad, how long is it, and how many square yards does it contain?
16. A wall is 120 bricks long, 17 high, and 3 thick, how many bricks does it contain?
17. In 27 bales of cloth, each containing 15 pieces, and each piece 15 yards, how many yards?
18. Shew that the product of 3846 and 705 is equal to the quotient of 51517170, when divided by 19.
19. Write the above fact in figures and signs.
20. The sum of 368979 and 335342 is equal to the sum of the products of 85 and 709, and of 11501 and 56. Give the value of these quantities, and write the expression in figures and signs.
21. A book contains 215 pages, 55 lines in a page, and 45 letters in a line; how many lines and letters in the book?
22. The 12th part of a number is 7563; what is the number itself?
23. A certain number when divided by 75 is 8907; what is the number?

24. The number 6742, when multiplied by a second number, becomes 5771152, what is the multiplier?
25. The dividend is 75992, the remainder 242, and the quotient 202 what is the divisor?
26. Express in signs these words, "the difference between the quotients of 37200 divided by 496, and of 45696 divided by 238, is 117."
27. If the quotient, divisor, and remainder be given, how do you find the dividend? Ex. Find the dividend, when the quotient is 345, the divisor 178, and remainder 27.
28. If the dividend be 4487234752, and the quotient 64064, what is the divisor?
29. By how much is the sum of 13459 and 756 greater than the sum of 1001 and 897?
30. How much greater is the product of 1894 and 325, than their sum?
31. From one million I take away 1000, and divide the remainder into 25 equal parts; how many in each part?
32. Fifty persons contribute 29 articles each; forty others give 27 each; and ten others 17 each; how many in all?
33. Express the result of $36 \div 4 \times 9 + 17$.
34. From three thousand and one take 299; multiply the remainder by 75 and 25 successively; what is the result?
35. Write out in words, $184 + 36 - 201 + 99 = 245 - 83 + 49 - 93$. What is the value of each of these equal quantities?
36. A book contains 275 pages of large type, with 35 lines in a page, and 45 letters in a line; and 97 pages of small type, with 55 lines in a page, and 67 letters in a line; how many lines and letters are contained in the book?
37. If the divisor be 375, the quotient 4685, and the dividend 1756976, find the remainder, without dividing.
38. The Creation took place 4004 B.C.; how many periods of 177 years, from that time to the end of the 60th year of the 17th century?
39. In a regiment consisting of 875 men, there are 5 officers to every 120 privates; how many officers in all? and how many privates to one officer?
40. Explain the method of dividing by a number which is formed by the multiplication of two numbers, each less than 12. Shew how to form the complete remainder.

OBSERVATIONS INTRODUCTORY TO

REDUCTION AND COMPOUND RULES.

29. We have so far been dealing only with the numbers described in Arts. (1) to (9); that is, with *whole* numbers; and the smallest number mentioned has been 1, or unity We now come to numbers which are either less than 1, or are between any two adjoining numbers, as between 2 and 3, 7 and 8, &c. Of this sort are the numbers which express the value of the familiar quantities, seven pence halfpenny, two yards and three quarters, &c. Such numbers are called *fractional.* But as a complete explanation of fractions would be generally found too difficult for pupils who have only just mastered the simple rules, we shall therefore give the names and meaning only of those fractional quantities which are most commonly used in Reduction and the Compound Rules. They are *one quarter; two quarters,* or *one half;* and *three quarters.* These common divisions may be thus explained.

Take a line ABCDE, one inch long, and divide it into two equal parts at C. Next, divide AC into two equal parts at B, and CE into two equal parts at D. The whole line AE will now have been divided into 4 equal parts, AB, BC, CD, DE, which are commonly called *quarters.* Also, if from A to B is *one* quarter, from A to C will be *two* quarters, from A to D will be *three* quarters, and from A to E will be *four* quarters, which make up the whole AE. The line AC which we see is *two* quarters, is generally called one-half.

Hence the three principal divisions to be remembered are

one quarter,	two quarters, or, one-half,	three quarters,

and they are thus written in figures:

$\frac{1}{4}$,	$\frac{2}{4}$, or $\frac{1}{2}$,	$\frac{3}{4}$.

Thus, $7\frac{1}{4}$ inches is read seven and a quarter inches, or seven inches and a quarter.

If, instead of dividing one inch, I had divided one penny, or any other single article, into 4 equal parts, I should have written the fractional parts in precisely the same manner. Also, a penny has been divided into 4 smaller coins, called farthings: and since these 4 coins are *quarters* of 1 penny, therefore the word *qrs.*, which is short for quarters, is often used to represent farthings.

Hence, since farthings are quarters of 1 penny, we shall have

1 far.	or *one* qr. of a penny		$= \frac{1}{4}$ of a penny,
2 far.	or *two* qrs.	„	$= \frac{2}{4}$, or $\frac{1}{2}$ of a penny,
3 far.	or *three* qrs.	„	$= \frac{3}{4}$ of a penny.

Instead of writing the words *of a penny*, as I have done, we write the letter *d.*;* thus $\frac{1}{4}$d. means one-fourth of a penny: and 7d. means 7 pence; so $7\frac{3}{4}$d. means 7 pence and 3 quarters of a penny, or 7 pence three farthings.

30. A quantity which consists of several kinds or denominations is called a *Compound* quantity. Thus, 25 pounds, 14 shillings, and 3 pence, is called a compound quantity; and it is written thus; £25 14s. 3d.; a space being placed between the pounds, shillings, and pence, that they may be kept distinct.

* This letter *d* is the first letter of the Latin word *denarius*, the Roman *penny*, as it is sometimes called; but the coin was in reality equal to about $7\frac{1}{2}$ pence of our money.

REDUCTION.

PART I.

31. Reduction is the changing of quantities which are expressed by numbers, from one or more denominations, to one or more others, so that the actual values of the quantities shall remain unaltered.

Before the method of performing these changes can be understood, it is necessary for the pupil to learn what are called the TABLES of Money, Weights, and Measures. We give one of the simplest for the sake of working examples with it.

2 farthings	= 1 halfpenny,	or $\frac{1}{2}$d.
2 halfpence or 4 farthings	= 1 penny,	or 1d.
12 pence	= 1 shilling,	or 1s.
20 shillings	= 1 pound sterling,	or £1.

32. As an Example of the process of reduction, let it be required to reduce £50 to shillings.

Now we know that £1 contains 20 shillings; therefore, for every pound in the £50 we must have 20 shillings; that is, we must have in all 20 times as many shillings as we have pounds, or, 20 times 50 pounds. If, then, we multiply the £50 by 20, we have as the product the number of shillings in £50, namely 1000. The work will be as above.

$$\begin{array}{r} \pounds \\ 50 \\ 20 \\ \hline 1000 \text{ shillings.} \\ \hline\hline \end{array}$$

Again, if it be required to reduce the £50 to pence;

then, since there are 12 pence in 1 shilling, there will be 12 times as many pence as there are shillings. We have already seen that there are 1000 shillings in £50. If, therefore, we multiply the 1000 shillings by 12, we shall have a product of 12000, which is the number of pence in 1000 shillings, or in £50.

$$\begin{array}{rl} \pounds & \\ 50 & \\ 20 & \\ \hline 1000 & \text{sh.} \\ 12 & \\ \hline 12000 & \text{d.} \\ \hline\hline \end{array}$$

And if we had further to reduce the £50 to farthings; then, since there are 4 farthings in 1 penny, if we multiply this 12000 pence by 4, we shall have 4 times as many farthings as pence, or 48000 farthings in the £50.

The whole work of the above example is represented in the margin; and it teaches how to reduce quantities of a *higher* name, as pounds, to quantities of a similar kind, but a *lower* name, as shillings, pence, and farthings.

$$\begin{array}{rl} \pounds & \\ 50 & \\ 20 & \\ \hline 1000 & \text{sh.} \\ 12 & \\ \hline 12000 & \text{d.} \\ 4 & \\ \hline 48000 & \text{far.} \\ \hline\hline \end{array}$$

33. In like manner, if we had to reduce any other quantity, as tons, to any lower denominations, as hundredweights, quarters, pounds, &c., we should multiply by the numbers given in the TABLES which join the successive denominations. Thus, in reducing tons to hundred weights, the multiplier is 20, because there are 20 cwts. in 1 ton; from hundred weights to quarters, the multiplier is 4, since there are 4 quarters in 1 cwt.; from quarters to lbs. it is 28, since there are 28 lbs. in 1 quarter; and so on.

Exs. 24.

1. Reduce £75 to pence.
2. Reduce £135 to farthings.
3. Reduce 520 guineas to pence.
4. Change 1075 moidores to pence.

5. How many farthings in a £10 note?
6. Reduce 15 cwt. to ounces.
7. Reduce 13 tons to drams.
8. How many drams (Apothecary) in 15 lbs.?
9. Reduce 17 lbs. Troy to pennyweights and grains.
10. Reduce 750 yards to nails.
11. How many seconds in five weeks?
12. Find the number of sheets of paper in 755 reams.

34. Sometimes the quantity given to be reduced is a compound quantity, as £25 13s. $6\frac{3}{4}$d., to be reduced to farthings.

Here we proceed to multiply by 20, 12, and 4, as in the Example just worked; but when we reduce to shillings, we add the 13s. into the line of shillings; so, in reducing to pence, we add the 6d. to the line of pence; and lastly, we add the 3 farthings into the line of farthings. The whole work will be most easily understood as in (E), but we commonly write it as in (F).

$$
\text{(E)}\quad
\begin{array}{l}
\begin{array}{lll} \text{£} & s. & d. \\ 25 & 13 & 6\frac{3}{4} \end{array} \\
\underline{\quad 20} \\
513 \text{ sh.} = \text{£}25\ 13\text{s.} \\
\underline{\quad 12} \\
6162 \text{ d.} = \text{£}25\ 13\text{s. } 6\text{d.} \\
\underline{\qquad 4} \\
\underline{\underline{24651}} \text{ f.} = \text{£}25\ 13\text{s. } 6\frac{3}{4}\text{d.}
\end{array}
\qquad\qquad
\begin{array}{l}
\begin{array}{lll} \text{£} & s. & d. \\ 25 & 13 & 6\frac{3}{4} \end{array} \\
\underline{\quad 20} \\
513 \text{ sh.} \\
\underline{\quad 12} \\
6162 \text{ d.} \\
\underline{\qquad 4} \\
\underline{\underline{24651}} \text{ f.}
\end{array}
\quad\text{(F)}
$$

And in like manner for any other compound quantity. Hence, for reducing quantities to a lower denomination, we have this

Rule. Multiply the highest denomination by the number given in the tables as connecting it with the next lower; and if in the quantity to be reduced, there be any units of this lower denomination, add them to the product; repeat this step for every succeeding denomination, till the quantity is reduced to the required name.

Exs. 25.

1. Reduce £75 15s. 6d. to pence.
2. Reduce 19 guineas 18s. 7½d. to farthings.
3. Convert 157 moidores 13s. 4d. to pence.
4. Change 3255 crowns 2s. 6¾d. into pence and farthings.
5. How many drams in 3 tons 0 cwt. 2 qrs. 17 lbs. 14 oz. 5 drs.?
6. How many grains in 17 lbs. 3 dwts. 15 grains?
7. Express 75 gals. 3 qts. 1 pt. of wine in half pints.
8. Change 175 lbs. 7 oz. 3 drs. 2 scr. into scruples.
9. In 117 sacks 11 pks. 1 gal. 3 qts., how many quarts?
10. Reduce 377 Eng. ells 4 qrs. 3 nls. to nails.
11. How many minutes in 365 days 5 hrs. 48 min.?
12. Find the number of poles in 817 mls. 5 fur. 25 poles.

35. Ex. III. Reduce £315 15s. 4d. to crowns and two-pences. Here, the denominations are not those usually found in Tables of Money; but since we may observe that there are 4 crowns in £1, and 30 twopences in one crown, we therefore multiply, first by 4, and then by 30.

Also, since in 15s. there are three crowns, I add this 15s. into my first product, viz. of crowns, not as 15, but as 3: and in the next product, viz. of twopences, I add the 4d. not as 4, but as 2. The work is

£	s.	d.
315	15	4
4		
1263 crowns		
30		
37892 twopences.		

Exs. 26.

1. Reduce £345 15s. to crowns.
2. Change £5001 17s. 6d. to half-crowns.
3. Convert 1755 guins. 19s. 6d. to sixpences.
4. How many groats in £371 18s. 8d.?
5. What number of twopences shall I receive for 175 moid. 23s. 10d.?
6. How many parcels of 4 oz. in 17 cwt. 2 qrs. 17 lbs.?
7. Bring 3 weeks 6 days 19 hrs. into spaces of 10 minutes.
8. How many parcels of 6 sheets are contained in 175 reams 15 quires of paper?
9. Find the number of quarter gallons in 3 hhds. 29 gals. 1 qt. of wine.

10. Reduce 115 cwt. 97¼ lbs. to parcels of 8 oz.
11. How many spaces of 4 inches in 1 mile 750 yds. 8 in.?
12. Reduce 1355 Eng. ells 3 qrs. 4½ in. to half quarters of a yard?

All these Examples have required only multiplication, because in every case we had to change our larger coins into smaller, and therefore we required *more* in number. Hence we multiplied.

36. Now let it be required to perform an operation of reduction just the reverse of that employed in the above Examples; for instance,

Ex. IV. To reduce 1225 farthings to pounds.

Since 4 farthings = 1 penny, we shall have but 1 penny for every 4 of the 1225 farthings; therefore the whole number of pence therein will be found by seeing how often 4 is contained in 1225, that is, by dividing it by 4; and we find that there are 306 pence, and a remainder 1, which is of course 1 farthing, or ¼d., since it is one of the 1225 farthings.

Again, if it be required to bring the 1225 farthings, or 306 pence, to shillings; since there will be only one shilling for every 12 of the pence, we shall obtain the number of shillings in 306 pence by dividing the 306 by 12; this gives 25 shillings, and a remainder 6, which is of course 6 pence. If we wish to reduce to pounds, we have in like manner only 1 pound for every 20 shillings, and must therefore divide the 25 by 20: this gives a quotient £1, and 5 over, which is 5 shillings. The complete quotient, with all the remainders, is £1 5s. 6¼d. The operation stands thus.

```
      farthings
  4)  1225
   12) 306  ¼d.
   2,0) 2,5  6d.
        £1 5s. 6¼d.
```

37. If the quantity given to be reduced were of any other name,—as ounces avoirdupois, to be brought to cwts.,—we must first divide by 16, to bring ounces to lbs., since 16 oz. = 1 lb.; then by 28, to bring lbs. to quarters, since 28 lbs. = 1 quarter; and lastly by 4, to bring quarters into cwts., since 4 qrs. = 1 cwt.

Hence, if we have to reduce a quantity from a lower denomination to a higher, we have this

Rule. Divide the proposed quantity by the number which is given in the tables, as connecting it with the next higher name; and place the remainder, if any, on a line with the quotient, and with its name attached to it. Perform similar operations of division, till the quantity is reduced to the required name. When the last quotient is obtained, bring down in a line with it all the remainders in their proper order, beginning with the highest in value.

Exs. 27.

1. In 34758 pence, how many pounds?
2. In 75389 farthings, how many guineas?
3. Change 137456 pence for moidores.
4. How many lbs. Troy in 756843 grains?
5. Reduce 2374598 seconds to days and weeks.
6. In 41063897 drams, how many tons?
7. What number of quarters is contained in 7410683 pints?
8. How many days in 89765321 seconds?
9. Reduce 7589432 pints of wine to hogsheads.
10. Reduce 13897564 minutes to years.
11. Find the number of lbs. Apoth. in 38596 grains.
12. How many barrels of ale in 47891 half pints?

38. Sometimes, as in Ex. V., the denominations given or required may not be those generally found in the tables: we have then only to find the divisors which connect every two successive denominations, and be careful to observe *the nature* of the remainders.

Ex. V. Reduce 38975 groats to crowns and pounds.

Here, since 3 groats = 1 shilling, I must first divide by 3, and the remainder will be groats. Also, since 5 shillings = 1 crown, I must next divide by 5, and the remainder will be shillings. And, since 4 crowns = £1, I must next divide by 4, and the remainder will be crowns.

The work is as follows.

```
        groats
     3) 38975
     5) 12991  2 gr.
      4) 2598  1 sh.
         £649  2 cr.,   or £649 2 cr. 1 sh. 2 groats.
```

We have two answers, £649 11s. 8d., or 2598 cr. 1s. 8d.

Exs. 28.

1. Change 34275 groats into moidores.
2. How many crowns in 145893 farthings?
3. How many sixpences and half-crowns in 58976 farthings?
4. Change 34589 half-crowns into pounds.
5. Convert 148235 twopences into moidores.
6. Change 348796 groats into seven-shilling pieces.
7. Change 38976 drams (Avoirdupois) into portions of 7 lbs. each.
8. How many spaces of 3 hours are contained in 49539600 seconds?
9. In 458976 parcels of 4 sheets of paper, how many reams?
10. How many measures of 3 hogsheads are contained in 48976 pints of wine?
11. In 654321 parcels of 8 oz., find the number of cwts.

NOTE. If a pupil, in attempting an Example in reduction, be in doubt whether to use the first or the second Rule, that is, whether to multiply or divide, he must remember that he has only to ask himself one question: Will the answer which I have to find be a number *greater* or *less* than the number which I have to reduce? If it is to be *greater*, I must of course multiply; if *less*, I must divide. Thus, to bring £10 to farthings, I must of course *multiply*, because there will plainly be more farthings than *ten* in £10; and if I have to reduce 1250 pence to pounds, I

must *divide*, because there are evidently fewer pounds than 1250, in 1250 pence.

39. In some of the Examples under weights and measures we shall find it necessary to multiply and divide by such quantities as $5\frac{1}{2}$, and $30\frac{1}{4}$.

I will give an Example of each of such cases.

Ex. VI. Reduce 3 fur. 17 po. 2 yds. 1 ft. to feet.

$$\begin{array}{rl}
 & 3\text{ f. } 17\text{ p. } 2\text{ y. } 1\text{ ft.} \\
 & \underline{40} \\
 & 137\text{ po.} \\
 & \underline{5\frac{1}{2}} \\
 & 687 \\
\frac{1}{2}\text{ of } 137 = & \underline{68\frac{1}{2}} \\
 & 755\frac{1}{2}\text{ yds.} \\
 & \underline{3} \\
 & \underline{\underline{2267\frac{1}{2}}}\text{ feet.}
\end{array}$$

The first multiplication by 40 is quite plain. Now to multiply a quantity by $5\frac{1}{2}$ is to repeat it 5 times and half a time; therefore $5\frac{1}{2}$ times 137 will be found by multiplying the 137 by 5, and then adding to this product one half of 137: thus working, and adding in the 2 yds., I obtain $755\frac{1}{2}$ yds. as the whole product. In bringing these yards to feet by multiplying by 3, I remember that the $\frac{1}{2}$ yd. is 1 foot and a half; so adding it in, as well as the 1 foot in the top line, I obtain $2267\frac{1}{2}$ as the number of feet in 3 fur. 17 po. 2 y. 1 ft.

Had the multiplier been $5\frac{1}{4}$, instead of $5\frac{1}{2}$, then instead of taking one-*half* of the 137, I should have taken one *quarter*. The following Example will illustrate this.

Ex. VII. Reduce 3 ro. 23 po. 14 sq. yds. to square feet.

$$\begin{array}{rl}
 & 3\text{ ro. } 23\text{ po. } 14\text{ sq. yds.} \\
 & \underline{40} \\
 & 143\text{ po.} \\
 & \underline{30\frac{1}{4}} \\
 & 4304 \\
\frac{1}{4}\text{ of } 143 = & \underline{35\frac{3}{4}} \\
 & 4339\frac{3}{4}\text{ sq. yds.} \\
 & \underline{9} \\
 & \underline{\underline{39057\frac{3}{4}}}\text{ sq. ft.}
\end{array}$$

Multiplying by 4, and $30\frac{1}{4}$, I obtain $4339\frac{3}{4}$ square yards; and since the $\frac{3}{4}$ sq. yd. is equal to $6\frac{3}{4}$ square feet, I have added it in as $6\frac{3}{4}$, when multiplying by 9 to reduce to square feet.

40. Now let it be required to *divide* by these same numbers $5\frac{1}{2}$ and $30\frac{1}{4}$. The following Examples will require such divisions.

Ex. VIII. Reduce 384976 feet to furlongs.

```
     feet.
  3) 384976
 5½) 128325  1 ft.
 2        2
 11) 256650
  4,0) 2333,1  9 hf. yds.
       583 fur. 11 po. 4½ yds. 1 ft.
```

Here, proceeding to reduce to yards, poles, and furlongs successively, I divide by 3, $5\frac{1}{2}$, and 40. The first division is simple. In the second division I have to see how often $5\frac{1}{2}$ yds. are contained in 128325 yds. Now I cannot divide by $5\frac{1}{2}$, as it stands; but if I bring both divisor and dividend into half yards, viz. 11 and 256650, it will be precisely the same, whether I see how often $5\frac{1}{2}$ yds. are contained in 128325 yds., or 11 *half* yds. in 256650 *half* yards.

Performing the division, I have as quotient 23331 poles; and because the dividend 266650, was half yards, therefore the remainder 9 was 9 half yards, or $4\frac{1}{2}$ yds. Dividing as usual by the 40, to bring poles into furlongs, I have the complete answer 583 fur. 11 po. $4\frac{1}{2}$ yds. 1 ft.

41. The following Example will show how to divide by $30\frac{1}{4}$, and will need no explanation.

Ex. IX. Reduce 785447 sq. feet to acres.

```
                 sq. feet.
  yds.        9) 785447
  30¼             87271  8 sq. ft.
   4                  4
  121 qrs. { 11) 349084 qrs.
           { 11)  31734  10 }
            4,0)  288,4  10 } 120 quarters, or 30 yds.*
                   4) 72  4 poles
                      18 ac. 0 ro. 4 po. 30 sq. yds. 8 sq. ft.
```

* The reader who is acquainted with fractions will perceive that this method is

Exs. 29.

1. How many square inches are contained in 3 ro. 35 po. 25 yds. 8 ft.?
2. Reduce 15 m. 7 fur. 8 po. 3 yds. to yards.
3. Convert 185 degrees of 69½ miles into yards.
4. In 1815 coins, each worth 5¼ guineas, how many pence?
5. Convert 37584 dollars, each worth 4½ shillings, into halfpence.
6. Convert 60000 barley corns into fathoms.
7. In 37589 inches, how many fathoms?
8. Reduce 13859 yds. to miles.
9. How many leagues in 478321 feet?
10. How many acres in 349876 square poles?
11. Reduce 6897543 square inches to roods.
12. In 4596328 perches, how many square miles?

42. But there is another class of Examples in Reduction which require both the above processes of Multiplication and Division to be used in the same question. As a simple Example of this kind, let us take

Ex. X. To reduce 1000 guineas to pounds.

Now this question really is; "How many times is £1 contained in 1000 guineas?" To answer this, I must divide 1000 guineas by £1: but I cannot do this, till I bring both divisor and dividend to the same name. The highest coin of which they both consist is shillings; I therefore reduce them both to shillings and then divide.

The operation is most clearly shown thus,—

$$\begin{array}{rr}
 & \text{guineas.} \\
 & 1000 \\
 & 21 \\
\hline
\pounds 1 & 1000 \\
20 & 2000 \\
\hline
2,0\,\text{sh.}) & 2100,0\,\text{sh.} \\
\hline
 & 1050\ \text{pounds.}
\end{array}$$

---

identical with that pursued in division of fractions.

$$\text{Thus, } 87271 \text{ yds.} \div 30\tfrac{1}{4} = \frac{87271}{\frac{121}{4}} \text{po.} = \frac{87271 \times 4}{121} \text{po.}$$

$$= \frac{349084}{121} \text{po.} = 2884\tfrac{120}{121} \text{ po.}$$

43. In this Example we have had to reduce the given quantity, guineas, and the required quantity, pounds, only one step,—namely, to shillings. But sometimes the numbers will require to be reduced more than one step, as for instance, if it be required to change this 1000 guineas into half-crowns.

	guineas.
	1000
	21
	1000
	2000
s. *d.*	
2 6	21000 sh.
12	12
3,0 pence)	25200,0 pence.
	8400 hf. crowns.

I have here to see how often a half-crown will go in 1000 guineas. Since 1000 guineas and 1 half-crown both exactly consist of pence, I bring both to pence, and they become 252000d. and 30d. Dividing the greater by the less, I have as quotient 8400, which is the required number of half-crowns.

	guineas.
	1000
s. *d.*	21
2 6	21000 sh.
2	2
5 sixp.)	42000 sixp.
	8400 hf. crowns.

Again, since 2s. 6d. and 1 guinea both consist of *sixpences*, I might have reduced them both to sixpences, instead of pence, and the work would then have been shorter. For such Examples, we may therefore lay down the following

Rule. Find the greatest denomination or kind of which both quantities exactly consist; reduce them both to that denomination, and divide the greater by the less.

44. I give one more Example of this kind, on account of the difficulty that pupils sometimes have in applying the above Rule.

Ex. XI. In £453 16s. 8d., how many pieces of coin, each 3s. 4½d.?

Here the coin or denomination of which both the quantities consist is half-pence. I therefore reduce both to half-pence, and, as before, divide the larger number by the smaller: The remainder 31 must of course be half-pence; so that the required number of coins is 2689; and 15½d. remain.

```
                         £    s.  d.
                        453  16   8
                         20
  s.  d.               9076 sh.
  3   4½                 12
 12                   108920 d.
 40 d.                    2
  2                 9) 217840 hf. pence.
 81 hf. pence       9)  24204  4 } 31 hf. pence.
                         2689  3
```

Exs. 30.

1. In £50 15s. 6d., how many coins each 4s. 6d.?
2. Reduce 175 guineas to pieces each worth 2s. 7½d.
3. Find how many coins of 6s. 8d. can be obtained out of £217 19s. 8d.
4. I exchange 375 pieces of 7s. 6d. each for coins worth 22s. 9d. each; how many shall I obtain?
5. Divide 1875 yds. 3 qrs. into pieces, each 3¼ nails?
6. How many portions of 1¼ oz. can be obtained from 1 cwt. 1 qr. 6lbs?
7. What number of intervals of 2¼ feet are there in 1 mile 800 yds.?
8. Find how many intervals of 19½ seconds there are in a week.
9. How many distances each 1¾ furlongs in a degree?
10. Convert 375 Eng. ells into portions of 4½ inches.
11. How many subscribers of 27s. 6d. each will be required to raise £1925.

45. We have now given Examples of all the *principles* which we need learn in order to work any question in Reduction; but there are such various forms in which Reduction can occur, that it will be advisable to work some additional questions which will illustrate the difficulties. It is better, however, to defer these questions till after the Compound Rules are mastered, because in many of them a knowledge of these rules is of use; also, the pupil will, by his increased experience, be more able to contend with difficult examples.

COMPOUND RULES.

ADDITION OF COMPOUND QUANTITIES, OR

COMPOUND ADDITION.

46. In adding Compound Quantities, we must remember, as in Simple Addition, that like quantities must be added together, as farthings to farthings, pence to pence, and so on.

Hence, if it be required to find the sum of any compound quantities, as £73 2s. 9½d.; £25 8s. 4¾d.; £68 3s. 11¼d.; £76 17s. 7d.; £5 14s. 5¾d.; we must place the quantities under one another so that the farthings may be in a row, as also the pence, shillings, and pounds: and the sum will stand thus:

£	s.	d.
73	2	9½
25	8	4¾
68	3	11¼
76	17	7
5	14	5¾
249	7	2¼

Beginning at the right-hand column, and adding the farthings, we find their sum to be 9: this, by reduction to pence, gives 2 pence and 1 farthing over: put down the 1 farthing (thus, ¼) and carry the 2 pence to the next column, which consists of pence. Adding it, as in Simple Addition, we find its sum to be 38 pence, which by reduction gives 3 shillings, and 2 pence. Put down the 2 pence, and carry the 3 shillings to the next row which consists of shillings. Adding again, we find the row

of shillings to amount to 47 shillings, or, by reduction, to 2 pounds, 7 shillings: put down the 7s. and carry the £2 to the next column, which consists of pounds. The sum of this last column we find to be £249; place this under the pounds. The complete answer is therefore, 249 pounds seven shillings and twopence farthing, or £249 7s. $2\frac{1}{4}$d.

47. The method would be precisely the same, if the quantities to be added were expressed in any other table, weight, or measure; only that, in carrying from any column to the next, we should have to use the reductions proper to the denominations we were adding: and the method of performing these reductions has been explained under the head of Reduction.

Hence, for adding together compound quantities, we have this

Rule. Arrange the quantities under one another, so that all those of the same kind may be in upright rows. Begin at the right-hand, and find the sum of the first row, as in Simple Addition: see how many units of the next higher name are contained in the sum, put down the remainder, if any, and carry those units to the next row; proceed in like manner with each column to the end, and the sum of the last column to the left write down in full.

Exs. 31. Find the value of

	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
1.	16	8	$9\frac{1}{2}$ +	1106	19	$7\frac{1}{4}$ +	915	10	0 +	3	8	$7\frac{3}{4}$
	+5	6	$8\frac{1}{2}$ +	1415	5	$2\frac{3}{4}$.						
2.	858	7	$6\frac{1}{4}$ +	1984	19	$11\frac{1}{2}$ +	347	5	$9\frac{1}{4}$ +	3426	17	10
	+549	9	$9\frac{3}{4}$ +	1685	12	$8\frac{3}{4}$.						
3.	345	7	$9\frac{1}{2}$ +	685	11	2	+3899	14	$11\frac{3}{4}$ +	7645	8	$7\frac{1}{4}$
	+598	19	$2\frac{1}{2}$ +	1386	7	4.						
4.	3476	8	$9\frac{1}{4}$ +	11063	10	9	+9876	15	$11\frac{1}{2}$ +	10010	3	$0\frac{3}{4}$
	+7856	16	$7\frac{1}{4}$ +	15892	18	$8\frac{3}{4}$.						

£ s. d. £ s. d. £ s. d. £ s. d.

5. 13487 12 2 + 9875 18 9¼ + 34267 5 6½ + 1899 9 9¾
+24682 11 7½ + 13897 15 9.

6. 89645 13 3 + 745627 19 7¼ + 99314 16 5¼ + 3875 0 11
+45683 16 8 + 1976 14 5.

7. 38976 5 4½ + 7043 17 9 + 689 16 4¾ + 14582 9 11¼
+38764 17 10 + 429 19 9½ + 1080 0 7¾.

8. 41987 16 7 + 112785 14 3½ + 98979 19 8¼ + 4568 16 11¾
+3897 17 3¼ + 31456 11 0 + 4289 7 3½.

9. 10689 15 6 + 39345 8 9¾ + 4786 13 7 + 98764 14 3¾
+111468 17 11½ + 3487 19 10¼ + 87562 17 7.

10. 67489 10 7¾ + 149876 19 9 + 348754 17 4½ + 689 11 0½
+87532 9 11 + 4986 1 7¾ + 15019 3 6¼ + 875 10 10

LONG MEASURE.

yds. ft. in. b.c. yds. ft. in. b.c. yds. ft. in. b.c. yds. ft. in. b.c.

11. 5 2 9 2 + 16 1 11 1 + 18 0 7 0 + 25 1 8 2 +
17 2 6 1 + 6 2 9 0.

12. 75 2 11 2 + 187 0 9 1 + 34 1 8 0 + 93 2 7 1 +
106 1 4 2 + 85 2 6 1.

mls. yds. ft. in. mls. yds. ft. in. mls. yds. ft. in. mls. yds. ft. in.

13. 185 25 2 7 + 17 809 1 8 + 361 73 0 11 + 95 145 2 4
+84 698 2 3 + 603 45 2 9.

14. 684 116 2 6 + 1785 385 1 8 + 907 47 0 10 + 64 903 1 9
+7832 86 0 7 + 986 345 2 5.

fur. po. yds. ft. fur. po. yds. ft. fur. po. yds. ft. fur. po. yds. ft.

15. 37 35 3 2 + 18 17 1¼ 1 + 109 30 4½ 2 + 75 29 2 1
+1846 18 3½ 0 + 49 4 4¼ 0 + 168 31 3 2.

16. 185 15 4 1 + 76 25 3 2 + 349 35 2 0 + 608 17 0 1
+705 19 5 2 + 986 23 3 2 + 432 11 2 0.

TROY WEIGHT.

lbs. oz. dwts. grs. lbs. oz. dwts. grs. lbs. oz. dwts. grs. lbs. oz. dwts. grs.

17. 17 5 16 20 + 135 8 15 19 + 89 9 19 23 + 604 11 7 7
+73 7 10 18 + 496 10 13 19.

18. 345 11 17 21 + 1029 9 18 17 + 687 8 4 19 + 4321 5 7 14
+864 4 9 3 + 387 2 11 12.

lbs. oz. dwts. grs. lbs. oz. dwts. grs. lbs. oz. dwts. grs. lbs. oz. dwts. grs.

19. 879 4 17 15 + 1000 10 6 11 + 754 9 18 17 + 16 11 19 5 + 1910 7 13 9 + 875 8 5 14.

20. 715 11 17 19 + 684 9 18 5 + 1932 8 14 16 + 45 7 15 22 + 507 4 13 20 + 18 0 11 17.

AVOIRDUPOIS WEIGHT.

lbs. oz. drs. lbs. oz. drs. lbs. oz. drs. lbs. oz. drs. lbs. oz. drs.

21. 101 8 2 + 74 12 4 + 53 10 6 + 20 14 9 + 63 13 13 + 39 7 15 + 103 9 7.

22. 75 3 11 + 176 11 15 + 819 14 14 + 47 9 3 + 160 7 11 + 918 15 8 + 456 10 9.

cwts. qrs. lbs. oz. cwts. qrs. lbs. oz. cwts. qrs. lbs. oz. cwts. qrs. lbs. oz.

23. 15 3 27 15 + 175 2 25 13 + 1987 1 13 11 + 432 0 19 9 + 375 3 16 7 + 1689 2 18 8.

24. 350 3 16 9 + 75 2 15 11 + 917 1 25 15 + 6542 0 23 13 + 689 2 27 10 + 750 3 20 14 + 1897 2 20 12.

tons cwts. lbs. oz. tons cwts. lbs. oz. tons cwts. lbs. oz. tons cwts. lbs. oz.

25. 75 19 17 14 + 389 14 63 13 + 2648 15 97 12 + 750 9 100 11 + 684 8 15 7 + 3968 7 45 8 + 589 6 111 10 + 642 18 87 9

26. 387 17 45 9 + 49 18 75 15 + 604 3 89 14 + 138 15 3 11 + 1796 8 10 7 + 423 9 44 8 + 897 11 101 3 + 1145 19 19 2

APOTHECARIES' WEIGHT.

oz. drs. sc. grs. oz. drs. sc. grs. oz. drs. sc. grs. oz. drs. sc. grs.

27. 11 7 2 19 + 8 6 1 17 + 9 4 0 13 + 17 0 1 11 + 35 3 2 8 + 86 2 0 9 + 97 1 1 12.

28. 17 5 0 17 + 139 7 2 18 + 44 6 1 10 + 65 3 0 19 + 63 2 2 11 + 245 1 1 13 + 178 0 0 14.

bs. oz. drs. sc. lbs. oz. drs. sc. lbs. oz. drs. sc. lbs. oz. drs. sc.

29. 117 11 6 2 + 73 10 7 1 + 1094 6 5 2 + 685 4 3 1 + 734 11 4 0 + 99 3 2 1 + 108 9 1 2.

30. 375 9 7 2 + 649 4 6 1 + 832 11 3 0 + 1048 8 7 1 + 756 7 5 2 + 89 6 4 2 + 635 5 2 0.

CLOTH MEASURE.

yds. qrs. nls. in. yds. qrs. nls. in. yds. qrs. nls. in. yds. qr. nl. in.

31. 15 3 2 2 + 75 2 3 $1\frac{1}{2}$ + 389 0 2 2 + 60 1 1 $1\frac{3}{4}$ 14 2 0 2 + 175 3 2 $0\frac{1}{4}$ + 87 1 3 $1\frac{1}{4}$.

yds. qrs. nls. in. yds. qrs. nls. in. yds. qrs. nls. in. yds. qrs. nl. in.

32. 375 3 2 1¼ + 408 2 3 2 + 96 1 0 1¼ + 235 0 1 2
87 2 3 0¾ + 591 3 2 1½ + 62 1 0 1.

WINE MEASURE.

hhds. gals. qts. pts. hhds. gals. qts. pts. hhds. gals. qts. pts. hhds. gals. qts. pts.

33. 60 45 2 0 + 3 36 3 1 + 14 7 2 0 + 28 60 1 1 +
179 49 3 0 + 14 37 2 1 + 5 25 1 0.

34. 45 15 3 0 + 236 45 2 1 + 87 57 1 0 + 95 16 3 0 +
215 7 2 1 + 64 19 3 0 + 93 27 2 1.

35. 15 50 2 0 + 175 53 1 1 + 64 49 3 1 + 815 7 2 1 +
76 18 1 0 + 193 25 3 1 + 42 37 2 1.

36. 75 35 2 1 + 137 43 1 1 + 94 17 3 0 + 216 9 1 1 +
189 51 2 0 + 76 19 3 0 + 39 24 0 1.

ALE AND BEER MEASURE.

hhds. gals. qts. pts. hhds. gals. qts. pts. hhds. gals. qts. pts. hhds. gals. qts. pts.

37. 236 45 2 1 + 45 15 3 0 + 87 50 1 0 + 115 7 2 0 +
95 16 3 1 + 64 19 3 1 + 93 27 2 0.

hhds. bar. kil. gals. hhds. bar. kil. gals. hhds. bar. kil. gals. hhds. bar. kil. gals.

38. 17 1 1 14 + 85 1¼ 0¾ 15 + 193 0¾ 1½ 9 + 207 1 1¼ 6 +
79 0½ 1¼ 17 + 101 0 0 8.

SQUARE MEASURE.

ac. ro. po. yds. ac. ro. po. yds. ac. ro. po. yds. ac. ro. po. yds.

39. 75 3 19 7 + 329 2 25 18 + 4869 1 16 6 + 459 0 35 5
+ 78 2 18 4 + 385 1 17 3 + 1217 0 18 2¾ + 876 3 19 3

40. 118 2 11 25 + 457 3 39 17 + 11892 0 7 4 + 8972 1 27 20
+ 3145 1 37 30 + 9864 2 19 10 + 4382 3 20 15½ + 4 3 2 2¼

sq. m. ac. yds. sq. m. ac. yds. sq. m. ac. yds. sq. m. ac. yds.

41. 1358 345 3758 + 964 27 897 + 875 495 1684 + 4809 605 329
+ 293 87 401 + 687 327 2348 + 5904 95 3729 + 1873 119 495

CUBIC MEASURE.

sol. yds. ft. in. sol. yds. ft. in. sol. yds. ft. in. sol. yds. ft. in.

42. 387 18 1000 + 9126 25 895 + 45 7 1643 + 821 19 27
+ 3437 11 5 + 89 6 1519 + 1000 26 372.

43. 4186 15 874 + 379 9 910 + 25 18 35 + 5804 6 1681
+ 1427 11 25 + 983 10 984 + 467 17 39.

PAPER.

	reams	qui.	shts.	reams	qui.	shts.	reams	qui.	shts.	reams	qui.	shts.
44.	75	19	23 +	468	7	17 +	1937	16	18 +	46	11	5 +
	289	9	19 +	310	8	22 +	898	13	21.			
45.	875	18	17 +	9832	5	18 +	459	19	20 +	1684	11	5 +
	359	10	19 +	1875	15	21 +	348	15	23.			

COMPOUND SUBTRACTION.

48. Here, as in Simple Subtraction, the quantities to be subtracted must be taken from others of the same kind; and, therefore, we arrange the two quantities as in Compound Addition, putting the less under the greater.

Ex. Let it be required to find the difference of £325 19s. $4\frac{3}{4}$d. and £253 7s. $6\frac{1}{2}$d.

Placing them as we have just directed, and beginning at the right-hand, we take $\frac{1}{2}$d. or 2 farthings from $\frac{3}{4}$d., or 3 farthings, and the difference 1 farthing, or $\frac{1}{4}$d., we put down under the column of farthings. Proceeding to the pence, we cannot take the 6 pence in the lower line from the 4 pence in the upper: we must, therefore, *borrow*, as it is called, from the next higher name, which is in this case shillings; we take 1 shilling, or 12 pence, from the 19 shillings, and add it to the 4 pence in the top line, making 16 pence; we now subtract the 6d. from the 16d., and have the remainder 10d., which is to be placed under the column of pence. And when we have borrowed in any subtraction, we must for the reasons given in Simple Subtraction, carry *one to* the next row to the left, and then subtract. We

£	s.	d.
325	19	$4\frac{3}{4}$
253	7	$6\frac{1}{2}$
£72	11	$10\frac{1}{4}$

shall thus subtract 8s. from 19s., and have a remainder 11s. In the pounds, we find the difference of the two rows, precisely as in Simple Subtraction, to be 72. Hence the whole difference is £72 11s. 10¼d.

49. The same alterations must be made in performing the subtraction as are made in Compound Addition, when the compound quantities consists of any other kind than money. Thus, in subtracting the pence, in the above Ex., we borrowed 12 pence, because 12 pence = 1 shilling; so, if the Example had been in Avoirdupois weight, and while subtracting in ounces, we were obliged to borrow, we should have borrowed 16, because 16 ounces make 1 pound, which is an unit of the next higher name. And similarly for any other weight or measure.

Hence, if it be required to find the difference of two compound quantities, we have this

Rule. Place the less number under the greater, so that quantities of the same name may be under one another. Begin at the right-hand, and take the lower number from the upper, if possible; but if the lower number be greater than the upper, take one unit of the next higher name, reduce it to the denomination in which you are now subtracting, add it to the upper figure, and then subtract, placing the difference underneath. Carry 1 to the lower figure of the next name, and proceed in exactly the same manner to the last figure on the left-hand.

Exs. 32.

	£	s.	d.		£	s.	d.		£	s	d.		£	s.	d.
1.	85	9	7½	–	75	16	9	5.	8972	18	6½	–	4389	10	8¾
2.	432	19	8	–	375	0	11¼	6.	18759	11	9	–	9867	18	7¼
3.	1827	5	6	–	1103	18	9¼	7.	38974	15	6	–	9368	16	11½
4.	4268	19	0	–	575	19	10½	8.	14589	11	9¼	–	7892	10	11¾

	£	s.	d.		£	s.	d.
9.	45860	3	6¼	–	8977	7	10½
10.	46932	17	0	–	7145	18	7¼
11.	897654	18	11	–	69596	18	11½
12.	1489765	14	8	–	13609	17	9¼

TROY WEIGHT.

	lbs.	oz.	dwts.	grs.		lbs.	oz.	dwts.	grs.
13.	175	1	16	13	–	89	10	13	20
14.	8345	6	17	9	–	689	4	18	21
15.	5689	4	13	11	–	3870	8	9	17
16.	14896	8	11	10	–	9738	10	16	23

AVOIRDUPOIS WEIGHT.

	tons	cwts.	qrs.	lbs.		tons	cwts.	qrs.	lbs.
17.	2345	11	2	17	–	879	18	3	26
	cwts.	qrs.	lbs.	oz.		cwts.	qrs.	lbs.	oz.
18.	7189	2	15	8	–	349	3	19	12
	cwts.	lbs.	oz.	drs.		cwts.	lbs.	oz.	drs.
19.	3459	101	11	8	–	783	99	14	15
20.	3195	85	7	11	–	839	103	15	8

CLOTH MEASURE.

	yds.	qrs.	nls.	in.		yds.	qrs.	nls.	in.
21.	3894	2	1	1¼	–	986	3	3	2
	E. ells	qrs.	nls.	in.		Fr. Ells	qrs.	nls.	in.
22.	175	3	2	1	–	68	4	3	2
	Fr. ells	qrs.	nls.	in.		Fr. ells	qrs.	nls.	in.
23.	346	1	1	0	–	89	5	3	2
	Fl. ells.	qrs.	nls.	in.		Fl. ells	qrs.	nls.	in.
24.	4185	2	1	1	–	376	2	3	1½

WINE MEASURE.

	bar.	gals.	qts.	pts.		bar.	gals.	qts.	pts.
25.	1375	12	2	0	–	889	15	3	1
26.	2387	24	1	0	–	748	30	3	1
	hhds.	gals.	qts.	pts.		hhds.	gals.	qts.	pts.
27.	8976	35	3	0	–	987	45	2	1
28.	2892	50	1	1	–	983	34	2	0

SQUARE MEASURE.

	sq. yds.	ft.	in.		sq. yds.	ft.	in.	
29.	7181	3	45	–	923	8	104	
30.	8432	7	125	–	2785	8	37	

	ac.	ro.	sq. po.	sq. yds.		ac.	ro.	sq. po.	sq. yds.
31.	375	2	19	25	–	17	3	35	30
32.	876	3	15	$18\frac{1}{2}$	–	437	3	31	20

TIME.

	dys.	hrs.	min.	sec.		dys.	hrs.	min.	sec.
33.	245	13	14	53	–	67	19	41	17
34.	327	17	13	34	–	248	6	50	45

	wks.	dys.	hrs.	min.		wks.	dys.	hrs.	min.
35.	315	4	17	39	–	76	5	22	35
36.	178	3	3	57	–	109	6	19	59

DRY MEASURE.

	wys.	qrs.	bush.	pks.		wys.	qrs.	bush	pks.
37.	73	4	5	3	–	45	1	7	2
38.	179	0	4	2	–	138	4	6	3

	lasts	wys.	qrs.	bush.		lasts	wys.	qrs.	bush.
39.	325	1	4	3	–	89	1	4	7
40.	1827	0	3	2	–	809	1	3	6

COMPOUND MULTIPLICATION.

50. As Simple Multiplication was shewn to be merely a shorter method of performing Simple Addition, so when we have learnt how to add compound quantities of a similar kind, we shall have no difficulty in multiplying compound quantities by any multiplier whatever. And first let the multiplier be not greater than 12; and let it be required to multiply £358 4s. $7\frac{3}{4}$d. by 5. We may perform the required operation both by addition and multiplication, and explain the second method from the first.

£	s.	d.
358	4	$7\frac{3}{4}$
358	4	$7\frac{3}{4}$
358	4	$7\frac{3}{4}$
358	4	$7\frac{3}{4}$
358	4	$7\frac{3}{4}$
£1791	3	$2\frac{3}{4}$

£	s.	d.
358	4	$7\frac{3}{4}$
		5
£1791	3	$2\frac{3}{4}$

Beginning with the farthings in either of the two sums, we have the sum of the farthings, or five times $\frac{3}{4}$d. = 15 farthings, which = 3 pence, and 3 farthings over; put down the 3 farthings, and carry the 3 pence; so, also, 5 times 7d. = 35d. and with 3d. carried = 38d., or 3s. 2d.; 5 times 4s. = 20s., and with 3s. carried = 23s., or £1 3s.; and by Simple Multiplication we have 5 times £358, with the £1 carried, = £1791.

51. What has been said in Compound Addition about carrying, when the given quantities consist of any other than pounds, shillings, and pence, applies in Compound Multiplication; for we have shewn the two processes of Addition and Multiplication to be the same.

Hence, for multiplying any compound quantity by a multiplier under 12, we have this

RULE. Place the multiplier under the lowest denomination in the multiplicand, and multiply that name by the multiplier. See, as in Compound Addition, how many units of the next higher name are contained in this product; put down the remainder, if any, and carry these units to the next name. Multiply in like manner each denomination of the multiplicand: and when the highest product has been found, write it down in full.

52. Now let the multiplier be between 12 and 144. And first, let it be a number which can be exactly split into two numbers, for instance 72. This is equal to 8 times 9. If,

therefore, I multiply by 8, the first product will be equal to 8 times the multiplicand. If, now I multiply this product by 9, then the second product will be 9 times the former one, or 72 times the top line, and therefore be the required amount.

£	s.	d.	
358	4	$7\frac{3}{4}$	
		8	
2865	17	2	= 8 times
		9	
£25792	14	6	= 72 „

But if the multiplier cannot exactly be split into two numbers each under 12, as for instance 76, we must choose the next number below, which *can* be so split; in this case we take 72; and since $76 = 72 + 4$, we may multiply by 72, or 8×9 as before, and then take 4 times the top line; thus obtaining 72 times, and 4 times the given quantity; the two products added together will give 76 times the multiplicand, as in the Example here given.

£	s.	d.	
358	4	$7\frac{3}{4}$	
		8	
2865	17	2	= 8 times
		9	
25792	14	6	= 72 „
1432	18	7	= 4 „
£27225	13	1	= 76 „

Exs. 33. Form the following products.

	£	s.	d.	
I.	7	18	$10\frac{1}{2}$	by 5, 6, 7, 8.
II.	13	9	$7\frac{3}{4}$	by 7, 8, 9, 10.
III.	456	18	$7\frac{1}{4}$	by 9, 10, 11, 12.

Exs. 34.

	£	s.	d.				£	s.	d.	
1.	138	19	4	× 35.		5.	498	17	$10\frac{1}{2}$	× 84.
2.	725	6	$8\frac{1}{2}$	× 24.		6.	2784	13	$1\frac{3}{4}$	× 96.
3.	1829	2	11	× 40.		7.	3345	9	$10\frac{1}{2}$	× 108.
4.	1785	15	$9\frac{1}{4}$	× 56.		8.	7864	17	8	× 120.

53. Next, let the multiplier be any whole number greater than 144, as 256.

Since 256 = 200 + 50 + 6, if therefore we multiply the given quantity by 200, and by 50, and by 6, and add together these three products, we shall find the product of £358 4s. 7¾d. by 256.

Now, 200 = 10 × 10 × 2; therefore if we multiply the multiplicand by 10, and that product by 10, and this second product by 2, we shall have the third product the same as if we had multiplied the multiplicand at once by 200:

To obtain the product of the multiplicand by 50, we multiply the first product by 10, since 50 = 5 × 10: then multiplying the top line by 6, and adding the three products just obtained, we have the product of £358 4s. 7¾d. by 256, viz. £91707 9s. 4d.

£	s.	d.			
358	4	7¾			
		10			
3582	6	5½	=	10 times the top line	
		10			
35823	4	7	=	100 times	,,
		2			
71646	9	2	=	200 times	,,
17911	12	3½	=	50 times	,,
2149	7	10½	=	6 times	,,
£91707	9	4	=	256 times	,,

54. Similarly, if we had a multiplier of four figures, as 7538, we should have to multiply three times by 10, which would give 1000 times the top line; then by the 7, to make 7000 times.
Also the third line, which equals 100 times the top line, must be multiplied by 5— .... 500 ,,
and the second line, which equals 10 times the top line, must be multiplied by 3— .... 30 ,,
and the top line itself by 8— .... 8 ,,
Therefore, the sum of all the four products so formed will be equal to 7538 times.

And similarly for any other number.

55. It seems unnecessary to state a separate Rule for multiplying by such numbers as we have been just using in the above Examples, since the process of multiplication, for forming the products to be added together, has been explained in the Rule already given for multiplying compound quantities by any number not exceeding 12.

Exs. 35.

1.	8	9	$7\frac{1}{2}$	×	23
2.	11	19	8	×	39
3.	14	13	$7\frac{1}{4}$	×	47
4.	21	2	$0\frac{3}{4}$	×	53
5.	35	17	11	×	61
6.	42	8	$8\frac{3}{4}$	×	75
7.	101	9	10	×	83
8.	215	7	$8\frac{3}{4}$	×	104
9.	45	5	$9\frac{1}{4}$	×	137
10.	178	11	10	×	174
11.	216	13	4	×	180
12.	189	6	8	×	196
13.	1	17	$9\frac{1}{4}$	×	237
14.	10	19	$11\frac{1}{2}$	×	348
15.	42	3	$6\frac{1}{4}$	×	375
16.	2	7	$9\frac{3}{4}$	×	573
17.	15	7	$9\frac{1}{4}$	×	943
18.	3	17	$7\frac{1}{4}$	×	1103
19.	3	0	$7\frac{1}{2}$	×	1215
20.	4	9	$7\frac{1}{2}$	×	3201
21.	31	16	11	×	4375
22.	1	11	6	×	7235
23.	16	16	$6\frac{1}{2}$	×	4520
24.	1	7	11	×	11829

TROY WEIGHT.

	lbs.	oz.	dwts.		
25.	15	9	17	×	35
26.	75	8	13	×	57
27.	117	11	19	×	89

	lbs.	oz.	dwts.	grs.		
28.	14	11	19	13	×	137
29.	45	9	14	16	×	785
30.	170	8	18	17	×	8923

AVOIRDUPOIS WEIGHT.

	cwts.	lbs.	oz.	drs.		
31.	14	75	14	6	×	74
32.	35	54	8	5	×	801
33.	147	108	4	13	×	945

	tons.	cwts.	lbs.	oz.		
34.	45	17	101	14	×	175
35.	73	14	96	11	×	432
36.	185	11	10	9	×	819

APOTHECARIES' WEIGHT.

	oz.	drs.	sc.	grs.		
37.	11	7	2	15	×	215
38.	9	5	1	7	×	307
39.	10	6	0	19	×	95

	lbs.	oz.	drs.	sc.		
40.	11	4	7	2	×	574
41.	35	11	6	1	×	809
42.	79	10	5	0	×	199

LONG MEASURE.

	mls.	fur.	po.	yds.			fur.	po.	yds.	ft.	
43.	175	6	35	4	× 89	46.	15	38	5	2	× 587
44.	216	5	19	3	× 117	47.	175	17	3	1	× 2017
45.	384	7	27	2	× 438	48.	83	37	4	2	× 7845

CLOTH MEASURE.

	yds.	qrs.	nls.			E. ells.	qrs.	nls.	in.	
49.	476	3	3	× 73	52.	78	3	2	1	× 487
50.	894	2	1	× 194	53.	805	2	3	2	× 534
51.	1727	1	2	× 216	54.	648	3	2	2	× 1749

ALE AND BEER MEASURE.

	hhds.	gals.	qts.			bar.	gals.	qts.	pts.	
55.	114	51	1	× 98	58.	67	15	3	1	× 415
56.	268	47	3	× 375	59.	395	31	2	0	× 1209
57.	897	15	2	× 4021	60.	427	27	3	1	× 877

WINE MEASURE.

	hhds.	gals.	qts.			hhds.	gals.	qts.	pts.	
61.	87	59	3	× 65	64.	45	17	2	1	× 407
62.	275	60	2	× 183	65.	306	43	3	0	× 196
63.	349	15	1	× 235	66.	742	37	2	1	× 384

SQUARE MEASURE.

	sq. yds.	sq. ft.	sq. in.			ac.	ro.	sq. po.	sq. yds.	
67.	75	5	75	× 73	70.	75	3	25	17	× 117
68.	117	7	108	× 85	71.	127	2	35	25	× 245
69.	237	8	125	× 97	72.	345	1	17	30	× 367

CUBIC MEASURE.

	sol. yds.	ft.	in.			sol. yds.	ft.	in.	
73.	83	15	1000	× 35	75.	305	20	584	× 115
74.	215	17	896	× 72	76.	785	25	1425	× 327

WOOL WEIGHT.

	sacks	tods	lbs.			packs	lbs.	
77.	45	11	25	× 275	79.	875	215	× 489
78.	138	10	27	× 604	80.	1000	175	× 563

TIME.

	hrs.	min.	sec.			dys.	hrs.	min.	sec.	
81.	17	57	45	× 875	83.	119	11	30	17	× 1823
82.	23	45	59	× 904	84.	235	17	37	25	× 1987

COMPOUND DIVISION.

56. Ex. I. To divide £189 8s. 4d. by 8.

The £189 can be divided precisely as in Simple Division, and the remainder is £5. To divide this by 8, bring it to shillings; this gives 100 sh., and with the 8s. in the dividend = 108 sh.: dividing by 8, as in Short Division, we have 13s., and 4s. over. Bring this to pence; it = 48d., and with the 4d. in the dividend = 52d.: dividing by 8, the quotient is 6d., and the remainder 4d.: bring this to farthings; it = 16 farthings, which divided by 8 gives 2 farthings, or 1 halfpenny; and the complete quotient is £23 13s. 6½d.

	£	s.	d.
8)	189	8	4
	£23	13	6½

57. The same alterations that we described in Compound Addition, Subtraction, and Multiplication, are to be made here, when the compound quantity consists of any other than pounds, shillings, pence, and farthings. For instance, if we had for a dividend a quantity consisting of tons, cwts., qrs., &c.; then, after dividing the tons, we should have to reduce the remaining tons to cwts., and add in the cwts. already in the dividend; so also any remaining cwts. would have to be reduced to qrs.; the remaining qrs. to lbs.; and so on.

Hence, when we have to divide a compound quantity by a number under 12, we have the following

RULE. Divide the highest denomination as in Short Division; if there be a remainder, reduce it to the next name, and add to it the units there may be of this next name in the dividend, and divide again. Proceed in like manner through all the denominations, treating the remainders after each division, exactly as the first remainder. But if, after any division, there be no remainder, then divide, if possible, the units of the succeeding denomination; but if these units be not equal in number to the divisor, place a cipher underneath as quotient, and treat these units as a remainder to be reduced to the next denomination.

Exs. 36.

Divide

	£	s.	d.	
I.	88	2	6	by 3, 5, 7.
II.	268	3	$1\frac{1}{2}$	by 3, 5, 6, 10.
III.	517	11	0	by 4, 6, 8, 12.

TROY WEIGHT.

	lbs.	oz.	dwts.	grs.			lbs.	oz.	dwts.	grs.	
1.	18	9	15	16	÷ 5	4.	117	7	14	20	÷ 10
2.	175	11	7	21	÷ 7	5.	359	11	19	6	÷ 11
3.	308	9	16	5	÷ 9	6.	1827	10	0	12	÷ 12

AVOIRDUPOIS WEIGHT.

	cwts.	lbs.	oz.	drs.			tons	cwts	qrs.	lbs.	
7.	75	15	8	15	÷ 6	9.	45	11	3	20	÷ 8
8.	315	94	13	12	÷ 7	10.	117	9	2	17	÷ 9

58. When the divisor exceeds 12, but is a number which can be exactly formed by the multiplication of two numbers each less than 12, then we can divide by the two numbers successively. Thus, if the divisor be 27, or 9×3, we divide first by the 9 and then by the 3, as was shewn (26) in

Simple Division. Also, the two remainders must be formed into one, as was shewn in the same article. *

Ex. II. Divide £189 8s. 4d. by 27.

	£	s.	d.		
3)	189	8	4		
9)	63	2	$9\frac{1}{4}$	1	$\Big\}$ 22 rem^r †
	£7	0	$3\frac{1}{2}$	7	

Exs. 37. Find the required quotients in the following Examples.

	£	s.	d.				£	s.	d.		
1.	616	17	6	+	35	7.	893	17	11	+	45
2.	804	9	$4\frac{1}{4}$	+	90	8.	1356	8	$10\frac{1}{2}$	+	72
3.	1025	2	0	+	48	9.	4589	16	$8\frac{1}{4}$	+	99
4.	987	6	$4\frac{1}{2}$	+	56	10.	8764	14	$10\frac{1}{2}$	+	100
5.	594	18	9	+	63	11.	5783	19	$1\frac{3}{4}$	+	121
6.	1023	7	$6\frac{1}{4}$	+	36	12.	6897	11	$0\frac{1}{2}$	+	144

LONG MEASURE.

	mls.	fur.	po.	yds.				faths.	yds.	ft.	in.		
13.	15	5	30	3	+	27	16.	75	1	2	9	+	84
14.	48	6	35	2	+	35	17.	117	0	1	10	+	90
15.	96	7	13	4	+	81	18.	875	1	2	11	+	110

TIME.

	yrs.	wks.	dys.	hrs.				yrs.	dys.	hrs.	min.		
19.	75	43	3	15	+	36	22.	144	14	17	45	+	64
20.	96	51	6	17	+	49	23.	903	245	14	18	+	72
21.	107	18	4	11	+	55	24.	1000	75	19	30	+	121

59. But when the divisor exceeds 12, and cannot be broken up as in the last article, it is necessary to divide by a process similar to that of Simple Long Division; but as

* The learner will see hereafter that the best mode of forming the complete remainder will be—not to have farthings in the quotient, and a remainder besides; but to express as a fraction of a penny the whole of the quantities remaining after the pence in the quotient.

† The more correct form of the quotient, as described in the previous note, will be £7 0s. $3\frac{1}{2}$[illegible]d.

the only difference between Compound Long Division and Compound Short Division is, that in the former case the reducing of the remainders is performed on the paper, and in the latter case in the memory, we shall not write out a new rule for Long Division, but merely give one Example to shew the method of working.

Ex. III. To divide £3178 11s. 7½d. by 754.

```
        £     s.  d.   £  s.  d.
754)  3178   11  7½  (4   4  3¾
      3016
      ----
       162
        20
      ----
      3251 (4s.
      3016
      ----
       235
        12
      ----
      2827 (3d.
      2262
      ----
       565
         4
      ----
      2262 (3 farthings
      2262
      ====
```

Here we see the division after each reduction by 20, 12, and 4, to be precisely of the same kind as in Simple Division.

Exs. 38. Find the required quotients in the following Examples.

	£	s.	d.				£	s.	d.		
1.	45	18	9	÷	117	11.	1745	9	10½	÷	875
2.	236	17	6½	÷	235	12.	2834	10	7	÷	908
3.	435	9	8¼	÷	384	13.	3479	12	9¾	÷	1423
4.	1027	16	9	÷	379	14.	6842	14	3	÷	1785
5.	1847	11	7¾	÷	645	15.	999	8	5¼	÷	1827
6.	8923	14	6	÷	893	16.	10101	10	4	÷	3459
7.	11023	3	4½	÷	1001	17.	4586	16	6½	÷	6083
8.	8976	18	5	÷	986	18.	3897	13	8	÷	8756
9.	7643	15	4¼	÷	1237	19.	7854	11	10¾	÷	9802
10.	8962	14	11	÷	1461	20.	6923	7	11	÷	1143

SQUARE MEASURE.

	ac.	ro.	po.	yds.			sq. mls.	ac.	ro.	po.	
21.	1175	3	15	15	+ 870	24.	11485	480	3	17	+ 785
22.	207	2	20	17	+ 927	25.	9746	523	2	20	+ 983
23.	199	1	27	19	+ 1345	26.	10409	610	1	29	+ 1425

CUBIC MEASURE.

	sol. yds.	ft.	in.			sol. yds.	ft.	in.	
27.	8963	20	1000	+ 3024	29.	15684	19	897	+ 2568
28.	11429	15	1684	+ 4837	30.	83746	26	1432	+ 11984

31. If 721 persons earn £626 7s. 4½d., how much is that to each?
32. The expenses of a railway are £10299 1s. 8d. per annum; how much per day?
33. Find the price per ounce of a piece of gold weighing 375¼ oz., and costing £1548 18s. 9d.
34. A person owes £1000, and has only £758 6s. 8d.; how much can he pay for every pound which he owes?

60. There is one other kind of Examples which may be found in Compound Division, viz. when it is required to find how often one compound quantity is contained in another.

Ex. IV. How often is £3 15s. 3¼d. contained in £26 6s. 10¾d.?

Here we must reduce both quantities to the same name, farthings: and the question becomes one of Simple Division, viz., How often are 3613 farthings contained in 25291 farthings? and the result is, 7. But such questions are more properly treated under Reduction: and some examples of the kind will be found in the following pages.

REDUCTION.

PART II.

61. I will now give one instance of each kind of the more difficult Examples that involve Reduction and the Compound Rules.

Ex. I. How many times must I take a stride of 2 ft. 9 in., in walking 7 miles ?

This question is similar to the examples in (43) and (44), and in plain language means—How often is the quantity 2 ft. 9 in. contained in 7 miles, or in seven times 1760 yds. ?

Bringing both quantities into portions of 3 inches, I have the annexed work, where I multiply the 2 feet by 4, because there are in one foot four parts, each three inches; and the 12320 yards by 12, because there are in one yard 12 parts, each three inches.

	yds.
	1760
	7
ft. in. 2 9	12320 yds.
4	12
11 portions)	147840 portions
	13440 steps.

Ex. II. How many times will a coach wheel revolve, in going 175 miles, if its circumference be $15\frac{1}{2}$ feet ?

Comparing this question with the previous one, and putting the circumference of the wheel in place of the length of the man's step, the questions are seen to be of the same kind.

The work will be

```
                          miles.
                           175
                           1760
                          10500
                          1225
ft                        175
15½                       308000 yds.
 2                             6
31 half ft.          31) 1848000 hf. ft. (59612
                         155
                          298
                          279
                           190        Ans. 59612 times, and 28 half ft.
                           186             or 14 ft. remaining.
                            40
                            31
                             90
                             62
                             28
```

An example the opposite of the two last will be as follows.

Ex. III. A man takes 3744 strides in walking a distance of 1 m. 1516 yds.; what is the length of each step ?

I have here merely to divide the whole distance by the number of strides, by Compound Long Division, as follows :

```
            m.  yds.
            1  1516
            1760
            3276 yds.
               3
    3744)  9828 ft. (2 ft. 7½ in.
           7488
           2340
             12
          28080 (7 in.
          26208
           1872
              2
           3744 (1 hf. in.
           3744
```

The correctness of the answer of this last Example may be proved by working the following question.

How far will a man walk, if he takes 3744 strides, each 2 ft. $7\frac{1}{2}$ inches?

This question will of course be worked by Compound Multiplication, and the answer will be 1 m. 1516 yds.

Ex. IV. In 55 purses, each containing a guinea, a moidore, a sovereign, a crown, and a groat, how many pounds sterling?

I must here find the value of the whole of the coins in one purse, and then multiply the sum by 55. The work will be as here shown.

$$\begin{array}{rrrl}
\pounds & s. & d. & \\
1 & 1 & 0 & \\
1 & 7 & 0 & \\
1 & 0 & 0 & \\
 & 5 & 0 & \\
 & & 4 & \\
\hline
3 & 13 & 4 & \\
 & & 5 & \times 11 = 55. \\
\hline
18 & 6 & 8 & = 5 \text{ times.} \\
 & & 11 & \\
\hline
201 & 13 & 4 & = 55 \text{ times.} \\
\hline\hline
\end{array}$$

Ex. V. How many crowns, half-crowns, shillings, and groats, amount to £99 16s. 4d.; taking of each an equal number?

The sum of all these coins must be found, and I must then see, by division, how often that sum is contained in the £99 16s. 4d.; the quotient will be the required number of coins of each kind.

The sum of the coins is 8s. 10d.; since, therefore, my dividend is £99 16s. 4d., and my divisor 8s. 10d, I must reduce both to pence, and the whole work will be as follows;

```
 s. d.
 5  0
 2  6            £   s.  d.
 1  0            99  16  4
    4            20
 ----            -------
 8 10            1996 sh.
12                 12
 ----          -----------
106d.        )  23956d.  (226
 ====           212
                ---
                 275
                 212
                 ---
                  636
                  636          Ans. 226 of each.
                  ===
```

The question which is the reverse of this would be,

In 226 purses, each containing a crown, a half-crown, a shilling, and a groat, how many pounds? And this question is of the same kind as Ex. IV.

Ex. VI. I changed £27 14s. 6d. for pieces of 15d., 10d., 9d., and 4d., taking of each an equal number: how many of each had I?

This question will be found to be the same as V., if I put it in the following form:—How many coins of 15d., 10d., 9d., and 4d., amount to £27 14s. 6d., taking of each an equal number? The work is

```
  d.             £   s.  d.
 15              27  14  6
 10              20
  9             -----
  4              554 sh.
 ---              12
 38d.        ) -----------
 ====           6654d.  (175 pieces, and 4d. remain.
                38
                ---
                285
                266
                ---
                 194
                 190
                 ---
                   4
                 ===
```

Some of the Examples given below will involve various weights and measures; but they can all be worked upon the principles of the questions just shown. In the first

twelve Examples, two questions are given upon each of the above six Examples in order; but the remainder are mixed, and it must be left to the judgment of the learner to discover which among these six is the mode of working. And it will be a very good exercise for a beginner to endeavour in such questions to find out the reverse question, as I have done in Exs. (III.) and (V.)

Exs. 39.

1. The interval between the tollings of a bell is 7½ seconds, how many times will it be heard in 1 hour 45 min. 30 sec.?
2. I take a stride of 2 ft. 8½ in., but put my stick to the ground only every other step; how often will it touch the ground if I walk 17¼ miles?
3. The circumference of a wheel is 5¾ feet; how many times will it turn in 175¾ miles?
4. The piston of a steam engine travels 18 miles 396 yds. 1 ft. 2 in.; how many times must it have oscillated, if its stroke be 2 ft. 7 in.?
5. What is the circumference of a wheel which revolves 1570 times in 1 mile 28 yds. 2 in.?
6. A bridge, containing 75 arches and 76 piers, measures 1 furlong 156 yds. 2 ft.; what is the length of each arch, if each pier is half the length of an arch?
7. What amount will be obtained from 357 subscribers, each contributing a guinea, a crown, a half-crown, and a shilling?
8. 1175 casks contain each 3 gallons, 3 quarts, 3 pints, and 3 half-pints; how much do they all hold?
9. How many parcels of 1¾ lbs., 2½ lbs., and 10 lbs., can be obtained out of a cask of sugar weighing 7 cwt. 2 qrs. 11 lbs., taking of each an equal number?
10. A field of 68 acres 3 roods is divided into equal numbers of plots of 1 rood 25 poles, and of 2 roods 35 poles; how many will there be?
11. I exchanged 355 guineas, worth £1 6s. 6d. each, for one-pound notes, crowns, and groats, taking of each an equal number; how many of each had I; and what was their value?
12. An estate of 375 acres, and another of the same size, worth twice as much, was exchanged for equal numbers of allotments of 3 acres, of 2 roods, and of 25 perches, all of the former quality; how many portions should I receive in all?

13. There are 1000 subscribers to a charity, each giving a guinea, a pound, and a crown; and 525, each giving a half-crown, a shilling, and sixpence; how much is raised in all?

14. In the clothing of a regiment, 655 suits are made each containing 3 yds. 1 qr. 3 nls. of cloth, and 1 yd. 3 qrs. 1 nl. of lining; how many yards of material are required for the whole?

15. A certain vessel contains a hogshead, a barrel, a kilderkin, and a quart; how many such can be filled out of 30758 gallons?

16. From 3 cwt. 2 qrs. 14 lbs. I take away 1 lb. 1 oz. and 1 dram; how many times can I do this, and what will remain?

17. How many intervals of 3 minutes, 2 min., $\frac{1}{2}$ min., $\frac{1}{4}$ min., can be made out of a fortnight, and of each an equal number?

18. A man walks up a hill in 7 min. $35\frac{1}{2}$ sec., and down again in 5 min. 13 sec.; how many times can he repeat this in 7 hrs. 2 min. $40\frac{1}{2}$ sec.?

19. A piece of land is required, which can be divided into allotments of 3 roods, 2 roods, 1 rood, 35 perches, and 25 perches; there are to be 127 of each allotment; how many acres must the land contain?

20. A peal of 10 bells has 2700 changes rung in 3 hours, what is the average length of interval between the stroke of each bell, if all the bells are rung at every change?

Exs. 40.

MISCELLANEOUS EXAMPLES.

1. If a spoon weigh 15 dwts. 11 grs., how many dozen of such spoons can be formed out of 122 oz. 9 dwts. 1 gr.?

2. How many loaves of 4 lb., 2 lb., and $1\frac{1}{2}$ lb., and of each an equal number, can be made out of 12 sacks of flour, each weighing 240 lbs.?

3. Find the difference in hours between 3 yrs. 10 months, 3 wks. 6 dys., and 4 yrs. 9 months 2 wks. and 4 dys.

4. Find the age of a person whose pulse has beat 589,764,385 times, at the rate of 70 per minute.

5. *A* was born at 10 o'clock p.m., on Sept. 26, 1845; and *B* at 3 a.m., on April 15th, 1846; how much older in hours is *A* than *B*?

6. An estate costing £17897 6s. 4d. is divided into 5876 allotments; what is the value of each share?

7. The diameter of a crown is $1\frac{1}{2}$ inches; how many will it take to reach $11\frac{1}{2}$ miles?

8. Three thousand and eighty-nine subscribers contribute £3 13s. 4d. each, how much will they raise in all?
9. A piece of cloth 500 yds. 2 qrs. in length is cut up to form coats, each requiring 1 yd. 3 qrs. 2 nls.; how many coats can be obtained from it?
10. How many subscribers, of £2 11s. 7½d. each, will be required to purchase an estate worth £11953 15s. 4½d.?
11. A square whose side is 385 inches, is divided into oblongs, 7 inches by 5, how many will there be?
12. How many intervals of 5 minutes 35 seconds are in a leap year?
13. I exchange 4375 yds. for pieces of 3 qrs. 2 nls.; how many did I receive?
14. Find the number of cubic yards in 2,877,580 inches?
15. How often must the sum of 5s. 0½d., 7s. 7d., and £1 17s. 4½d., be repeated to make £200?
16. What would be the length of an acre of ground, if its breadth were 60½ yds.?
17. What is the amount of £3 4s. 4½d. repeated 800 times?
18. Find the difference between £34 15s. 9½d. + £78 7s. 6d. and £135 10s. 9d. − £84 17s. 10½d.
19. The number of solid feet or inches, in a block of wood, having all its sides oblong, is found by multiplying the length, breadth, and thickness; find the number of solid feet in a block 245 inches long, 39 broad, and 17 thick.
20. If 6½ millions of visitors entered the Crystal Palace in 26 weeks, what was the average attendance per day?
21. How many acres in a field 387 yds. long, and 275 yds. broad?
22. Sound travels about 1100 feet per second; what is the interval between the flash and the sound, when the storm is 4 miles off?
23. In 25 bales, each containing 24 pieces, and each piece 45½ yards, how many lengths of 4 yds. 2 qrs. 3 nls.?
24. If a spoon cost 7s. 9½d., how many dozen can be bought for £44 8s. 3d.?
25. A man who owes £2348, pays 12s. 9½d. for every pound which he owes; how much does he pay in all?
26. The cost of gilding is 4½d. per square inch; find the expense of gilding a box, of which the dimensions are 7 in., 9 in., and 15 inches.
27. A book requires 25½ sheets of paper; how large an edition can be printed from 184 reams 2 quires 7 sheets?
28. Fifteen hundred men contribute £1 3s. 6½d. each, and as many children half as much; how much money is raised?
29. How much per day is 1000 guineas per leap year?

30. If a pint contains 2728 barleycorns, how many will it take to fill a sack?

31. A man spends £155 5s. 7d. per year; how much will he lay by in 37 years, out of £200 per annum?

32. The divisor is £32 6s. 8d., the remainder 10s. 6d., and the quotient is 375; what is the dividend?

33. Among how many persons can I divide £1764 15s., so that they may each have £1 2s. $7\frac{1}{2}$d.?

34. If the sun's light comes to us in 8 min. 58 sec., and the distance is 95,000,000 miles, what is the rate of the light per second?

35. On a railway, the rails weigh 70 lbs. per yard, and the chairs, weighing 14 lbs. each, are placed at intervals of 18 inches, how many tons of iron are employed in making 20 miles of a double line of rails?

36. If 1 lb. Avoirdupois is equal to 14 oz. 11 dwt. 16 grs. Troy, how many lbs. Troy in 6 cwt.?

37. Thirty-five cheeses weigh 17 lbs. 3 oz. each; how many pieces of 4 oz., 6 oz., 1 lb., and $1\frac{1}{2}$ lbs., can be cut from them, taking of each an equal number?

38. Five bells of different tones are successively struck at intervals of 3 minutes, 2 min., 1 min., $\frac{1}{2}$ min., and $\frac{1}{4}$ min.; how many times could I hear the whole round of tones in 9 hrs. 27 minutes?

39. In the last question, which bell should I have last heard at the end of 4 hrs. 58 min.?

40. How many intervals of 3 min., $35\frac{3}{4}$ sec. in half a century?

41. A wall is $8\frac{1}{2}$ feet high, 236 feet long, and 17 inches thick; there are in it two doorways each 6 feet by 4 feet; how many bricks would be required for it, each containing 204 solid inches?

42. Divide £357 12s. 2d. among 3 men, 4 women, and 6 children, giving to each man twice as much as to a woman, and four times as much as to a child.

43. A mixture is made of 4 gallons at 3s. 9d., 5 gals. at 4s. 6d., and 11 gals. at 6s. 8d.; what is the value of a gallon of the mixture?

44. A court yard, 150 ft. square, is surrounded by a walk 24 feet broad; and a grassplot occupies the remainder; find the area of the walk and grassplot.

45. In a foot-race, *A* gains on *B* at the rate of 5 yds. in 1 min. 50 secs., how soon will he be half a mile a-head?

ON THE PRINCIPLES OF THE SIMPLE RULES.

1. What do you mean by Numeration? Give examples of its use?
2. Explain how numbers greater than 10 can be represented, though there are but ten different figures.
3. What do you mean by the term "digits?"
4. Express by signs the addition of the numbers 365, 4000, and 18, with the subtraction of 1728 and 496. Give the result.
5. Describe an Addition Table; and shew how by its aid you add together two quantities, one of which is more, and the other less, than 10.
6. Explain the common process of borrowing employed in Subtraction, and shew what is the correct mode of making allowance for it.
7. How do you prove a sum in Subtraction?
8. Write down in signs—"the sum of 17 and 8 is equal to the difference between 36 and 11."
9. Write down in signs—"the product of 18 and 12 is equal to the quotient of 2376 divided by 11."
10. Shew why you can multiply by 10, 100, 1000, &c. more easily than by any other numbers.
11. If the quotient, remainder, and dividend are known, how will you find a divisor? Construct such an example, and find the divisor.
12. Explain how a Multiplication Table is made.
13. Shew how it can be used as a Division Table.
14. If the floor of an oblong room were covered with square tiles, all of the same size, how would you find the number of them without counting them all?
15. If you knew how many bricks were used in paving a floor, and how many bricks the floor was in length, how would you find the number in the breadth?
16. Shew that Short Division is the same as Long Division, only that the work is performed mentally.
17. How do you *prove* a Division Sum?
18. What is the meaning of the terms *divisor*, *dividend*, and *quotient*?
19. Explain the process of forming the complete remainder, when you divide by a composite divisor. Ex. Divide 325 shillings among 72 people.
20. Shew how to divide by such numbers as 20, 300, 4000, &c., and explain the correctness of the remainder.

ON REDUCTION AND THE COMPOUND RULES.

1. What do you mean by *Compound* Rules?
2. What is meant by Tables of Weights and Measures? Write out "Long Measure."
3. In a Compound Addition sum, of money, what other numbers may be found besides the whole numbers found in the Simple Rules
4. Write down *one*-farthing or one-quarter, *two* farthings, *three* farthings, in figures.
5. How otherwise can you write two farthings, and by what other name would you then call it?
6. Explain, without working the questions, the mode of operation in the following cases:
   (1) Reduce 3 cwt. 3 qrs. $17\frac{1}{2}$ lbs. to half pounds.
   (2) Reduce 19275 farthings to pounds.
7. State the steps whereby you would find how many persons could receive each 3s. 4d. out of £20.
8. Explain the mode of multiplying any sum of pounds, shillings, and pence, by 3256.
9. Construct a question, which requires reduction by both processes of multiplication and division; and work it.
10. Explain the mode of multiplying by $5\frac{1}{2}$ and $30\frac{1}{4}$.
11. Shew how to divide by the same quantities.
12. Form an example the opposite to question 11 in Exs. 40.

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DECIMALS.

OUR numeration has thus far been carried from *right to left*, beginning with units, and proceeding tenfold *higher* each step, to *tens, hundreds, thousands*, &c. It may, however, be continued from *left to right*, below units, proceeding therefore tenfold *lower* each step; to *tenths, hundredths, thousandths*. And in order to mark the place of the units, a dot, or point (·), is placed after the figure which denotes units: so that all the figures to the left of the point represent either units, or numbers higher than units; and all to the right represent numbers lower than units. And any numbers, as 6666, (which we have read in words, 6 thousands, 6 hundreds, 6 tens, and 6,) may be continued any distance to the right, thus becoming 6666·666 &c.; and the part added will be read 6 tenths, 6 hundredths, 6 thousandths; or the whole three may be read in the lowest name, namely, 666 *thousandths*, just as any ordinary number, 666, would, when expressed in terms of its lowest name, *i. e.* of units, be 666 *units*.

The numbers below units, which have thus been added after the point, are called DECIMALS; they are said to occupy so many *decimal places;* and the point is called the decimal point.

The four operations of Addition, Subtraction, Multiplication, and Division, can be applied to decimals, as well as to common numbers, if the following four rules be observed.

ADDITION.

RULE. Arrange the numbers to be added, so that all the decimal points in them shall be in one vertical line; add, as in whole numbers, and in the result place the point below the row of points in the given numbers.

Exs. 1. ·765 + 3·418 + 96 + ·004 + 276·01 + 45·00835.
2. 7·101 + 61·703 + ·00827 + 1·76 + ·75 + ·0683 + 9.
3. ·005 + 700·1 + 6·523 + ·00415 + 76·56 + ·25 + ·0085.
4. ·000812 + 260 + 74·32 + ·00242 + 5 + ·000846.
5. ·674 + ·08342 + 2060 + 17·17 + 209·209 + ·8.
6. 43·1 + 163·002 + 72·183 + ·0084169 + 145·03 + ·8.

SUBTRACTION.

RULE. Place the less number below the greater*, so that the decimal points in them shall be in one vertical line; subtract, as in whole numbers, and in the result place the point below the points in the given numbers.

If the lower line should extend to the right beyond the upper, ciphers must be supplied in the upper line.

Exs. 1.	·4086382 − ·2709.	5.	4·107 − 3·1072685.
2.	416·2 − 208·008476.	6.	57·623 − 18·41095.
3.	705 − ·7684003.	7.	374·4160 − ·83927.
4.	·000418 − ·00003624.	8.	15·1064 − ·09846.

* That quantity is always the larger in which the number to the left of the point, *i. e.* the whole number, is the larger, whatever be the length of the number following the point. Thus of 3·275 and 1·28964, the former is the larger, because the 3 is greater than the 1. But if there be no whole numbers, the larger quantity will be found by placing the numbers, with their points one under the other, and observing which contains the largest number in the highest denomination; thus ·0375 is greater than ·02046, because 3 hundredths is greater than 2 hundredths; and ·175 is greater than ·0843, because 1 tenth is greater than 8 hundredths.

MULTIPLICATION.

RULE. Place the multiplier under the multiplicand just as in whole numbers; multiply as usual without taking any notice of the decimal points, and in the result place the point as many places from the right hand extremity, as there are decimal places in both the multiplier and multiplicand.

For example; if there are *three* places in the multiplicand and *two* in the multiplier, there must be *five* in the product; that is, the point must be put *five* places from the right-hand extremity.

These places should be counted from right to left; and if there be not as many figures in the product as the rule requires, ciphers must be added to the left, to make up the proper number.

Exs. 1.	·074109 × 23·3084.	5.	760·83 × ·00984.
2.	·00846 × ·00324.	6.	·750604 × ·089.
3.	562·73 × ·00008.	7.	·001019 × ·08074.
4.	19·835 × ·0084.	8.	800·416 × ·0087.

DIVISION.

RULE. Count the number of decimal places in the divisor; count also the same number of places in the dividend, beginning after the point, and make a mark (`) cutting off all the remaining figures of the dividend. If there be not as many places in the dividend as in the divisor, add ciphers till the number is made up, and then make the mark.

Divide, as in whole numbers; all the figures in the quotient obtained by using the figures in the dividend to the

left of the mark will be whole numbers. When these figures have been used, place the decimal point in the quotient, the remaining figures obtained by division will be decimals. About six places of decimals should be taken unless the division stops, by there being no remainder, before six places are obtained.

Ex. I. Divide 285·5172 by 45·8.

```
45·8) 285·5‘172 (6·234
      2748
       1071
        916
        1557
        1374
         1832
         1832
```

Here, since there is one decimal place in the divisor, the mark (‘) is placed after the first figure beyond the point; and as the divisor is contained 6 times in 2855, there is an integer 6 in the quotient, and the remaining figures are decimals.

It may happen, as in Ex. II. below, that the divisor is larger than that part of the dividend which is to the left of the mark; in this case there will be no whole numbers in the quotient. And if, even when the mark is passed, the divisor is still too large, a cipher must be placed in the quotient, after the point, for every figure that is used beyond that mark, until the dividend becomes large enough to contain the divisor, and therefore till a figure, other than 0, appears in the quotient.

Ex. II.

```
341·9) 2·56425 (·0075
       23933
        17095
        17095
```

Here *two* figures, 6 and 4, after the mark had to be used, before the figure 7 was obtained in the quotient; hence *two ciphers had to be* placed after the point in the quotient.

Exs. 1. ·0764183÷45.
2. 410·176÷·00824.
3. 18÷·00846.
4. 24196÷8·438.
5. ·0160828÷·00248.
6. 27841÷·08463.
7. ·004187÷·000632.
8. ·0031824÷73·8.
9. 83417÷1·843.
10. ·0080706÷·00821.
11. ·076384÷9·186.
12. ·300187458÷834.

ANSWERS TO EXS. IN DECIMALS.

Addition.

1. 421·20535.
2. 80·39057.
3. 783·45065.
4. 339·324078.
5. 2287·93642.
6. 424·1234169.

Subtraction.

1. ·1377382.
2. 208·191524.
3. 704·2315997.
4. ·00038176.
5. ·9997315.
6. 39·21205.
7. 373·57673.
8. 15·00794.

Multiplication.

1. 1·7273622156.
2. ·0000274104.
3. ·0450184.
4. ·166614.
5. 7·4865672.
6. ·066803756.
7. ·00008227406.
8. 6·9636192.

Division.

1. ·00169818$\dot{4}$.
2. 49778·64077 &c.
3. 2127·65957 &c.
4. 2867·5041479.
5. 6·485.
6. 328973·1773 &c.
7. 6·625.
8. ·0000431̇219$\dot{5}$.
9. 45261·53011 &c.
10. ·9830207064 &c.
11. ·008315262 &c.
12. ·000359937.

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A

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OF

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BY THE

REV. FREDERICK CALDER, M.A.

HEAD MASTER OF THE GRAMMAR SCHOOL, CHESTERFIELD.

NEW EDITION.

LONDON:
LONGMAN, GREEN, LONGMAN, AND ROBERTS.
1861.

Cambridge:
PRINTED BY C. J. CLAY, M.A.
AT THE UNIVERSITY PRESS.

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CONTENTS.

THE FOLLOWING SIGNS MUST BE VERY WELL KNOWN BEFORE THIS BOOK IS READ.

$+$ *plus*, placed between two numbers, shows that they are to be added together.

$-$ *minus*, between two numbers, shows that the latter number is to be subtracted from the former.

$\times$ *into*, or *multiplied by*, between two numbers, shows that the two are to be multiplied together.

$\div$ *by*, or *divided by*, between two numbers, shows that the former is to be divided by the latter.

$=$ *equal to*, between two quantities, shows that the quantities on each side of it are equal.

$\therefore$ stands for *therefore*.

$\because$ *because*, or *since*.

$>$ *greater than*, placed between two quantities, shows that the former is greater than the latter.

$<$ *less than*, placed between two quantities, shows that the former is less than the latter.

$\sim$ placed between two quantities, shows that the smaller of the two is to be subtracted from the larger.

Note.—The figures enclosed in parentheses, as (8) in p. 4, line 22, refer to previous articles in the work.

ARITHMETIC.

BEFORE commencing the consideration of Fractions, it will be necessary to explain certain properties of numbers, which enter very much into the treatment of fractions.

MEASURES AND MULTIPLES.

1. Of two numbers, the larger is said to be a *multiple* of the smaller, when the larger can be divided exactly, *i. e.* without remainder, by the smaller; and the smaller number is said to be a *measure* of the larger. Thus, since 12 can be divided exactly by 4, 12 is called a multiple of 4, and 4 a measure of 12. So, 16 is a multiple of 8, and 8 a measure of 16.

2. Also, one number is said to be a *common* multiple of two or more numbers, when it can be divided exactly by the two or more numbers. Thus, 24 is a common multiple of 6, 4, and 2, because it can be divided exactly by those numbers; and 12 is the *least* common multiple of 6, 4, and 2, because it is the *least* number that can be divided exactly by those three divisors.

3. A number is said to be a common measure of two or more numbers when it will exactly divide those numbers; and the greatest number which will so divide them is called

their greatest common measure. Thus, 2, 4, 6, 12, are common measures of 24 and 36; and 12 is their greatest common measure.

We shall for the future write G. C. M. for greatest common measure, and L. C. M. for least common multiple.

4. A number is called a *prime* number when it cannot be divided exactly by any number greater than 1. Thus, 3, 5, 7, are prime numbers, because they have no divisor greater than 1; we also call such numbers *primes.*

5. Two or more numbers are said to be prime to one another, when they cannot *all* be divided exactly by any number greater than 1. Thus, 7, 8, 9, are prime to each other: but such numbers are not necessarily themselves prime numbers; for 8 can be divided by 2 and 4; and 9 can be divided by 3; but 8 and 9 have no divisor common to both.

6. All numbers not prime are called *composite;* because they are composed of two or more prime numbers multiplied together. Thus, 6 is a composite number, being formed by the multiplication of the prime numbers 2 and 3.

7. The prime numbers which, when multiplied together, form a composite number, are called *factors* of that number. Thus, 2, 2, 2, 3, are factors of 24, because $2 \times 2 \times 2 \times 3 = 24$. So the factors of 36 are 2, 2, 3, 3, for $2 \times 2 \times 3 \times 3 = 36$.

8. To break up a composite number into its prime factors, divide it by the smallest prime which will divide it without remainder; and continue dividing by prime divisors until the last quotient is 1. The number will be found equal to the product of all the divisors—that is, all the divisors are its factors.

Ex. To break 120 into prime factors:

$$\begin{array}{r|r} 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$\therefore$ 2, 2, 2, 3, 5, are its prime factors, or, $120 = 2 \times 2 \times 2 \times 3 \times 5$.

When, therefore, I say—break any number, as 120, into its factors, I mean, exhibit it in the above form. This form is called an equation: thus, $7 \times 5 = 35$ is an equation; and 7×5 is called the left-hand *side*, and 35 the right-hand *side*, of the equation.

Exs. 1. Resolve into prime factors

1. 1050.	3. 1485.	5. 1820.	7. 64.	9. 2310.	11. 5724.
2. 2625.	4. 1155.	6. 4802.	8. 4389.	10. 6342.	12. 1168.

Obs. If any one number be divisible by a second, we must have all the factors of the second number among the factors of the first: thus, if 36 be divisible by 12, all the factors of 12 will be among the factors of 36, or 36 must contain all the factors of 12.

9. Also, if I wish to divide 35 by 5, (since 35 is the same as 7×5), I may divide 7×5 by 5; and since I know that the quotient is 7, I can now observe that that quotient is found by taking away the divisor 5 out of the product 7×5: so also in the equation $7 \times 3 \times 5 = 105$, to divide the 105 by 5, take away the 5 out of the factors on the left-hand side, and there remains 7×3, or 21, as quotient; to divide by 7, take away 7, and 3×5, or 15, is quotient; or, we can divide by the product of two factors at once: thus, to divide by 15, take away 3×5, and the quotient is 7. This is a very quick method of dividing a composite number by any one or more of its factors, especially if the number be large.

1—2

Thus, since $2520 = 2 \times 2 \times 2 \times 7 \times 3 \times 3 \times 5$;
$\text{or} = 8 \times 7 \times 9 \times 5$;

therefore, if I wish to divide 2520 by any measure of it, as 24, I take away 24, or $2 \times 2 \times 2 \times 3$, from its factors, and the product of the remaining factors, $7 \times 3 \times 5 = 105$, gives the quotient. To divide 2520 by 63, I remove 7×9, and the quotient $= 8 \times 5 = 40$.

LEAST COMMON MULTIPLE.

10. It is required to find the L. C. M. of 2, 3, 5, 6, 9, 10, 12, 18, 20.

Now, of the above numbers, we observe that 2, 3, 6, and 9 are divisors, or measures, of 18; every number, therefore, which is a multiple of 18, will also be a multiple of 2, 3, 6, 9. So likewise, every multiple of 20 will be a multiple of 5 and 10; therefore, every multiple of 18 and of 20 will be a multiple of 2, 3, 6, 9, 5 and 10. If, then, I find the L. C. M. of the three remaining numbers 12, 18, and 20, that multiple will be the L. C. M. of 2, 3, 5, 6, 9, 10, 12, 18, 20. Now, $12 \times 18 \times 20$, or 4320 gives *one* common multiple of 12, 18, and 20, since this product is plainly divisible by 12, 18, and 20; but this is not the *least* C. M.

11. To find the L. C. M., break the numbers 12, 18, 20, into their prime factors, as in (8), and place a comma between each set of factors, thus:

12,	18,	20,
$2 \times 2 \times 3$,	$2 \times 3 \times 3$,	$2 \times 2 \times 5$.

Now, by (8 OBS.), in order that the L. C. M. may contain 12, it must contain all the factors of 12; ∴ it must contain 2, 2, 3,
so, to contain 18, ... 2, 3, 3,
and to contain 20, ... 2, 2, 5;

that is, in the required L. C. M. I want two twos, two threes, and one 5; but in the product of $12 \times 18 \times 20$, or of $2 \times 2 \times 3 \times 2 \times 3 \times 3 \times 2 \times 2 \times 5$, I have five twos and three threes; therefore I have three twos and one 3 which I do not require; and, omitting these, the remaining factors, multiplied together, give $2 \times 2 \times 3 \times 3 \times 5 = 180$; where it may be seen upon trial that this row of factors contains all that is necessary for 12, 18, and 20, and no more; and therefore the product of these factors is the least number divisible by 12, 18, and 20, or is their L. C. M.

12. The whole work should be written out as follows, where the mark (–) is placed over every number which is afterwards to be omitted:—

$$\bar{2}, \bar{3}, \bar{5}, \bar{6}, \bar{9}, \overline{10}, 12, 18, 20.$$

$$2 \times 2 \times \bar{3}, \qquad \bar{2} \times 3 \times 3, \qquad \bar{2} \times \bar{2} \times 5,$$

$$\therefore \text{L. C. M.} = 4 \times 9 \times 5 = 20 \times 9 = 180 \ldots\ldots\ldots\ldots (A).$$

OBS. It is worth notice that, in multiplying the numbers which are to form the L. C. M., a little dexterity will generally enable the pupil to find the product mentally; for instance, the numbers very often contain one quantity ending in 5, and one even number; and these can be more readily multiplied than any others, as is seen in line (A).

Ex. II. To find the L. C. M. of 7, 16, 32, 21, 56, 42.

$$\bar{7}, \overline{16}, 32, \overline{21}, 56, 42,$$

$$2 \times 2 \times 2 \times 2 \times 2, \qquad \bar{2} \times \bar{2} \times \bar{2} \times \bar{7}, \qquad \bar{2} \times 3 \times 7,$$

$$\therefore \text{L. C. M.} = 32 \times 21 = 672.$$

In this example, I first reject 7 and 21, because 42 contains them; next I reject 16, because 32 contains it; then, of the prime factors, I preserve five twos for the number 32, and reject the others: I preserve one 7, which is required for 56 and 42, and one 3 for 42. In preserving the five twos, it is better to keep them all together, rather than have some in one set of factors, and some in another.

Exs. 2. Find the L. C. M. of

1. 2, 3, 4, 8.
2. 3, 5, 9, 16, 20.
3. 8, 11, 5, 35, 21.
4. 7, 28, 35, 42, 63.
5. 4, 5, 8, 24, 40, 120.
6. 5, 7, 9, 12, 15.
7. 13, 14, 56, 63, 72.
8. 1, 2, 3, 4, 5, 6, 7, 8, 9.
9. 12, 33, 55, 27, 18.
10. 7, 11, 13, 17.
11. 8, 9, 10, 11, 12, 15.
12. 10, 14, 21, 28, 35.
13. 18, 20, 24, 36, 48.
14. 27, 36, 45, 42, 16.

GREATEST COMMON MEASURE

13. We have seen in (3) that the G. C. M. of two or more numbers is the largest divisor which can exactly be contained in those numbers.

Where the numbers are not large, this G. C. M. may be found by breaking them into their prime factors; and if any factors are contained in all the numbers, the product of these will be the G. C. M.

Thus, to find the G. C. M. of 36, 27, 144; breaking up into factors, we have

$$\begin{array}{ccc} 36, & 27, & 144, \\ 2\times2\times3\times3, & 3\times3\times3, & 2\times2\times2\times2\times3\times3, \end{array}$$

Here it is plain that in these sets of factors two threes and no other number are common to all; that is, 3×3, or 9, is common to all the numbers 36, 27, 144,—and is therefore their G. C. M.

14. But where the numbers are large, and they cannot be easily broken into factors, the following rule is to be *used:*—

Take two of the proposed numbers, and divide the greater by the less: if there be a remainder, make that remainder a new divisor, and take the former divisor as a new dividend, and continue this process until there be no remainder: *the last divisor will be the* G. C. M.

If there be a third number, go through the same work with this third number, and the G. C. M. of the other two; then, as before, the last divisor will be the G. C. M. And the same process must be continued if there be 4, 5, &c., or any amount of numbers, of which we have to find the G. C. M.

We will try this rule upon the three numbers taken above, viz. 36, 27, and 144.

```
27) 36 (1                    9) 144 (16
    27                           9
    --                          --
     9) 27 (3                   54
        27                      54
        --                      --
```

∴ 9 is the G. C. M. of 27 and 36: and 9 is the G. C. M. of 27, 36, and 144, as was shewn above.

Ex. II. Find the G. C. M. of 324, 456, 728.

```
324) 456 (1                       12) 728 (60
     324                              720
     ---                              ---
     132) 324 (2                        8) 12 (1
          264                               8
          ---                              --
           60) 132 (2                       4) 8 (2
               120                             8
               ---                            --
                12) 60 (5
                    60
                    --
```

∴ 12 is the G. C. M. of 324, 456; and 4 is the G. C. M. of 324, 456, 728.

15. If the last divisor be 1, we then learn that the numbers have no common divisor greater than 1; *i. e.* they are prime to each other.

OBS. Even when the divisors are less than 12, it is better to divide by long, rather than short division, because the remainders are thereby placed in the most convenient

situation for continuing the process. This rule for finding the G. C. M. cannot be proved true without the use of algebra; but the *method* of proof is shewn below*.

16. When we have to find the L. C. M. or G. C. M. of any numbers which are not large, we may often, after a little experience, *see* the answer, without going through the work.

Thus, if it were required to find the L. C. M. of 3, 4, 6, and 8, a pupil would soon learn that 24 was the required number. The easiest method of performing this operation, without writing, is to multiply in one's head the largest of the numbers given, by 2, 3, 4, and so on, until a number be found which will contain all the other numbers. So, in 3, 4, 6, and 8, if I try $2 \times 8 = 16$, it will not hold the 6; but $3 \times 8 = 24$ will hold the 6 as well as the 3 and 4, and therefore is the required L. C. M.

17. Again, to find the G. C. M. of 4, 8, 12: it is easy to *see* that no number greater than 4 can go in 4, 8, and 12, or that 4 is their G. C. M.

When we can thus *see* the answer without working, we are said to find the L. C. M. or G. C. M. by *inspection*.

---

* When any number measures two others, it will be found to measure any number composed either of the *sum* of any multiples of those numbers, or their *difference*.

For example, since 6 is a common measure of 18 and 24, it will be a measure of $(3 \times 18) + (4 \times 24)$ which $= 150$; or of $(7 \times 18) - (3 \times 24)$ which $= 54$. This rule can be *proved* true by algebra, and may be seen to be correct in the particular Ex. just given.

Making use of this fact, and referring to the first portion of Ex. II., I observe that every C. M. of 324 and 456 must be a measure of $(456 - 324)$ or of 132; also that every C. M. of 132 and 324 must be a measure of $(132 + 324)$, or of 456; hence the common measures of 324 and 456 are precisely the same as those of 132 and 324; and therefore the *greatest* C. M. of 324 and 456 is the same as the *greatest* C. M. of 132 and 324. Again, I see that 60 and 132 are derived from 132 and 324, just as these latter numbers were derived from 324 and 456; hence it is plain that the G. C. M. of 132 and 324 is also the G. C. M. of 60 and 132, and still further, that it is the same as the G. C. M. of 12 and 60; hence the G. C. M. of the original numbers 324 and 456 is shewn to be the same as that of 12 and 60; but the G. C. M. of 12 and 60 is 12; hence 12 is the G. C. M. of 324 and 456.

Exs. 3. Find the G. C. M. of

1.	348 and 390.	5.	1836 and 1845.
2.	510 „ 595.	6.	4775 „ 10959.
3.	413 „ 343.	7.	715, 781, and 1067.
4.	217 „ 643.	8.	189, 216, „ 729.

Shew that 327 and 529, also that 189 and 727 are prime to each other.

OBS.—THE FOLLOWING ABBREVIATIONS WILL SOMETIMES BE USED.

Den^r. . . .	*for* Denominator.	Rem^r . . .	*for* Remainder.
Num^r . . .	„ Numerator.	Diff^e . . .	„ Difference.
Fr^n	„ Fraction.	Comp^d . . .	„ Compound.
Mult^n. . . .	„ Multiplication.	Red^d . . .	„ Reduced.
Add^n . . .	„ Addition.	Com. . . .	„ Common.
Sub^n . . .	„ Subtraction.	Quan^y. . .	„ Quantity.
Div^n.	„ Division.	Imp^r . . .	„ Improper.
Quot^t . . .	„ Quotient.	Frac^l . . .	„ Fractional.
Ex.	„ Example.	Art..	„ Article.

FRACTIONS.

18. **DEF.** By the term *Unit*, we are to understand a single article of any kind, as 1 inch, 1 yard, 1 penny, 1 ounce, &c.

DEF. *Unity* is merely another name for the figure 1.

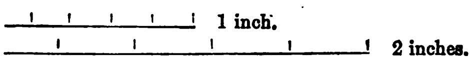
19. A fraction is a part or parts of a number, or quantity, supposed to be broken into any number of equal portions. If, then, the *unit* be divided into 4, 5, or 6 equal parts, one of these parts will be called one-fourth, one-fifth, or one-sixth, and is thus written—$\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$. So also, if any *two* of these parts be taken, the quantities thus taken will be called *two*-fourths, *two*-fifths, and *two*-sixths, *i. e.* of the unit or of *one*, and be written $\frac{2}{4}$, $\frac{2}{5}$, $\frac{2}{6}$.

20. The number below the line, which shews into how many parts the *unit* was broken, is called the *denominator*, because it expresses the denomination, or kind of parts, as *fourths*, *fifths*, *sixths*. The upper number, which *enumerates*, or counts, how many parts are taken, is called the *numerator*. Thus, in the fraction $\frac{2}{5}$, 5 is the denominator, 2 the numerator.

Obs. Such a fraction as I have been describing is called a *Vulgar Fraction*. We shall afterwards find that *Vulgar* Fractions can be expressed in another form, and are then called *Decimal* Fractions, or more commonly *Decimals*.

21. Again, any fraction, as $\frac{2}{5}$, may represent other quantities, besides being two-fifths of *one*, as we have just explained.

Fig. 1.



For if we divide the quantity 1, as 1 inch, into 5 equal parts (see Fig. 1), and also divide the quantity 2 into 5 equal parts, then one of the *latter* parts will be twice as great as one of the *former*; or *two* of the former and *one* of the latter have the same value. Now, the former or smaller is one-fifth of 1; and the latter or larger is one-fifth of 2; and, since two of the former = one of the latter, therefore two-fifths of 1 = one-fifth of 2, or $\frac{2}{5}$ of 1 = $\frac{1}{5}$ of 2; and both these quantities are expressed by the fraction $\frac{2}{5}$. Similarly, $\frac{6}{7}$ means *six*-sevenths of *one*, or *one*-seventh of *six*; $\frac{3}{4}$*s.* means three-fourths of 1 shilling, or one-fourth of 3 shillings, each of which will, on trial, be found to be 9*d.*

22. From what has been said, it will be seen that the den^{r} of a fraction shews into how many parts the unit is divided; since, then, $\frac{6}{7}$ is proved to mean one-seventh of 6,

that is, it is equal to 6 divided by 7; we hence see that a number placed as a den^r, underneath any other number as a num^r, shews that this den^r is to be taken as a divisor of the num^r; thus, $\frac{15}{7}$ implies that 15 is to be divided by 7: but as long as I do not work out the div^n, that division is said only to be *expressed*. Thus again, in the fraction $\frac{23}{7}$, we understand that 23 *is to be divided* by 7.

We shall hereafter see more clearly than now, that if 23 is divided by 7, the fraction $\frac{23}{7}$ may be called the *quotient*.—(See Appendix, Art. *Fractional Quotient.*)

23. A fraction is called a *proper* fraction when the num^r is less than the den^r; thus, $\frac{3}{8}$, $\frac{4}{8}$, are called *proper* fractions, because they really represent a part or parts of a unit, and are less than the whole unit.

24. A fraction whose num^r is equal to, or is greater than the denom^r, is called an *improper* fraction: Thus, $\frac{5}{5}$, $\frac{12}{7}$, are called *improper* fractions, because they are not in reality *parts* of a unit broken up—*i. e.* are not less than the whole unit—but they are either one complete unit, or more than one.

25. Any whole number may be made to appear as an improper fraction with any required den^r, by multiplying and dividing the whole number by that denominator. Thus, $3=\frac{3\times5}{5}=\frac{15}{5}$: if the required den^r be 7, $3=\frac{21}{7}$.

26. A mixed number is one formed of a whole number and a fraction, as $3\frac{2}{5}$, which is read three and two-fifths—that is, three units, and two-fifths of another unit; just as 3*s*. 7*d*. means 3 shillings and $\frac{7}{12}$ of another shilling, and might be written $3\frac{7}{12}$*s*., where the unit is one shilling.

27. A fraction consisting of two or more frac

connected by the word *of* placed between them, is called a *compound* fraction; as $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{8}{9}$: and a *complex* fraction is one in which either the num^r or den^r, or both, are fractions; as

$$\frac{2\frac{3}{4}}{4},\qquad \frac{3}{7\frac{1}{2}},\qquad \frac{1\frac{1}{3}}{3\frac{2}{5}},\qquad \frac{\frac{8}{9}\text{ of }1\frac{2}{3}}{7\frac{1}{5}},\text{ \&c.}$$

This last fraction is thus read: eight-ninths of one-and-two-thirds, divided by seven-and-one-fifth.

28. To multiply a fraction by any whole number, we multiply the num^r and retain the same den^r.

Thus, if it be required to multiply $\frac{5}{12}$ by 4,—just as 4 times 5 *pence* = 20 *pence*, so 4 times *5-twelfths* = 20-*twelfths*, or $\frac{20}{12}$;

$$\therefore\ \frac{5}{12}\times 4 = \frac{20}{12},\text{ which} = \frac{5\times 4}{12};\text{ hence,}$$

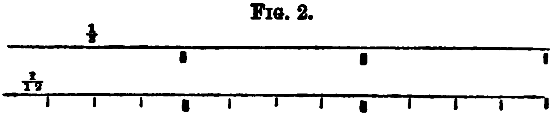
To multiply a fraction by a whole number, the num^r is multiplied by the number, and the den^r is not altered.

29. Again, to multiply a fraction by any whole number, *if possible*, divide the den^r by this multiplier, and leave the numerator unaltered. By this rule we should have

$$\frac{5}{12}\times 4 = \frac{5}{12\div 4} = \frac{5}{3}.$$

To prove this, let any unit be divided—1st, into 3 equal parts, and 2ndly, into 12 equal parts (see Fig. 2); then

FIG. 2.



each of the larger parts is called $\frac{1}{3}$, and each of the smaller $\frac{1}{12}$. Now, one of the *thirds* is four times as large as one of

the *twelfths*, therefore 5 of such thirds are 4 times as large as 5 of the twelfths;

$$\therefore \frac{5}{3} = 4 \text{ times } \frac{5}{12} = 4 \times \frac{5}{12};$$

or, in the former shape, $\frac{5}{12} \times 4 = \frac{5}{3}$, which $= \frac{5}{12 \div 4}$; hence,

In multiplying by a whole number, the den^r must be divided, and the num^r left unaltered.

I subjoin a specimen of the form in which these Exs. should be worked.

$$\frac{5}{18} \times 6 = \frac{5}{18 \div 6} = \frac{5}{3}.$$

$$\frac{5}{18} \times 8 = \frac{5 \times 8}{18} = \frac{40}{18}.$$

Exs. 4.

I. Multiply $\frac{5}{8}$ by 2, 3, 4, 5, 6, 7, 8, 9, successively.

II. Multiply $\frac{12}{24}$ by 3, 5, 6, 8, 9, 12, successively.

30. To divide a fraction by any whole number, divide the num^r, if possible, and keep the same den^r.

Thus, if it be required to divide $\frac{12}{13}$ by 4, just as 12 *pence* divided by 4 would give a quot^t 3 *pence*, so 12 *thirteenths* divided by 4 would give a quot^t 3 *thirteenths;*

$$\therefore \frac{12}{13} \div 4 = \frac{3}{13} \text{ which} = \frac{12 \div 4}{13}; \text{ hence,}$$

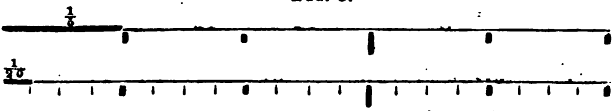
The num^r is divided by the given divisor, and the den^r unaltered.

But if we cannot divide the num^r of the fr^n by the whole number, then we must multiply the den^r by this divisor. By this method $\frac{3}{5} \div 4$ would $= \frac{3}{5 \times 4} = \frac{3}{20}$.

To prove this, observe in the fractions $\frac{3}{5}$ and $\frac{3}{20}$, that in the first fr^n the unit is divided into 5 equal parts,

Fig. 3,) and therefore each $=\frac{1}{5}$; and, in the second, the same unit is divided into 20 equal parts, and therefore each $=\frac{1}{20}$:

FIG. 3.



now it is plain, from Fig. 3, that *one*-fifth, when divided by 4, becomes *one*-twentieth; so *three*-fifths divided by 4 become *three*-twentieths;

$$\therefore \ \frac{3}{5} \div 4 = \frac{3}{20}, \text{ which} = \frac{3}{5 \times 4}; \text{ hence,}$$

We multiply the den^{r} by the given divisor, and leave the num^{r} unaltered.

The following are specimens of the mode of working.

$$\frac{9}{10} \div 3 = \frac{9 \div 3}{10} = \frac{3}{10}.$$

$$\frac{9}{10} \div 6 = \frac{9}{10 \times 6} = \frac{9}{60}.$$

OBS. Since it rarely happens that we can divide any num^{r} or den^{r} by any chance number, we therefore, in multiplying a fr^{n}, have, as a general rule, to multiply the num^{r}; and in dividing a fr^{n}, to multiply the den^{r}.

Exs. 4.

III. Divide $\frac{8}{9}$ by 2, 3, 4, 5, 6, 7, 8, 9, successively.

IV. Divide $\frac{36}{49}$ by 2, 3, 4, 7, 8, 9, 18, successively.

31. We have now shown that to multiply a fr^{n} by a whole number, we multiply the *num^{r}*, and to divide a fr^{n} we multiply the *den^{r}*; therefore if we multiply both num^{r} and den^{r} by the same quan^{y}, we shall have both multiplied and divided the fr^{n} by the same number; but since to multiply a quan^{y} by any number, and then to divide by the same, leaves

the original quan^r unaltered, therefore to multiply both num^r and den^r of a fr^n will not alter its value:

$$\text{Thus, } \frac{3}{5}=\frac{3\times4}{5\times4}=\frac{12}{20};$$

and by observing Fig. 3, we shall see that since 3 of the large divisions, or *fifths*, = 12 of the small divisions, or *twentieths*, we have by inspection the same result, viz. $\frac{3}{5}=\frac{12}{20}$.

32. So also, since by dividing the num^r we *divide* the whole fr^n, and by dividing the den^r we *multiply* the same fr^n; therefore, if we divide both num^r and den^r by the same quantity, the fr^n will remain unaltered in value:

$$\text{Thus, } \frac{12}{20}=\frac{12\div4}{20\div4}=\frac{3}{5};$$

by a process opposite to that in the last Art.

TO REDUCE A FRACTION TO ITS LOWEST TERMS.

33. We have shown that both num^r and den^r of a fr^n can be divided by the same number without altering the value of the fr^n; and if the num^r and den^r cannot be exactly divided by any whole number, the fr^n is said to be in its lowest terms, because it is expressed in the lowest possible numbers.

Thus, $\frac{3}{4}$, $\frac{5}{6}$ are fr^{ns} in their lowest terms, because 3 and 4 in the first fr^n, and 5 and 6 in the second fr^n, have no common divisor, or C. M.

But $\frac{8}{12}$ is not in its lowest terms, for both num^r and den^r can be divided by 4; and the fr^n will then be $\frac{2}{3}$. So also, in fr^n $\frac{15}{25}$, 5 is G. C. M. of num^r and den^r; and the fr^n in its lowest terms $=\frac{3}{5}$.

It is plain, therefore, that if any fr^n be not in its lowest

terms, and we wish so to reduce it, both num^r and den^r must be divided by the greatest possible divisor, *i. e.* by their G. C. M.

In working a sum of this kind we should express the operation thus:

$$\frac{15}{25}=\frac{15\div 5}{25\div 5}=\frac{3}{5},$$

but when, in future, it is necessary to use this operation, it may be thus written, $\frac{15}{25}=\frac{3}{5}$, the intermediate step being omitted.

34. When the num^r and den^r of a fr^n are both small numbers, the G. C. M. may be often found by *inspection*, or sometimes *some* C. M. may be found, though not the greatest; and, by dividing by it, the fr^n may *partly* be reduced; and then a C. M. of the fr^n so reduced may be found, and a second division performed, till we reduce the fr^n, if not to the lowest, at any rate to lower terms.

Thus, $\frac{35}{245}=\frac{7}{49}=\frac{1}{7}$ is now reduced to lowest terms.

It is worth notice, that every number ending in 5 or 0 must have 5 for a divisor, and every even number can be divided by 2.

But if mere inspection will not tell us the G. C. M. of num^r and den^r, it must be found by the method already given (14) for finding the G. C. M. of two given numbers.

Exs. 5. Reduce to their lowest terms

1. $\frac{24}{40}$	3. $\frac{275}{425}$	5. $\frac{1197}{2471}$	7. $\frac{985}{1379}$	9. $\frac{459}{1560}$	11. $\frac{343}{539}$
2. $\frac{85}{153}$	4. $\frac{1368}{3096}$	6. $\frac{2241}{2700}$	8. $\frac{5705}{6559}$	10. $\frac{1111}{9090}$	12. $\frac{891}{549}$

TO REDUCE AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER.

35. Ex. To reduce $\frac{17}{5}$ to a whole or mixed number. We have before shown (22) that this fr^n expresses that 17 is to

be divided by 5; now, if the 17 were exactly divisible by 5, the quotient would be a whole number; but if not, as in this case, then since 15 is the largest multiple of 5 below 17, we can divide the 15 by 5, and the quotient is 3; but the division of the remaining 2 by the 5 cannot be *performed:* it must therefore be *understood,* by placing the divisor 5 underneath the 2 as a den^r, and thus $\frac{2}{5}$ may be said to be the quot^t of 2 when divided by 5: the whole quotient will, therefore, be 3 and $\frac{2}{5}$; and the operation may be thus written—

$$\frac{17}{5}=\frac{15+2}{5}=\frac{15}{5}+\frac{2}{5}=3+\frac{2}{5},$$

or, as it is generally written, $=3\frac{2}{5}$.

Also, since 5-fifths $=1$, therefore 15-fifths $=3$, and 17-fifths must $=3$ and 2-fifths, or $3\frac{2}{5}$, as just shown.

36. In writing the whole operation as fully as has been done above, it is necessary to find what is the rem^r after the num^r has been divided by the den^r. It must then be placed as the rem^r 2 has been placed in the above Ex., and the 15 is of course found by subtracting this rem^r from the whole num^r. Where the num^r and den^r are small, this rem^r may be found without working any div^n sum on the paper; but if the numbers are large, a div^n sum must be worked before the above operation can be shown.

Thus: Ex. 2. Reduce $\frac{327}{19}$ to a whole or mixed number.

$$\begin{array}{l} 19\,)\,327\,(17\frac{4}{19} \\ \underline{19} \\ 137 \\ \underline{133} \\ \underline{4} \\ 19 \end{array}$$

Here, the rem^r being 4, the largest multiple of 19 below 327 is 323, which contains 19, 17 times,

therefore we have $\frac{327}{19} = \frac{323+4}{19} = \frac{323}{19} + \frac{4}{19} = 17\frac{4}{19}$;

or, as usually written, $\frac{327}{19} = 17\frac{4}{19}$.

Exs. 6. Reduce to whole or mixed numbers

1. $\frac{94}{7}$	3. $\frac{288}{15}$	5. $\frac{12211}{113}$	7. $\frac{10001}{99}$	9. $\frac{1759}{176}$	11. $\frac{1749}{55}$
2. $\frac{82}{11}$	4. $\frac{603}{28}$	6. $\frac{5576}{17}$	8. $\frac{9999}{100}$	10. $\frac{8431}{844}$	12. $\frac{4001}{87}$

TO REDUCE A MIXED NUMBER TO AN IMPROPER FRACTION.

37. Ex. To reduce $3\frac{2}{5}$ to an improper fr^{n}. That we may explain this operation properly, we must observe, that in order to express two or more numbers in one sum, these must all be of one kind or denomination; thus, in a simple addition sum, we add units to units, tens to tens, &c.; and in a compound addition sum we add together like quantities, as pounds to pounds, shillings to shillings, &c.

If, therefore, I have to express in one sum 3 units and 2 fifths, these two quantities must be reduced to the same kind. Now, I cannot reduce the fifths to a whole number, because two-fifths are less than unity; I must, therefore, reduce the 3 units to fifths, that is, I must express 3 as a fr^{n} with a den^{r} 5.

Thus, by (25) $3 = \frac{3 \times 5}{5} = \frac{15}{5}$; adding then the 15-fifths to the two-fifths, I have the sum 17-fifths, or $\frac{17}{5}$.

The whole operation may be thus expressed:

$$3\frac{2}{5} = \frac{3 \times 5 + 2}{5} = \frac{15+2}{5} = \frac{17}{5}.$$

38. It will be seen that this Ex. is exactly the reverse of that in (35); and we may observe, that the num^{r} of the improper fr^{n}, viz. 17, is obtained by multiplying the whole

number 3 by the den^r 5, and adding the former num^r 2 : thus, in working the above Ex., I should say 5 times 3 are 15 and 2 are 17.

When, then, we have in future Ex^s to reduce a mixed number, $3\frac{2}{5}$, to an improper fr^n, we should merely write $3\frac{2}{5} = \frac{17}{5}$, where all the intermediate work can be performed mentally, when the numbers are not large. We could thus write $9\frac{7}{12} = \frac{115}{12}$, because 12 times 9 are 108 and 7 are 115.

Exs. 7. Reduce to improper fractions

1. $1\frac{6}{7}$	3. $13\frac{2}{13}$	5. $130\frac{43}{114}$	7. $175\frac{1}{175}$	9. $145\frac{10}{111}$
2. $4\frac{1}{3}$	4. $276\frac{5}{29}$	6. $100\frac{161}{193}$	8. $45\frac{121}{343}$	10. $77\frac{7}{90}$

TO REDUCE FRACTIONS TO A COMMON DENOMINATOR.

39. It has been shown (37) that numbers, whether whole or fractional, cannot be added together, without being reduced to the same denom^n or kind.

We must, therefore, show how to reduce fractions to a common den^r; and then they will be of the same denomination.

$$\text{Ex.} \quad \frac{3}{4}, \ \frac{5}{6}, \ \frac{7}{8}, \ \frac{9}{10}.$$

Now, in order that the fractions, when reduced, may still consist of as small numbers as possible, we shall always find their *least* common den^r.

Also, since we can alter the form of fractions only by multiplying or dividing both num^r and den^r, we must therefore multiply these present den^{rs}, each one by some number, so that they all shall be changed into the new den^r; and for this purpose the new den^r must be divisible by all these den^{rs}, and therefore must be their L. C. M.

Working according to (12) we find the L. C. M. of 4, 6, 8, 10, to be 120. We now choose such multipliers as shall make 4, 6, 8, 10, become 120: these will be found by dividing 120 by the 4, 6, 8, 10; and are 30, 20, 15, 12.

Now, we know (31) that if we multiply the den^r of a fr^n by any number, we must also multiply the num^r by the same number, or else the fr^n will be altered in value. We therefore work thus:

$$\frac{3}{4}=\frac{3\times 30}{4\times 30}=\frac{90}{120}$$
$$\frac{5}{6}=\frac{5\times 20}{6\times 20}=\frac{100}{120}$$
$$\frac{7}{8}=\frac{7\times 15}{8\times 15}=\frac{105}{120}$$
$$\frac{9}{10}=\frac{9\times 12}{10\times 12}=\frac{108}{120}$$

and these fractions may be thus written, $\frac{90,\ 100,\ 105,\ 108}{120}$ showing at once that they have the same den^r.

40. In working sums of this kind, we should have to write down the process of finding the L. C. M. of all the den^rs, unless we could see it by *inspection*, which cannot be often done, when the numbers are large.

If it can be done, we must then merely write as the first line of the work L. C. M. = 120, or any number that it may chance to be.

Exs. 8. Reduce to their least common denominator

1. $\frac{13}{18}, \frac{4}{5}, \frac{7}{9}, \frac{11}{40}, \frac{15}{16}$	4. $\frac{3}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}$	7. $\frac{7}{15}, \frac{27}{35}, \frac{5}{72}, \frac{51}{63}, \frac{29}{84}$
2. $\frac{1}{7}, \frac{3}{11}, \frac{5}{21}, \frac{9}{56}, \frac{3}{14}$	5. $\frac{2}{3}, \frac{4}{5}, \frac{7}{8}, \frac{11}{12}, \frac{83}{120}$	8. $\frac{14}{17}, \frac{1}{11}, \frac{11}{18}, \frac{17}{33}$
3. $\frac{11}{15}, \frac{13}{16}, \frac{14}{17}, \frac{17}{18}$	6. $\frac{3}{4}, \frac{13}{20}, \frac{11}{18}, \frac{27}{70}, \frac{41}{90}$	9. $\frac{11}{12}, \frac{59}{60}, \frac{17}{24}, \frac{1}{120}$

TO ADD TOGETHER ANY NUMBER OF FRACTIONS.

41. These may be all proper fractions, or some of them may be proper, some improper, and some mixed numbers.

We have already seen that in order to add fractions together, we must reduce them to a com. den^r. Now, this has been done in the last two Art^s; and therefore we have only to conclude the operation in the last Ex. by performing the operation of adding the fractions so prepared. Now we observe, that whatever number of quantities of any kind are to be added together, the *kind* or denomination is not thereby changed: thus, 7 *months* added to 5 *months* would be 12 *months;* so, 4-*twelfths* + 3-*twelfths* = 7-*twelfths*, which, expressed in figures, gives $\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$; hence it is plain that we have only to add the num^{rs} together, and retain the *same* den^r; therefore, so doing in the last Ex., we have

$$\text{sum of the fr}^{\text{ns}} = \frac{90+100+105+108}{120}=\frac{403}{120}$$
$$=3\tfrac{43}{120} \text{ by (35).}$$

If the proper fr^n $\frac{43}{120}$ had not been in its lowest terms, it must have been reduced to that state before the sum could be called complete.

42. If some of the fr^{ns} to be added together be mixed numbers, we must first add the frac^l parts precisely as above; and to this sum then add the amount of the whole numbers. Thus, if the Ex. which we have just worked had been as follows, $3\frac{3}{4}+\frac{5}{6}+6\frac{7}{8}+15\frac{9}{10}$ (where the insertion of the whole numbers 3, 6, and 15, is the only alteration), we should then have, as before,

$$\text{sum of the fractional parts} = 3\tfrac{43}{120};$$

therefore, adding in the whole numbers, we have the whole sum $= 3+6+15+3\frac{43}{120}=27\frac{43}{120}$.

We will now work a second example completely.

Find the sum of $5\frac{2}{3}+1\frac{3}{4}+\frac{4}{7}+4\frac{5}{6}+\frac{7}{8}$.

$$\overline{3},\ \overline{4},\ 7,\ 6,\ 8,$$
$$7,\ \overline{2}\times 3,\ \ 2\times 2\times 2,$$
$$\text{L. C. M.}=7\times 3\times 8=21\times 8=168.$$

$$\frac{2}{3}=\frac{2\times 56}{3\times 56}=\frac{112}{168}$$
$$\frac{3}{4}=\frac{3\times 42}{4\times 42}=\frac{126}{168}$$
$$\frac{4}{7}=\frac{4\times 24}{7\times 24}=\frac{96}{168}$$
$$\frac{5}{6}=\frac{5\times 28}{6\times 28}=\frac{140}{168}$$
$$\frac{7}{8}=\frac{7\times 21}{8\times 21}=\frac{147}{168}$$

therefore, sum of frac[l] parts $=\dfrac{112+126+96+140+147}{168}$

$=\dfrac{621}{168}=3\frac{117}{168}=3\frac{39}{56}$ [in lowest terms]

and whole sum $=5+1+4+3\frac{39}{56}=13\frac{39}{56}$.

Obs. If there be any improper fractions in the proposed example, they may be reduced to mixed numbers before we begin to reduce to L. C. D.; or, if they are not very large, we may proceed with them as with the proper fractions.

Exs. 9. Find the value of

1. $\frac{3}{4}+\frac{4}{5}+\frac{5}{12}+\frac{11}{16}$
2. $\frac{7}{9}+\frac{5}{8}+\frac{11}{50}+\frac{14}{27}$
3. $\frac{5}{9}+\frac{7}{8}+\frac{11}{16}+\frac{18}{20}+\frac{1}{2}$
4. $\frac{3}{5}+\frac{17}{18}+\frac{4}{25}+\frac{7}{27}$
5. $10\frac{3}{8}+1\frac{5}{12}+\frac{7}{10}+4\frac{13}{16}$
6. $17+1\frac{3}{11}+\frac{5}{7}+13\frac{15}{28}$
7. $5\frac{7}{11}+3\frac{8}{9}+\frac{15}{22}+17\frac{8}{35}+1\frac{7}{10}$
8. $7\frac{1}{12}+8\frac{1}{13}+9\frac{1}{14}+10\frac{1}{15}$

SUBTRACTION.

43. The examples under this head will be of different kinds.

1st. When there are two fr[ns], both proper, as $\frac{3}{10}-\frac{1}{4}$, I.

2nd. When there are two fr[ns], one or both of which are mixed numbers, as $7\frac{3}{4}-3\frac{2}{5}$, II.; $8\frac{2}{5}-2\frac{3}{7}$, III.; $8-2\frac{5}{7}$, IV.

If there be any examples containing improper fr[ns], such examples may either be worked as I.; or, by reducing the improper fr[ns] to mixed numbers, we bring them under II., III., or IV.

The difference between II., III., and IV. consists in this, that

in II. the former fracl part $\frac{3}{4}$ is > the latter $\frac{3}{5}$;
in III. ,, ,, ,, $\frac{2}{5}$ is < ,, $\frac{3}{7}$; (A)
in IV. ,, ,, ,, is 0, and $\therefore < \frac{4}{7}$.

44. Under the head of Subtraction will also come examples in which there are more than two fractions; and wherein we have to find the difference between the sum of one set of fractions, and the sum of another set, as in the following

$$\text{Ex. } 5\tfrac{2}{3} + 4\tfrac{3}{8} - \tfrac{1}{2} + 3\tfrac{5}{6} - 7\tfrac{3}{7};$$

wherein the signs + and − show that I am to take both $\frac{1}{2}$ and $7\frac{3}{7}$, that is, the sum of $\frac{1}{2}$ and $7\frac{3}{7}$, from the sum of $5\frac{2}{3} + 4\frac{3}{8} + 3\frac{5}{6}$.

DEF. If any numbers, either whole or frac[l], are joined together by one or more of the signs +, −, &c., the whole set of numbers thus connected is often termed an *expression*.

45. Just as it has been shown that fractions cannot be added together until they are reduced to a C. D., so it will appear that one fr[n] cannot be subtracted from another till the two fr[ns] are reduced to a C. D. Thus, if I had to take 3 pence from 1 shilling, I must bring the 1*s.* to pence, and then take 3*d.* from 12*d.*, obtaining a rem[r] 9*d.*

Taking, then, Ex. I. $\frac{3}{10} - \frac{1}{4}$, we may see, by *inspection*, that L.C.D. $= 20$;

$$\text{therefore } \frac{3}{10} = \frac{3 \times 2}{10 \times 2} = \frac{6}{20}; \ \frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20};$$

$$\text{and } \quad \frac{3}{10} - \frac{1}{4} = \frac{6-5}{20} = \frac{1}{20}. \qquad \text{(B)}$$

When the L. C. D. is not large, say under 150, a pupil will, with a little experience, soon be able at once to write down the line (B), without working the two previous operations on paper.

46. It was said in line (A) $\frac{2}{5} < \frac{3}{7}$; a beginner cannot see this at once; but he may do so very readily—thus: we already know that no two quantities can be compared, to see which is the larger, unless they be of the same kind; so, two fr^ns cannot be compared unless reduced to a C. D. *Any* C. D. will, however, do for the purpose of merely telling which of the two fr^ns is the larger; now the product of the two den^rs is always *one* C. D., but not always the *least;* therefore since 35 is a C. D. of $\frac{2}{5}$ and $\frac{3}{7}$,

$$\frac{2}{5}-\frac{3}{7}=\frac{2\times 7-5\times 3}{5\times 7}=\frac{14-15}{35}; \quad \text{(C)}$$

$$\text{and since } 14<15, \text{ therefore } \frac{14}{35}<\frac{15}{35}, \text{ or } \frac{2}{5}<\frac{3}{7}.$$

Now this result may be very rapidly obtained without any work on paper; for if we observe line (C), we notice that the numbers 14 and 15, which numbers alone I wish to compare, are found by multiplying the num^r of the 1st fr^n by the den^r of the 2nd, and the den^r of the 1st by the num^r of the 2nd. Thus, beginning with the num^r of the 1st, I say, *mentally,* $2 \times 7 = 14$: and again, $3 \times 5 = 15$; therefore, since the first product is less than the second, so also the first fr^n is less than the second.

Taking one or two more Exs. of this process—

1. To compare $\frac{3}{7}$ and $\frac{2}{9}$: I say, $3 \times 9 = 27$; $7 \times 2 = 14$; therefore the former fr^n is the larger.

2. To compare $\frac{3}{8}$ and $\frac{7}{12}$: I say, $3 \times 12 = 36$; $8 \times 7 = 56$; therefore the latter fr^n is the larger.

It will be noticed that the C. D. to which we have just now mentally reduced $\frac{3}{8}$ and $\frac{7}{12}$ is 96; but when we come to the *actual sub^n*, we shall not use *this* C. D., but the L. C. D. 24

47. In Exs. II., III., IV., we may bring the mixed numbers into improper fractions, and work them as in Ex. I.; but since, when the numbers are large, the work becomes heavy, it is better generally to employ the same process that we use in Comp[d] Sub[n]. Take as patterns the three following:

A		B		C	
s.	*d.*	*s.*	*d.*	*s.*	*d.*
18	9	18	2	18	0
15	3	15	5	15	5
3	6	2	9	2	7

Now, in these Exs. it will be seen, that where the number of pence in the upper line exceeds that in the lower line, as in (A), we can at once subtract the 3*d.* from the 9*d.*, and the 15*s.* from the 18*s.*; giving a complete rem[r] 3*s.* 6*d.* So in Ex. II., $7\frac{3}{4} - 3\frac{2}{5}$, since $\frac{3}{4} > \frac{2}{5}$, therefore I can take the $\frac{2}{5}$ from the $\frac{3}{4}$, and the 3 from the 7.

The difference of the frac[l] parts $= \dfrac{3}{4} - \dfrac{2}{5} =$ (when reduced to L. C. D.) $\dfrac{15-8}{20} = \dfrac{7}{20}$; and diff[e] of the whole numbers $= 7 - 3 = 4$; therefore, difference of the complete fractions

$$= 4 + \tfrac{7}{20},$$
$$= 4\tfrac{7}{20}.$$

The following Ex. is written as it should be worked.

Find the value of $9\frac{3}{8} - 5\frac{1}{5}$.

$$\text{L. C. D.} = 40.$$
$$\therefore \frac{3}{8} - \frac{1}{5} = \frac{15-8}{40} = \frac{7}{40}.$$
$$\text{And } 9 - 5 = 4.$$
$$\therefore 9\tfrac{3}{8} - 5\tfrac{1}{5} = 4\tfrac{7}{40}.$$

48. But in (B) and (C) the number of pence in the lower line is larger than in the upper, and in both cases I must borrow 1*s.* or 12*d.* from the 18*s.* in the upper line, and

add this $12d.$ to the pence in the upper line, if there be any; and when I have sub[d] the $5d.$ from the $14d.$ in (B), leaving rem[r] $9d.$, and from $12d.$ in (C), leaving rem[r] $7d.$, I subtract $15s.$ from $17s.$, (not $18s.$, because I have just borrowed $1s.$ from the $18s.$) and in both cases I have a rem[r] $2s.$; therefore the whole rem[r] in (B) is $2s.\ 9d.$, and in (C) is $2s.\ 7d.$

I now apply this process to Exs. III. and IV., viz. $8\frac{2}{5}-2\frac{3}{7}$, III.; and $8-2\frac{5}{7}$, IV. As, then, in the Comp[d] Sub[n] I borrowed $1s.$ from the higher den[n] $18s.$, and added it to the pence in the upper row, if there were any, so I borrow 1 from the 8, and add it to the former fr[n], if there be any; thus in III. I add the 1 to the $\frac{2}{5}$, and then subtracting $\frac{3}{7}$, I say—

$$1\frac{2}{5}-\frac{3}{7}=\frac{7}{5}-\frac{3}{7}=\frac{49-15}{35}=\frac{34}{35},$$

then, taking the 8 as 7, I have $7-2=5$; therefore, combining the results of the two subtractions, I have

$$8\tfrac{2}{5}-2\tfrac{3}{7}=5+\tfrac{34}{35}=5\tfrac{34}{35}.$$

If, however, it is desired to have the result in the form of an improper fraction, as will be found most convenient in Exs. which involve Mult[n], we may bring the two quantities to improper fractions, as mentioned in (47); thus

$$8\tfrac{2}{5}-2\tfrac{3}{7}=\frac{42}{5}-\frac{17}{7}=\frac{294-85}{35}=\frac{209}{35}.$$

The following Ex. is written as it should be worked.

Find the value of $17\frac{1}{2}-11\frac{5}{6}$.

$$\frac{1}{2}<\frac{5}{6}\ \therefore\ \text{borrowing 1, I have }1\frac{1}{2}-\frac{5}{6}=\frac{3}{2}-\frac{5}{6}=\frac{9-5}{6}=\frac{4}{6}=\frac{2}{3}.$$

$$\text{And }16-11=5.$$

$$\therefore\ 17\tfrac{1}{2}-11\tfrac{5}{6}=5\tfrac{2}{3}.$$

49. In Ex. IV., $8-2\frac{5}{7}$, I borrow 1 from the 8, but as I have no former fr[n] to which I may add this 1, as in Ex. III., therefore subtracting the second fr[n] $\frac{5}{7}$ from the 1, I have

$$1-\frac{5}{7}=(\text{when red}^{\text{d}}\text{ to L. C. D.})\ \frac{7}{7}-\frac{5}{7}=\frac{7-5}{7}=\frac{2}{7};\ \text{(D)}$$

and taking 7 instead of 8, as before, I have $7-2=5$; therefore, I should work as follows—

$$1-\frac{5}{7}=\frac{7}{7}-\frac{5}{7}=\frac{2}{7}.$$

$$\text{And } 7-2=5.$$

$$\therefore\ 8-2\tfrac{5}{7}=5\tfrac{2}{7}.$$

50. In the line (D) I may observe that when the number 1 is expressed as a fr^{n}, its num^{r} is made the same as the den^{r} of the fr^{n} which I have to subtract: thus, 7 is the new num^{r}, and is also the den^{r} of the original fr^{n}; whether, therefore, I subtract the smaller num^{r} from the new num^{r} or from the den^{r} of the smaller fr^{n} I obtain the same result, viz. a rem^{r} 2; and I might say at once $1-\frac{5}{7}=\frac{2}{7}$, where the 2 has been found by subtracting the num^{r} 5 from its own den^{r} 7. Working thus, I should have written the whole Ex. as follows:

$$8-2\tfrac{5}{7}=(7-2)+(1-\tfrac{5}{7})=5\tfrac{2}{7},$$

only that in common use I should not write the figures enclosed in the two parentheses.

51. Lastly: when there are more than two fractions, as in

Ex. v. $5\frac{2}{9}+4\frac{3}{8}-\frac{1}{2}+3\frac{5}{6}-7\frac{3}{7}$.

Here, omitting for the present the whole numbers, I have to subtract the sum of $\frac{1}{2}$ and $\frac{3}{7}$ from the sum of $\frac{2}{9}+\frac{3}{8}+\frac{5}{6}$. Since the two fr^{ns} to be subtracted are together less than the three from which I have to subtract them, no difficulty will occur, and I proceed as in an Ex. in Addition, taking care to arrange the fr^{ns} to be subtracted last:

$$\frac{2}{9}+\frac{3}{8}+\frac{5}{6}-\frac{1}{2}-\frac{3}{7};$$

$$9,\ 8,\ 6,\ \bar{2},\ 7.$$

$$3\times3,\quad 2\times2\times2,\quad \bar{2}\times\bar{3},\quad 7.$$

$$\text{L. C. D.}=9\times8\times7=72\times7=504.$$

$$\frac{2}{9}=\frac{2\times56}{9\times56}=\frac{112}{504}\qquad \text{See Art. (9).}$$

$$\frac{3}{8}=\frac{3\times63}{8\times63}=\frac{189}{504}$$

$$\frac{5}{6}=\frac{5\times84}{6\times84}=\frac{420}{504}$$

$$\frac{1}{2}=\frac{1\times252}{2\times252}=\frac{252}{504}$$

$$\frac{3}{7}=\frac{3\times72}{7\times72}=\frac{216}{504}$$

$$\therefore \frac{2}{9}+\frac{3}{8}+\frac{5}{6}-\frac{1}{2}-\frac{3}{7}=\frac{112+189+420-252-216}{504}$$

$$=\frac{721-468}{504}=\frac{253}{504}$$

therefore, bringing in the whole numbers with their proper signs,

$$\text{the whole expression}=5+4+3-7+\frac{253}{504}=12-7+\frac{253}{504}$$

$$=5\tfrac{253}{504}.$$

52. We will take one more example of this kind.

Ex. VI. $3\frac{4}{5}-4\frac{1}{2}+7\frac{2}{7}-\frac{3}{4}.$

Now here $\frac{1}{2}+\frac{3}{4}$ are $>\frac{4}{5}+\frac{2}{7}$; but since this cannot be seen by mere *inspection*, I proceed as in Ex. V., as though I did not know this, and provide for the difficulty at the proper time. Placing the frns to be subtracted last, I have

$$\frac{4}{5}+\frac{2}{7}-\frac{1}{2}-\frac{3}{4}$$

$$5,\ 7,\ \bar{2},\ 4,$$

$$\text{L. C. D.}=5\times7\times4=140.$$

$$\frac{4}{5}=\frac{4\times28}{5\times28}=\frac{112}{140}$$

$$\frac{2}{7}=\frac{2\times20}{7\times20}=\frac{40}{140}$$

$$\frac{1}{2}=\frac{1\times70}{2\times70}=\frac{70}{140}$$

$$\frac{3}{4}=\frac{3\times35}{4\times35}=\frac{105}{140}$$

$$\text{therefore } \frac{4}{5}+\frac{2}{7}-\frac{1}{2}-\frac{3}{4}=\frac{112+40-70-105}{140}$$

$$=\frac{152-175}{140}. \qquad \text{(E)}$$

Since, then, $175>152$, it now appears that I ought to have borrowed 1; but rather than begin the work again, I proceed thus: from the $\frac{152}{140}$ in line (E) I take as much of the $\frac{175}{140}$ as I can, viz. $\frac{152}{140}$, and there will remain $\frac{23}{140}$, which cannot be subtracted, and before which I place the sign $(-)$, to shew that it has yet to be subtracted; but if I now borrow 1, I can subtract this $\frac{23}{140}$ from the 1, and remember to count the first $(+)$ whole number in Ex. VI. (*i. e.* the 3) as one less than it now stands. The whole operation may be thus written, recommencing with the line before (E):

$$\frac{4}{5}+\frac{2}{7}-\frac{1}{2}-\frac{3}{4}=\frac{112+40-70-105}{140}$$

$$=\frac{152-175}{140}=\frac{-23}{140}.$$

$$\text{Therefore borrowing the 1, } 1-\frac{23}{140}=\frac{140}{140}-\frac{23}{140}=\frac{117}{140},$$

and writing 2 for 3 in Ex. VI., and inserting the whole numbers with their proper signs,

$$\text{the whole expression }=2+7-4+\frac{117}{140}=9-4+\frac{117}{140}$$

$$=5\tfrac{117}{140}.$$

Exs. 10. Exhibit in one term each of the following expressions:

1.	$\frac{5}{7}-\frac{2}{3}$	7.	$5\frac{3}{8}-4\frac{2}{7}$	13.	$38-27\frac{11}{16}$
2.	$\frac{22}{45}-\frac{5}{18}$	8.	$17\frac{9}{10}-11\frac{5}{12}$	14.	$3\frac{2}{5}+4\frac{3}{4}-7\frac{1}{2}$
3.	$\frac{29}{56}-\frac{5}{14}$	9.	$8\frac{1}{2}-4\frac{3}{5}$	15.	$18\frac{1}{4}-7\frac{11}{12}+\frac{5}{9}$
4.	$\frac{15}{16}-\frac{11}{48}$	10.	$7\frac{3}{4}-3\frac{11}{12}$	16.	$7\frac{1}{2}+4\frac{5}{6}-\frac{7}{12}+3\frac{1}{10}-5\frac{11}{15}$
5.	$\frac{25}{36}-\frac{13}{64}$	11.	$8-3\frac{7}{9}$	17.	$8\frac{4}{5}-6\frac{3}{7}-\frac{8}{9}+15\frac{1}{3}-3\frac{4}{27}$
6.	$\frac{18}{19}-\frac{17}{18}$	12.	$17-\frac{1}{2}$	18.	$18\frac{1}{3}-5\frac{2}{7}+4\frac{8}{15}-\frac{2}{5}-\frac{5}{21}$

MULTIPLICATION.

53. We will show, as we proceed, that the operation of multiplication in fractions may be expressed in two ways, and that the word *of* and the sign (×) placed between fr^{ns}, have the same meaning: thus, we have to prove that

$\frac{5}{7}$ of $\frac{2}{3}$ of $3\frac{3}{4}$ and $\frac{5}{7} \times \frac{2}{3} \times 3\frac{3}{4}$ have the same value. (F)

The pupil must here clearly call to mind the method of multiplying and dividing fractions by whole numbers. (31)

I will first begin with a pair of fractions, as $\frac{5}{7} \times \frac{2}{3}$.

It has been proved that $\frac{2}{3}$ means *one-third* of 2; therefore I have now to multiply $\frac{5}{7}$ by one-third of 2: hence the product ought to be one-third of the product found by multiplying by 2 alone: *i. e.* if I multiply by 2, and then take one-third of that product, I shall obtain the correct result.

Multiplying $\frac{5}{7}$ by 2, I have $\frac{5}{7} \times 2 = \frac{5 \times 2}{7}$;

and since to take one-third of any quantity is to divide it by 3, I divide this last fr^n by 3, and have

$$\frac{5}{7} \times \frac{2}{3} = \frac{5 \times 2}{7} \div 3 = \frac{5 \times 2}{7 \times 3}. \qquad (G)$$

Commencing now with the expression $\frac{5}{7}$ of $\frac{2}{3}$: we know that $\frac{5}{7}$ means 5 times one-seventh; therefore, if I have to take five-sevenths of any quantity, I first take *one*-seventh of it, and then multiply the quot^t so obtained, by 5: *i. e.* to take $\frac{5}{7}$ of $\frac{2}{3}$, I must divide the $\frac{2}{3}$ by 7, and multiply it by 5: hence, according to our rules of mult^n and div^n of fractions by whole numbers, I multiply the den^r of the $\frac{2}{3}$ by 7, and the num^r by 5, and have

$$\frac{5}{7} \text{ of } \frac{2}{3} = \frac{5 \times 2}{7 \times 3}. \qquad (H)$$

Comparing (G) and (H), I now perceive that

$$\frac{5}{7} \text{ of } \frac{2}{3} = \frac{5}{7} \times \frac{2}{3}.$$

54. The following explanation may, perhaps, be intelligible to some pupils:

In multiplying a quantity, whether whole or fractional, by a whole number, I repeat that quantity as many times as there are units in the multiplier:

Thus, 7×5, or 7 times 5, means that I am to repeat the 5 as many times as there are units in 7;

So, $7 \times \frac{5}{7}$ shows that I am to repeat $\frac{5}{7}$ as often as there are units in 7;

Also, $1 \times \frac{5}{7}$ shows that I am to repeat $\frac{5}{7}$ as often as there are units in 1, *i. e.* 1 time, or once:

Therefore, $\frac{2}{3} \times \frac{5}{7}$ ought to mean, that I am to repeat $\frac{5}{7}$ as often as there are units in $\frac{2}{3}$.

Now, $\frac{2}{3}$ does not contain 1 unit,—it only contains $\frac{2}{3}$ of 1 unit; therefore, $\frac{2}{3} \times \frac{5}{7}$ is $\frac{5}{7}$ repeated $\frac{2}{3}$ of once, *i. e.* it $= \frac{2}{3}$ of once $\frac{5}{7}$, or $\frac{2}{3}$ of $\frac{5}{7}$; and this is the required result, as in last article.

55. From what has been shown in (53), we may see that

$$\text{since} \quad \frac{5}{7} \times \frac{2}{3} = \frac{5 \times 2}{7 \times 3} = \frac{10}{21},$$

$$\text{and} \quad \frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21};$$

$$\text{therefore} \quad \frac{5}{7} \times \frac{2}{3} = \frac{2}{3} \times \frac{5}{7}:$$

and since the sign (×) and the word *of* between fr^{ns} have been shown to have the same meaning, we see that

$$\frac{5}{7} \text{ of } \frac{2}{3} = \frac{2}{3} \text{ of } \frac{5}{7}.$$

56. Proceeding with the expression (F) in (53), and changing $3\frac{3}{5}$ into an imp^{r} fr^{n}, we have

$$\frac{5}{7} \text{ of } \frac{2}{3} \text{ of } 3\tfrac{3}{5} = \frac{5 \times 2}{7 \times 3} \text{ of } \frac{18}{5};$$

and since $\frac{5 \times 2}{7 \times 3}$ is one fr^{n}, and $\frac{18}{5}$ another fr^{n}, I may use the

method of multiplying, which has been shown to be true for *two* fractions;

$$\text{therefore } \frac{5}{7} \text{ of } \frac{2}{3} \text{ of } 3\tfrac{3}{5} = \frac{5 \times 2 \times 18}{7 \times 3 \times 5}: \quad \text{(I)}$$

and if the expression in (F) had consisted of four fr^{ns} connected by *of* or (×), the first three might have been reduced to one fr^{n}, as in (I), and we should then have had but *two* fr^{ns}, which we have just shown how to reduce to one. Therefore the results proved in lines (F), (G), (H), (I), are true, whatever number of fr^{ns} may be contained in any compound fraction.

57. To return to (I). It has been proved that if a num^{r} and den^{r} have any common measure, both of them may be divided by this common measure; therefore if in the right-hand side of equation (I), we find any factors common to both num^{r} and den^{r}, these should be struck out before we complete the multiplication.

We see in this case that there is a factor 5 in the num^{r} and den^{r}; also, that 18 in the num^{r} and 3 in the den^{r} have a common divisor 3: dividing by these common factors,

$$\text{the expression stands thus, } \frac{\overset{1}{\cancel{5}} \times 2 \times \overset{6}{\cancel{18}}}{7 \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}}}.$$

The remaining factors in the num^{r} are 1, 2, 6; and in the den^{r} are 7, 1, 1; these form a fraction $\frac{2 \times 6}{7} = \frac{12}{7} = 1\frac{5}{7}$.

This process of striking out factors from num^{r} and den^{r} is called *cancelling*, and is to be continued as long as any factors can be found common to both num^{r} and den^{r}. When the numbers are large, this cancelling cannot be readily performed, except by those who have had some experience.

Obs. A factor 1 left in a num^{r} or den^{r} will not alter

the value of the other factors which remain, when we multiply together all those in the numʳ, and all those in the denʳ; and hence the figures 1 need not be written, as in the above Ex.; but it must be noticed, that if no other factors remain, the value of the numʳ or denʳ will consist of the product of as many figures 1 as there were factors in the numʳ or denʳ before cancelling; *i. e.* it will be $=1$. If, therefore, all the factors in any numʳ or denʳ have been cancelled, the numʳ or denʳ of the fraction after cancelling will be 1. But when this occurs in the denʳ, it need not be expressed, but the result written as a whole number.

Ex. II. $$8\tfrac{2}{5}\times 35\times \tfrac{17}{24}=\frac{\overset{7}{\cancel{42}}}{\cancel{5}}\times\frac{\overset{7}{\cancel{35}}}{1}\times\frac{17}{\underset{4}{\cancel{24}}}$$
$$=\frac{7\times 7\times 17}{4}=\frac{833}{4} \quad \text{(K)}$$
$$=208\tfrac{1}{4}.$$

Here 42 and 24 had a C. M. 6, and 5 and 35 had a C. M. 5.

The frⁿ $\frac{833}{4}$ in line (K) will be in its lowest terms, if all the factors common to numʳ and denʳ have been cancelled at the proper time.

Ex. III. $$\tfrac{7}{8}\text{ of }7\tfrac{1}{2}\text{ of }\tfrac{4}{15}\text{ of }\tfrac{2}{7}=\frac{\cancel{7}}{\underset{2}{\cancel{8}}}\times\frac{\cancel{15}}{\cancel{2}}\times\frac{\cancel{4}}{\cancel{15}}\times\frac{\cancel{2}}{\cancel{7}}.$$

Here there are no factors to be *seen* in the numʳ; but we must remember that in reality there have been 4 divisions of the numʳ by the factors 7, 15, 4, and 2; and therefore 4 quotᵗˢ, which in this case are each $=1$; therefore the real numʳ $=1\times 1\times 1\times 1$, or 1; so the quotᵗˢ in the denʳ are 2, 1, 1, 1, and their product $=2$; therefore the reduced fraction has 1 as numʳ and 2 as denʳ, *i. e.* it is $\frac{1}{2}$.

Exs. 11. Find the value of

1. $\frac{2}{3}\times\frac{4}{5}$ of $\frac{6}{7}$	5. $2\frac{6}{7}$ of $\frac{14}{15}$ of $5\frac{1}{3}$ of $7\frac{7}{8}$
2. $1\frac{3}{5}\times\frac{4}{27}\times 2\frac{7}{9}$ of $\frac{18}{49}$	6. $\frac{1}{2}$ of $\frac{1}{7}$ of $\frac{1}{9}$ of $4\frac{1}{2}$
3. $3\frac{2}{7}$ of $5\frac{4}{9}$ of $\frac{15}{23}$	7. $(3\frac{2}{5}\times 5\frac{1}{2}\times\frac{7}{9})-(\frac{1}{3}$ of $\frac{5}{12})$
4. $10\frac{3}{8}\times\frac{17}{24}$ of $\frac{15}{19}$ of $\frac{12}{34}$	8. $1-(\frac{3}{11}$ of $\frac{5}{9}$ of $\frac{18}{25})$

DIVISION.

58. The operation of Division may be expressed in two ways;

$$\text{Thus: } \frac{11}{12} \div \frac{5}{9}, \quad \text{and } \frac{\frac{11}{12}}{\frac{5}{9}}$$

have the same meaning, both being read $\frac{11}{12}$ divived by $\frac{5}{9}$.

We have now to inquire how to perform this division.

We know by (31) that $5 = 9 \times \frac{5}{9} = 9$ times $\frac{5}{9}$, or that 5 is 9 times as large as $\frac{5}{9}$. If, now, I divide $\frac{11}{12}$ by 5 (when my object was to divide by $\frac{5}{9}$), I shall be dividing by a quantity 9 times too large, and therefore my quotient will be 9 times too small; to obtain, then, the *true* quotient, I must multiply the first quot^t by 9; *i. e.* the correct quot^t is found by performing two operations—first, by dividing by 5, and secondly, by multiplying by 9; and performing these two operations at one step, we have

$$\frac{11}{12} \div \frac{5}{9} = \frac{11}{12} \times \frac{9}{5},$$

and by comparing the two sides of this equation, we see that the divisor $\frac{5}{9}$ has been inverted into $\frac{9}{5}$, and and the $(\div)$ has been changed into $(\times)$, or the div^n changed into multiplication.

If mixed or whole numbers are found in any expression which is to be simplified, we must, of course, reduce to imp^r fr^{ns}, and invert the divisor, as before. Thus

$$\frac{9}{14\frac{5}{8}} = \frac{\frac{9}{1}}{\frac{117}{8}} = \frac{9}{1} \times \frac{8}{117} = \frac{\cancel{9}}{1} \times \frac{8}{\underset{13}{\cancel{117}}} = \frac{8}{13}.$$

59. The following is a more complicated Ex. involving both Mult^n and Division.

Reduce to the simplest form $\dfrac{7\frac{3}{5}}{8\frac{5}{9}}$ of $\dfrac{6\frac{5}{12}}{8\frac{5}{6}}$.

Here we reduce mixed numbers to imp^r fr^{ns}—change *of* into ($\times$), and invert the divisors, as was shown in the last art.; and then, making two steps in the operation,

$$\text{the expression} = \dfrac{\frac{38}{5}}{\frac{77}{9}} \times \dfrac{\frac{77}{12}}{\frac{53}{6}} = \frac{38}{5} \times \frac{9}{77} \times \frac{77}{12} \times \frac{6}{53} \qquad \text{(L)}$$

$$= \frac{\overset{19}{\cancel{38}}}{5} \times \frac{9}{\cancel{77}} \times \frac{\cancel{77}}{\underset{2}{\cancel{12}}} \times \frac{\cancel{6}}{53} \qquad \text{(M)}$$

$$= \frac{19 \times 9}{5 \times 53} = \frac{171}{265}.$$

In working such Exs. I should not write down both (L) and (M), but perform the cancelling on the line (L), which would then become (M): but a pupil would have been confused at first, if he had not seen both lines written in full.

Exs. 12. Find the value of

1. $\frac{11}{12} \div \frac{2}{3}$
2. $\frac{3}{14} \div \frac{1}{7}$
3. $9 \div \frac{4}{5}$
4. $18\frac{1}{2} \div \frac{6}{7}$
5. $\dfrac{15\frac{3}{4}}{9}$
6. $\dfrac{11}{17\frac{8}{11}}$
7. $\frac{1}{2}$ of $\dfrac{3\frac{2}{3}}{16\frac{1}{5}}$
8. $\dfrac{18}{\frac{3}{5} \text{ of } 4\frac{2}{7}} \div 17\frac{1}{2}$
9. $\dfrac{1}{2} + \dfrac{1}{3 + \dfrac{1}{1\frac{1}{2}}}$
10. $\dfrac{1\frac{1}{12}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$

SIMPLIFICATION OF FRACTIONS.

60. Under this head is classed a variety of *expressions* intended to exemplify the operations previously illustrated; and the working of the examples under it depends entirely upon a correct and ready use of the methods of performing Addition, Subtraction, &c., as already explained. There is no difficulty in any of them, so long as a pupil will pay close attention to the *signs;* but I will point out one or two Exs.

wherein he is liable to err, though every such error must arise entirely from want of proper observation.

Ex. $4\frac{4}{11}$ of $7\frac{5}{16} + 2\frac{1}{2}$ of 11.

In working this, a pupil must take care that he reduces the whole expression on each side of the (+) to the simplest form, before he begins to perform the operation of Add^n, which that sign (+) bids him perform. The same caution would be necessary if there had been (−) instead of (+) in the above example.

I will work this Ex. as it ought to be written; and leave a large space on each side of the (+), so that I may run no risk of connecting the two parts incorrectly.

$$4\tfrac{4}{11} \text{ of } 7\tfrac{5}{16} \quad + \quad 2\tfrac{1}{2} \text{ of } 11$$

$$= \frac{\overset{3}{\cancel{48}}}{11} \times \frac{117}{\cancel{16}} \quad + \quad \frac{5}{2} \times \frac{11}{1}$$

$$= \qquad \frac{351}{11} \quad + \quad \frac{55}{2} \qquad\qquad [\text{L. C. D.} = 22]$$

$$= \qquad \frac{702 \; + \; 605}{22} = \frac{1307}{22} = 59\tfrac{9}{22}.$$

Ex. II. Simplify $\dfrac{3\frac{1}{2} \text{ of } \frac{5}{8}}{2\frac{5}{6} \text{ of } 3\frac{1}{3}} \div \dfrac{2\frac{2}{11} \text{ of } \frac{11}{12}}{3\frac{1}{9} \text{ of } 7\frac{5}{7}}$.

Now, the whole of the fr^n to the right of the sign (÷) is a divisor of the fr^n to the left of the sign; therefore this right-hand fr^n must be inverted, and the sign (÷) changed to (×): the expression will then be

$$\frac{3\frac{1}{2} \text{ of } \frac{5}{8}}{2\frac{5}{6} \text{ of } 3\frac{1}{3}} \times \frac{3\frac{1}{9} \text{ of } 7\frac{5}{7}}{2\frac{2}{11} \text{ of } \frac{11}{12}};$$

and this, by changing *of* into ×, and the mixed numbers into imp^r fr^{ns}, becomes

$$\frac{\frac{7}{2} \times \frac{5}{8}}{\frac{17}{6} \times \frac{10}{3}} \times \frac{\frac{28}{9} \times \frac{54}{7}}{\frac{24}{11} \times \frac{11}{12}},$$

and lastly,—inverting the fr^{ns} in the lower line, because they are divisors, and connecting all the fr^{ns} with mult^n signs, I have

the entire expression $= \frac{7}{2}\times\frac{5}{8}\times\frac{6}{17}\times\frac{3}{10}\times\frac{28}{9}\times\frac{54}{7}\times\frac{11}{24}\times\frac{12}{11}$ (N)

$$=\frac{\cancel{7}}{\cancel{2}}\times\frac{\cancel{5}}{\underset{4}{\cancel{8}}}\times\frac{\overset{3}{\cancel{6}}}{17}\times\frac{3}{\underset{2}{\cancel{10}}}\times\frac{\overset{7}{\cancel{\overset{14}{\cancel{28}}}}}{\cancel{9}}\times\frac{\overset{3}{\cancel{\overset{6}{\cancel{54}}}}}{\cancel{7}}\times\frac{\cancel{11}}{\underset{2}{\cancel{24}}}\times\frac{\cancel{12}}{\cancel{11}}\,*$$

$$=\frac{3\times3\times7\times3}{4\times17}=\frac{189}{68}=2\tfrac{53}{68}.$$

In working such Exs. I should omit line (N) for the reason given in (59).

I subjoin an Ex., wherein the whole quantity enclosed in ( ) parentheses, or brackets, as they are called, must be reduced to as simple a form as possible, before the ($\times$) or ($\div$) joining the expressions in brackets can be made use of.

Thus; if I have to simplify $(\frac{7}{8}-\frac{3}{10})\times(2\frac{3}{4}+3\frac{1}{9})$, I shall first reduce $\frac{7}{8}-\frac{3}{10}$ to the simplest form; then $2\frac{3}{4}+3\frac{1}{9}$ must be reduced in like manner, and these two results must then be multiplied together, because the sign ($\times$) is placed between the brackets. The whole work will stand thus;

$$\frac{7}{8}-\frac{3}{10}=\frac{35-12}{40}=\frac{23}{40}\qquad\text{I.}\qquad(\text{since L. C. D.}=40)$$

$$\text{and } 2\tfrac{3}{4}+3\tfrac{1}{9}=\frac{11}{4}+\frac{28}{9}=\frac{99+112}{36}=\frac{211}{36}\qquad\text{II.}$$

Therefore, multiplying I. and II., I have

$$\left(\frac{7}{8}-\frac{3}{10}\right)\times\left(2\tfrac{3}{4}+3\tfrac{1}{9}\right)=\frac{23}{40}\times\frac{211}{36}$$

$$=\frac{4853}{1440}=3\tfrac{533}{1440}.$$

Exs. 13. Reduce to the simplest forms

I. $\frac{3}{11}$ of $8\frac{2}{3}-\frac{5}{7}$; $\frac{3}{11}$ of $(8\frac{2}{3}-\frac{5}{7})$; $\frac{3}{11}\sim(8\frac{2}{3}$ of $\frac{5}{7})$

II. $\frac{2}{3}$ of $\frac{6}{7}\div\frac{8}{9}$ of $17\frac{1}{2}$; $\frac{8}{17}+(\frac{3}{5}$ of $7\frac{1}{2})\div\frac{9}{11}$; $(\frac{8}{17}+\frac{3}{5}$ of $7\frac{1}{2})\div\frac{9}{11}$

III. $\dfrac{2\frac{2}{3}\times(3\frac{1}{2}+\frac{7}{9})}{\frac{3}{5}\sim\frac{2}{3}}$; $\dfrac{7\frac{1}{4}\times3\frac{1}{3}}{6\frac{1}{2}\text{ of }\frac{11}{13}}\div\dfrac{\frac{5}{8}\text{ of }11}{\frac{1}{7}}$

IV. $\dfrac{\frac{3}{5}}{\frac{5}{9}}+\dfrac{\frac{5}{7}}{7\frac{1}{2}\text{ of }\frac{11}{12}}$; $(5\frac{2}{3}+3\frac{1}{4})\times(6\frac{1}{7}$ of $\frac{1}{56})$.

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* A pupil will not necessarily perform the cancelling in exactly the same order as above; but the result will be the same.

REDUCTION OF FRACTIONS.

DEF. Any quantity involving money, weight, &c.,—as 4 shillings, 5 pence, $\frac{3}{4}$oz., is called a *concrete* quantity; and any number not involving any such denominations,—as 6, 8, $\frac{4}{5}$, is called an *abstract* number.

61. It is here intended to express in positive terms such quantities as $\frac{5}{8}$*s.*, $\frac{11}{12}$ of 10*s.* 6*d.* &c.; *i. e.* to express concrete quantities, being frac[l] parts of any given den[n], in terms of lower den[ns].

Thus, $\frac{5}{8}$*s.* must be expressed in terms of pence, and frac[l] parts of a penny; $\frac{11}{12}$ of 10*s.* 6*d.* in terms of shillings, pence, and frac[l] parts of 1*d.*

So, also, $\frac{9}{11}$ of a ton would be expressed in cwts., qrs., lbs., oz., drs., and frac[l] parts of a dram.

It may happen that the proposed fr[n] can be expressed in an exact number of units of some one of the lower den[ns]; then, of course, there will be no frac[l] part of the last named den[n].

62. I will now show how to perform the operation intended in the last article.

It has been seen (21) that $\frac{5}{8}$*s.* means either *one*-eighth of 5 shillings, or *five*-eighths of one shilling. We may, therefore, use either of these two methods:—divide the 5*s.* by 8, as in Comp[d] Short Division, and the process will be thus—

$$\begin{array}{r} 8\,\underline{)\;5\quad 0\;} \\ \underline{\underline{\;7\tfrac{1}{2}d.}} \end{array}$$

or, treating the fr[n] as $\frac{5}{8}$ of 1*s.*, we may change *of* into ($\times$), reduce the 1*s.* to pence, and proceed thus—

$$\frac{5}{8}\text{ of }1s. = \frac{5\times 12}{8}d. = \frac{5\times \overset{3}{\cancel{12}}}{\underset{2}{\cancel{8}}}d. = \frac{15}{2}d. = 7\tfrac{1}{2}d.$$

This latter method seems more suitable to an Ex. which professes to belong to fr^{ns}, and is the one which I recommend, unless the den^{r} of the proposed fr^{n} be very large, so as to make the division to be performed rather difficult: in that case, to work by $Comp^{d}$ Long Div^{n} is preferable, as being more likely to be correct. I will take such an Ex. and work it through by both methods.

Express in positive terms $\frac{17}{26}$ of a ton.

Working fractionally, we have

$$\frac{17}{26} \text{ of } 1 \text{ ton} = \frac{17 \times \overset{10}{\cancel{20}}}{\underset{13}{\cancel{26}}} \text{ cwts.} = \frac{170}{13} \text{ cwts.} = 13\tfrac{1}{13} \text{ cwts.}$$

I now take the $frac^{l}$ part of the cwts., viz. $\frac{1}{13}$ of 1 cwt., and express it in the next lower den^{n}, viz. quarters, and so proceed, till either there be no $frac^{l}$ part left, or till I come to the last den^{n} in Avoirdupois weight.

$$\frac{1}{13} \text{ of } 1 \text{ cwt.} = \frac{1 \times 4}{13} \text{ qrs.} = \frac{4}{13} \text{ qrs. (\textit{i.e.} 0 qrs. in the final quotient.)}$$

$$\frac{4}{13} \text{ of } 1 \text{ qr.} = \frac{4 \times 28}{13} \text{ lbs.} = \frac{112}{13} \text{ lbs.} = 8\tfrac{8}{13} \text{ lbs.}$$

$$\frac{8}{13} \text{ of } 1 \text{ lb.} = \frac{8 \times 16}{13} \text{ oz.} = \frac{128}{13} \text{ oz.} = 9\tfrac{11}{13} \text{ oz.}$$

$$\frac{11}{13} \text{ of } 1 \text{ oz.} = \frac{11 \times 16}{13} \text{ drs.} = \frac{176}{13} \text{ drs.} = 13\tfrac{7}{13} \text{ drs.}$$

$$\text{therefore, } \frac{17}{26} \text{ of } 1 \text{ ton} = \overset{\text{cwts.}}{13}\ \overset{\text{qrs.}}{0}\ \overset{\text{lbs.}}{8}\ \overset{\text{oz.}}{9}\ \overset{\text{drs.}}{13\tfrac{7}{13}}.$$

Secondly, take $\frac{1}{26}$ of 17 tons; *i.e.* divide 17 tons by 26.

$$\begin{array}{rrl} & \text{tns.} & \\ & 17 & \\ & 20 & \\ \hline 26) & 340 & (13 \text{ cwt.} \\ & 26 & \\ \hline & 80 & \\ & 78 & \\ \hline & 2 & \end{array}$$

$$
\begin{array}{r l}
 & 2 \\
 & \underline{4} \\
26\,) & 8 \;(0 \text{ qrs.} \\
 & \underline{28} \\
26\,) & 224 \;(8 \text{ lbs.} \\
 & \underline{208} \\
 & 16 \\
 & \underline{16} \\
26\,) & 256 \;(9 \text{ oz.} \\
 & \underline{234} \\
 & 22 \\
 & \underline{16} \\
26\,) & 352 \;(13\tfrac{7}{13} \text{ drs.} \\
 & \underline{26} \\
 & 92 \\
 & \underline{78} \\
 & \frac{14}{26}=\frac{7}{13}
\end{array}
$$

therefore, as before, $\frac{17}{26}$ tons $= \overset{\text{cwts.}}{13}\ \overset{\text{qrs.}}{0}\ \overset{\text{lbs.}}{8}\ \overset{\text{oz.}}{9}\ \overset{\text{drs.}}{13\tfrac{7}{13}}$.

63. When Exs. under this Case involve money, it is well to carry the reduction no farther than pence and frac[l] parts of a penny: for, since farthings are themselves written as frac[l] parts of a penny, therefore if we carry the reduction to frac[l] parts of a farthing, the answer will appear somewhat complicated. I will work an Ex. which will illustrate this.

Express in positive terms $\frac{151}{240}s.$

$$\frac{151}{240}s. = \frac{151\times\cancel{12}}{\underset{20}{\cancel{240}}}d. = \frac{151}{20}d. = 7\tfrac{11}{20}d. \qquad \text{(O)}$$

$$\frac{11}{20}d. = \frac{11\times\cancel{4}}{\underset{5}{\cancel{20}}}f. \;=\frac{11}{5}f. = 2\tfrac{1}{5}f.$$

$$\text{therefore } \frac{151}{240}s. = 7\tfrac{1}{2}d.\ \tfrac{1}{5}f. \qquad \text{(P)}$$

If I compare $7\frac{11}{20}d.$ in (O) and $7\frac{1}{2}d.\ \frac{1}{5}f.$ in (P), I see that the former fr[n] involves less labour, and conveys to a person acquainted with fr[ns] the idea of the real value of the expression, at least as well as the other.

64. When a unit of any den^{n} can be divided exactly by any number, as 4, 5, 6, &c., such fourths, fifths, sixths, &c., are called *aliquot* parts of that unit.

Thus, since £1 or 20*s*. when divided by 3, 4, 5, 6, 8, 10, 12, gives quotients involving an exact number of shillings and pence; therefore thirds, fourths, fifths, &c., of £1 are called aliquot parts of £1.

For example

$\frac{1}{5}$£$=4s.$ $\quad\frac{2}{5}$£$=8s.$ $\quad\frac{4}{5}$£$=16s.$

$\frac{1}{8}$£$=2s.\ 6d.$ $\quad\frac{5}{8}$£$=12s.\ 6d.$ $\quad\frac{7}{8}$£$=17s.\ 6d.$

$\frac{1}{3}$£$=6s.\ 8d.$ $\quad\frac{2}{3}$£$=13s.\ 4d.$

So also, $\frac{1}{8}s.=1\frac{1}{2}d.$ $\quad\frac{5}{8}s.=7\frac{1}{2}d.$ $\quad\frac{7}{8}s.=10\frac{1}{2}d.$

And since 1 lb. avoirdupois, or 16 oz., can be divided exactly by 2, 4, 8; therefore halves, fourths, and eighths of 1 lb. can be at once expressed in oz.

Thus, $\frac{3}{8}$ lb. = 6 oz. $\quad\frac{7}{8}$ lb. = 14 oz.

and if the value of one or more of these aliquot parts of the denominations in common use be remembered, the operations performed in this Case will often be much shortened.

$$\text{Ex. } \frac{27}{160}\text{£} = \frac{27\times \cancel{20}}{\underset{8}{\cancel{160}}}s. = \frac{27}{8}s. = 3\tfrac{3}{8}s. = 3s.\ 4\tfrac{1}{2}d.$$

Exs. 14. Express in positive terms

I. $\frac{3}{8}$ of 10*s*.; $\quad\frac{7}{8}$ of 27*s*.; $\quad\frac{4}{7}$ of $\frac{4}{15}$ of 19*s*. 6*d*.

II. $\frac{9}{11}$ of 1 ton; $\quad\frac{5}{16}$ of a cub. ft.; $\quad\frac{7}{15}$ of a quarter of corn.

III. $\frac{5\frac{1}{3}}{7\frac{1}{2}}$ of $\frac{7}{8}$ of a hhd. of wine; $\quad\frac{5}{11}$ of 1 week; $\quad\frac{7}{16}$ of $365\frac{1}{4}$ days.

IV. $\frac{3}{8}$ of $\frac{5}{9}$ of a Fr. ell; $\quad\frac{1}{11}$ of $\frac{9}{11}$ of $\frac{1}{7\frac{1}{2}}$ of a square mile.

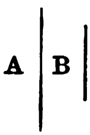
V. $\frac{7}{16}$ of a sq. yd.; $\quad 2\frac{1}{2}$ of $\frac{5}{9}$ of a lb. (Apoth.)

RATIO AND PROPORTION.

65. The next operation in Fractions will consist of the expression of one quantity in terms of, or as a frac^{l} part of, another of like nature. But before proceeding to attempt this operation, it will be advisable to discuss one of the most important relations in numbers, without which a pupil cannot understand the principle upon which such Exs. will be worked. This relation is termed RATIO.

And as this consideration of Ratio leads to the doctrine of PROPORTION, I have thought it well to continue the subject in one unbroken thread, and then refer to this explanation, when I come to the treatment of questions involving Proportion and its applications.

66. When two quantities are placed before us—as, for instance, the lines A, B—and we wish to compare their magnitude, we perceive that there is a certain relation between them, which we familiarly call the comparative size of them; and this comparison or relation is measured by seeing how often the one quantity contains the other.



Now, to see how often one quan^{y} contains the other—as, for example, how often 36 contains 5—we divide the former by the latter, *i. e.* the 36 by the 5, and the quotient is $\frac{36}{5}$; therefore, to see how often the line A contains the line B, I divide A by B: *i. e.* the relation of A to B is expressed by the fr^{n} $\frac{A}{B}$.

This relation is called the *ratio* of A to B, and is thus expressed, A : B.

If A > B, the fr^{n} expressing the ratio of A to B, *i. e.* $\frac{A}{B}$, will be an improper fr^{n} greater than 1;

If $A = B$, the fr^n will become $\frac{A}{A}$, or unity;

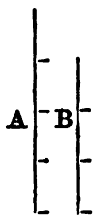
If $A < B$, the fr^n will become $\frac{A}{B}$, a proper fr^n.

Also, when the fr^n $\frac{A}{B}$ is greater than 1, the ratio of A to B is called a ratio of *greater* inequality, because $A > B$.

When the fr^n $\frac{A}{B} = \frac{A}{A} = 1$, the ratio is called a ratio of equality; and when the fr^n $\frac{A}{B}$ is a proper fr^n, the ratio is called a ratio of *less* inequality, because $A < B$.

67. If the num^r and den^r of a fr^n expressing ratio both consist of the same kind of *concrete* quantities, the ratio between these quantities will be an *abstract* number.

Ex. If the lines A and B represent 4 inches and 3 inches



respectively, then $A : B = \frac{A}{B} = \frac{4 \text{ in.}}{3 \text{ in.}} = \frac{4}{3}$; and if, instead of inches, these lines had represented 4 feet and 3 feet, or 4 lb. and 3 lb., the ratio of A to B would still be $\frac{4}{3}$, and of B to A would be $\frac{3}{4}$.

But if the num^r and den^r be of the same species of concrete quantity, but not expressed in the same denomination, the ratio cannot be represented by an abstract quantity, until they both be reduced to the same denomination. Thus, $\frac{4 \text{ feet}}{9 \text{ inches}}$ does not $= \frac{4}{9}$ feet, or $\frac{4}{9}$ in., or $\frac{4}{9}$, but it $= \frac{48 \text{ in.}}{9 \text{ in.}} = \frac{48}{9} = \frac{16}{3}$, an abstract number.

If quantities be not of the same nature, there can be no ratio between them; thus $\frac{4 \text{ feet}}{5 \text{ minutes}}$ is no ratio at all, since

we cannot compare the magnitude of 4 feet and of 5 minutes.

This article must be especially noticed, as upon it wil depend the mode of working the Exs. which have bee alluded to in (65).

68. We now see that any fraction, as $\frac{2}{5}$, has still further meaning, beside what was stated in (21), viz. *th ratio of* 2 *to* 5, or, as it is written, 2 : 5; and $\frac{2}{5}$ may be calle *a ratio.*

If, then, two other quantities be taken, as 4 and 10, an it is found that $\frac{4}{10} = \frac{2}{5}$, then we see that the ratio of 4 to 1 is equal to the ratio of 2 to 5, and this fact we expre either thus,

$$4 : 10 = 2 : 5, \quad \text{or } 4 : 10 :: 2 : 5;$$

the latter expression is thus read; 4 is to 10 as 2 is to 5.

When, therefore, two ratios, as $\frac{4}{10}$ and $\frac{2}{5}$, are place before us, and we learn that they are equal, we say that th quantities, 4, 10, 2, 5, are *Proportionals.*

69. We may now define Proportion to consist in th equality of two ratios. Of the above-mentioned numbe 4, 10, 2, 5; 4 and 5 are called the *extremes;* 10 and 2 ar called the *means,* because they are intermediate between th extremes.

Also, when 4 quantities are given, and we wish t ascertain whether they are proportionals or not, we mus see if the fr^{n} expressing the ratio of the 1st and 2nd = th fr^{n} expressing the ratio of the 3rd and 4th. Thus, if I tak the numbers 5, 6, 7, 8, and wish to try whether they a proportionals, I compare $\frac{5}{6}$, the ratio of the first pair, and the ratio of the second pair; and if these fr^{ns} are prove unequal, the above 4 quantities are not proportionals.

Now, it was shown in (50), that to compare two frac[l] quantities we must bring them to some Com. Den^r. Reducing the fr^{ns} $\frac{5}{6}$ and $\frac{7}{8}$ to a C. D. 48, we have to compare

$$\frac{5 \times 8}{48} \text{ and } \frac{6 \times 7}{48} \qquad \text{(Q)}$$

And since $\frac{40}{48}$ is not $= \frac{42}{48}$, therefore $\frac{5}{6}$ is not $= \frac{7}{8}$, and 5, 6, 7, 8, are not proportionals.

70. It will be seen in line (Q) that the test as to whether or not the four proposed quantities be proportionals, consists in multiplying, 1st, the two extremes 5, 8, and 2ndly, the two means 6, 7: and if these products be equal, the four quantities are proportionals; if they be not equal, the four quantities are not proportionals.

This operation may generally be performed *mentally:* thus, if I take 5, 8, 9, 16, I say $5 \times 16 = 80$, and $8 \times 9 = 72$; therefore these quantities are not proportionals. Again, if I take 4, 9, 5, $11\frac{1}{4}$, since the product of the extremes, $4 \times 11\frac{1}{4} = 4 \times \frac{45}{4} = 45$, and the product of the means, $9 \times 5 = 45$; therefore these last four quantities are proportionals. (See Appendix, Art. Ratio.)

71. Having now explained how to tell when four quantities are proportionals, we proceed to show how, when *three* numbers are given, we may find a fourth number, such that the other three and this fourth shall be proportionals; that is, to solve the following question:

If three numbers be given, as 5, 6, 10, what must be taken as a fourth number, such that the four, when taken in order, shall be proportionals?

Now, as I do not know (or, at least, the learner does not know), the required number, let the letter N stand for

or represent this number; and I have now to try and find what this N must be.

Since, then, 5, 6, 10, and N are required to be proportionals, therefore we must have the ratio between the first pair 5, 6, = that between the second pair 10, N; *i. e.*

$$\frac{5}{6}=\frac{10}{N}; \qquad \text{(R)}$$

Multiplying both sides by N, according to (28), we have $\frac{N\times 5}{6}=\frac{10}{1}=10$; next, multiplying both sides by 6, $N\times 5 = 10\times 6$; lastly, dividing by 5, according to (29), we obtain

$$N=\frac{6\times 10}{5} \qquad \text{(S)}$$

$$=\frac{6\times \overset{2}{\cancel{10}}}{\cancel{5}}=12.$$

Put 12 for N in (R), and we have $\frac{5}{6}=\frac{10}{12}$, which we know to be true, and therefore 5, 6, 10, 12, are proportionals*.

72. The result of this article must be most carefully noticed; for in observing (S) we learn that the required 4th number N was thus formed from the three former ones, 5, 6, 10: viz. that it is the value of this frⁿ—the product of the 2nd and 3rd terms, divided by the first; or it $=\frac{\text{2nd}}{\text{1st}}\times \text{3rd}$.

73. By observing the successive steps whereby N was obtained, we learn that when two fractions are equal, a factor of one denʳ, or the entire denʳ, may be transferred into the opposite numʳ, and conversely, the numʳ, or any factor of it, may be transferred into the opposite denʳ, without disturbing the equality. Hence also, when two fractions are equal,

---

* Or, since the product of the extremes N, 5 = that of the means 6, 10; I have $N\times 5=6\times 10$; $\therefore N=\frac{6\times 10}{5}$, as before.

they will be equal when inverted. A quick use of these properties is very serviceable in the solution of Algebraic Equations.

Exs. 15. Find a fourth proportional to each of the following sets of numbers:—

I. 3, 7, 15; 5, 2, 11; 9, 8, $\frac{1}{7}$;
II. $3\frac{1}{2}$, $7\frac{3}{4}$, $\frac{1}{5}$; $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; $\frac{1}{5}$, $8\frac{1}{2}$, $\frac{1}{16}$;
III. 19, 18, 17; 17, 18, 19.

74. We have shown that, in order that there may exist a ratio between two quantities, they must be of the same kind; since, then, there is to be a proportion among the three given quantities and the fourth, therefore between the 1st and 2nd there must be a ratio, and an equal ratio between the 3rd and 4th. Hence we see that the second pair must be of the same kind; that is, the 4th quantity, when found, ought to be of the same kind as the 3rd. Now, if we observe the fr^{n} $\frac{\text{2nd}}{\text{1st}} \times$ 3rd in the last article, we notice, that since it is allowed that the 2nd and 1st are alike in kind, therefore the $\frac{\text{2nd}}{\text{1st}}$ will be an abstract number (67); and therefore the 4th, which $= \frac{\text{2nd}}{\text{1st}} \times$ 3rd, $=$ 3rd multiplied by some abstract number; *i. e.* it will be the 3rd term repeated as many times as there are units in this number, and therefore will be of the same kind as that in which the third term was expressed.

Also, if the 2nd term be greater than the 1st, $\frac{\text{2nd}}{\text{1st}}$ will be a fr^{n} > 1, and this multiplier of the 3rd term will be greater than 1, and the 4th term greater than the 3rd; but if the 2nd term be less than the 1st, then $\frac{\text{2nd}}{\text{1st}}$ will be a proper fr^{n},

and the multiplier less than 1, and the 4th term less than the 3rd. (See Appendix, Art. *Proportion.*)

75. We may here notice a form of expression that we familiarly use, when we say, that two quantities are proportional to one another; as, for instance, that the amount of a servant's wages is proportional to the length of his service. Here, apparently, two quantities of different kinds are compared, viz. money and time; but, in reality, four quantities are implied, viz. two periods of time, and two amounts of wages. For since one year and one year's wages may be taken as fixed standards, by which we may measure other periods and other amounts; therefore I mentally compare any proposed length of service and the corresponding amount of wages with these fixed standards; and the original expression means this—that there is the same ratio between any given length of service and one year, that there is between the amount of wages for that service and one year's wages; or, that these two periods of service and two amounts of wages together form a proportion in this order,

Given time : 1 year :: wages for that time : wages for 1 year.

If we wish to ascertain whether two quantities are what is here called proportional to one another, according to the common usage of the words, we may try if doubling the one quantity causes the other to be doubled; as for example, in this instance—will the wages be doubled, if the time of service be doubled? Since this is the case, therefore the wages and time are proportional in the sense explained above.

The correct, though not common mode of expressing this proportion, is to say, that the amount of wages *varies as* the length of the time of service.

76. But sometimes this proportion occurs in a different form; as, when a man has a certain number of miles to walk in any time; then, if he quickens his rate of going, he can complete his task in less time. In this case it would be incorrect to say that the length of time was in proportion to the rate of walking; for, according to the test given above, I ask—if the rate be doubled, will the number of days be *doubled?* the answer is, no: on the contrary, it will be *halved.* There is here, then, evidently a proportion (taking the same word in the common usage,) but of a different kind, viz. that as one number is increased by multiplication, the other is correspondingly diminished by division.

As, in the first instance, it was said that the wages *varied as* the time, so, in this second example, the relation between the two quantities is correctly expressed by saying, that the number of days *varies inversely as* the rate of walking; or, as it is commonly said—in the first Ex., the wages were *directly* proportional to the time; and in the second Ex., the number of days is *inversely* proportional to the rate.

Proportion has here been shown to involve 4 quantities; these need not, however, be all different. Thus, if I find 3 quantities, whereof the ratio between the 1st and 2nd = that between the 2nd and 3rd, then the three quantities are said to be proportional: thus, 3, 6, 12 are proportionals,

because $\frac{3}{6} = \frac{6}{12}$, or $3 : 6 = 6 : 12$ (T)

Also, to find this 3rd propor[l] 12, when the two former ones are given, we must compare line (T) with an ordinary proportion containing 4 quan[s], and we shall see that 12, the required 3rd, is in the 4th place, and the 2nd quan[y], 6, is

repeated in the 3rd place: hence, the usual equation 4th $= \frac{\text{2nd}}{\text{1st}} \times \text{3rd}$, will become 3rd $= \frac{\text{2nd}}{\text{1st}} \times \text{2nd}$; and if 3 and 6 be the 1st and 2nd terms, we have 3rd $= \frac{6}{3} \times 6 = 12$.

Exs. 16.

Find a third proportional to each of the following pairs of numbers :—

IV. 8, 12; 12, 18; 15, 16; 20, 15;

V. $\frac{1}{3}, \frac{1}{4}$; $2\frac{1}{3}, \frac{3}{5}$; $\frac{1}{2}$ of $\frac{3}{5}$, $\frac{2}{7}$ of 3.

REDUCTION OF FRACTIONS.

77. We are now able to perform the operation alluded to in (65) viz. to express one quantity in terms of another, or as a frac[l] part of another. This fr[n] connecting the two quantities will show the ratio between them, and may be either proper or improper.

For example, to express 9*d.* in terms of £1, is to see what ratio 9*d.* bears to £1. Hence, according to what has been explained concerning ratio, we say,

$$9d. \text{ in terms of } £1 = 9d. : £1.$$

$$= \frac{9d.}{£1} = \frac{9d.}{240d.} = \frac{9}{240} = \frac{3}{80};$$

and transferring the £1 from the den[r] of the left-hand fr[n] to the num[r] of the right-hand fr[n] by (73), I have 9*d.* $= \frac{3}{80} \times £1$, or $\frac{3}{80}$ of £1. This result may also be obtained as follows:—

I have to express pence in terms of £1.

Now, 240 pence = £1; therefore, taking $\frac{1}{240}$th part of both sides—

$$1 \text{ penny} = \frac{1}{240} \text{ of } £1;$$

and multiplying by 9,

$$9 \text{ pence} = \frac{9}{240} \text{ of } £1;$$

$$= \frac{3}{80} \text{ of } £1;$$

and transferring the £1 from the right-hand numerator to the left-hand denominator, we have, as before,

$$\frac{9 \text{ pence}}{£1}=\frac{3}{80}$$

It will be observed, that by this second method we have an independent proof, that the ratio between two concrete numbers of like kind, as pence, pounds, &c., is an abstract number.

78. Either of the two methods just shown may be employed; but I prefer the former, because it more decidedly keeps the idea of ratio before the mind; and an Ex. so worked can be written out in a more condensed form than by the latter method. It will be noticed that the two concrete quantities are *both* to be reduced to some com^n den^n, in order that the ratio between them may be expressed: thus, in the last Ex., both were reduced to pence.

Ex. II. Express $7s.\ 7\frac{1}{2}d.$ in terms of $10s.\ 6d.$

Here I exhibit both numerator and denominator in terms of pence; and write

$$\frac{7s.\ 7\frac{1}{2}d.}{10s.\ 6d.}=\frac{91\frac{1}{2}d.}{126d.}=\frac{\frac{183}{2}}{126}=\frac{\overset{61}{\cancel{183}}}{2\times\underset{42}{\cancel{126}}}=\frac{61}{84}$$

$$\text{or,}=\frac{7\frac{5}{8}s.}{10\frac{1}{2}s.}=\frac{61}{\underset{4}{\cancel{8}}}\times\frac{\cancel{2}}{21}=\frac{61}{84}$$

I might have reduced both numerator and denominator to half-pence, but a pupil would not easily reduce $7s.\ 7\frac{1}{2}d.$ to half-pence mentally; whereas, by both the above methods, the whole work is fully shown.

Referring to Ex. I., we read—$\frac{9d.}{£1}=\frac{3}{80}$; and since, by (73), two equal fractions may be inverted, without disturbing the equality; therefore $\frac{£1}{9d.}=\frac{80}{3}$; or, changing the $9d.$ to the right-hand numerator,

$$£1=\frac{80}{3}\times 9d.;\ \text{or}=\frac{80}{3}\text{ of }9d.$$

Hence we see that in every Ex. where we have to express one quantity in terms of a second, we can express this second in terms of the first, by merely inverting the ratio which connected the first and second. Thus, in Ex. II.,

$$\text{since } 7s.\ 7\tfrac{1}{2}d. = \frac{61}{84} \text{ of } 10s.\ 6d.;$$

$$\text{therefore } 10s.\ 6d. = \frac{84}{61} \text{ of } 7s.\ 7\tfrac{1}{2}d.$$

79. Sometimes fractions may be involved in both num^r and den^r, as in

Ex. III. Express $1\frac{3}{4}$ of 2a. 1r. in terms of 3 acres $2\frac{1}{2}$ roods.

Here, bringing both quantities into roods, and changing *of* into ×, we have

$$\frac{1\frac{3}{4} \text{ of } 2a.\ 1r.}{3a.\ 2\frac{1}{2}r.} = \frac{\frac{7}{4} \times 9r.}{14\frac{1}{2}r.} = \frac{\frac{7}{4} \times 9}{\frac{29}{2}}$$

$$= \frac{7}{\cancel{4}_{2}} \times \frac{9}{1} \times \frac{\cancel{2}}{29} = \frac{63}{58}$$

This should be left as an improper fraction, because I can then read off this desired result—that

$$1\tfrac{3}{4} \text{ of } 2a.\ 1r. : 3a.\ 2\tfrac{1}{2}r. = 63 : 58;$$

or that the ratio of these two portions of land is that of 63 to 58.

As in (64) it was shown to be advisable to be able to express aliquot parts of any den^n in terms of lower den^ns, so, with a little experience, a pupil will, by a reverse process, mentally work easy Exs. in this Case, so as at once to see, that 5s. in terms of £1 $= \frac{1}{4}$; that 6s. 8d. : £1 $= \frac{1}{3}$, and so on: and a readiness in performing such simple reductions will often materially shorten the labour of more complicated Exs. Thus, to express $\frac{7}{8}$ of 13s. 4d. in terms of 10s.:—

By observing that 13s. 4d. $= \frac{2}{3}$£, and 10s. $= \frac{1}{2}$£, I have

$$\frac{\frac{7}{8} \text{ of } 13s.\ 4d.}{10s.} = \frac{\frac{7}{8} \text{ of } \frac{2}{3}£}{\frac{1}{2}£} = \frac{7}{\cancel{8}_{\cancel{4}_{2}}} \times \frac{\cancel{2}}{3} \times \frac{\cancel{2}}{1} = \frac{7}{6};$$

or, $\frac{7}{8}$ of 13s. 4d. $= \frac{7}{6}$ of 10s.

By the ordinary method I should have reduced both 13s. 4d. and 10s. to pence or fourpences, and the resulting fraction would have required much heavier reduction than mine has needed.

80. When either the num^r or den^r of the left-hand fr^n requires much reduction to a lower den^n, it is better to express by signs the multiplication requisite to perform this reduction, rather than to perform the mult^n, and put down the result; because, when either num^r or den^r is thus expressed in factors, it is in the best form for detecting the probability of any cancelling taking place, so as to present the resulting fr^n in its lowest terms.

For example, in reducing 5 drs. 1 sc. 15 grs. to the fraction of 1 lb., I must bring the 1 lb. to grains, and I write

$$\frac{5\text{dr. }1\text{sc. }15\text{grs.}}{1\text{lb.}} = \frac{16\text{sc. }15\text{grs.}}{1\times12\times8\times3\text{sc.}} = \frac{335\text{grs.}}{12\times8\times3\times20\text{grs.}} \qquad \text{(U)}$$

$$= \frac{\overset{67}{\cancel{335}}}{12\times8\times3\times\underset{4}{\cancel{20}}} = \frac{67}{96\times12} = \frac{67}{1152}.$$

But in this Ex. we may also notice that the last denomination 15 grs. may be very readily expressed as a fraction of the preceding denomination, viz. scruples, for 15 grs. $=\frac{15}{20}$ sc. $=\frac{3}{4}$ sc.; therefore, instead of the two latter fractions in line (U), I should write

$$= \frac{16\frac{3}{4}\text{sc.}}{12\times8\times3\text{sc.}} = \frac{\frac{67}{4}}{12\times8\times3} = \frac{67}{4\cdot12\times8\times3} = \frac{67}{12\times96}$$

$$= \frac{67}{1152}.$$

Of these five fractions the last four are merely reductions to a simpler form; and the number of steps which a pupil may have to write down in working Exs. similar to the one above will depend, partly on the magnitude of the numbers involved, and partly on his own quickness in working mentally.

I will give one more Ex. in which the work is condensed, but still the successive operations are intelligible.

Express $\frac{11}{12}$ of $4\frac{1}{4}d.$ in terms of $9s.\ 7\frac{1}{2}d.$

$$\frac{\frac{11}{12}\text{ of }4\frac{1}{4}d.}{9s.\ 7\frac{1}{2}d.} = \frac{\frac{11}{12}\times\frac{17}{4}d.}{115\frac{1}{2}d.} = \frac{\cancel{11}}{12}\times\frac{17}{\underset{2}{\cancel{4}}}\times\frac{\cancel{2}}{\underset{21}{\cancel{231}}} = \frac{17}{504};$$

$$\text{or, } \frac{11}{12}\text{ of }4\tfrac{1}{4}d. = \frac{17}{504}\text{ of }9s.\ 7\tfrac{1}{2}d.$$

Exs. 17.

1.	Express	1*s*. 3*d*.,......	and	17*s*. 6*d*.......	in terms of	£1.
2.	,,	17*s*. $7\frac{1}{3}$*d*.,...	,,	25*s*. $9\frac{1}{4}$*d*......	,,	1 guinea.
3.	,,	12*s*. $7\frac{5}{7}$*d*.,...	,,	2*s*. $11\frac{1}{9}$*d*......	,,	15*s*. 6*d*.
4.	,,	$\frac{5}{18}$ of a moidore,	,,	$3\frac{3}{4}$ groats.....	,,	$12\frac{1}{2}$ guineas.
5.	,,	$\frac{3}{7}$£,..........	,,	$\frac{1}{1\frac{2}{3}}$ of 7*s*.......	,,	1 farthing.
6.	,,	3 qrs. $1\frac{1}{3}$ nls.,	,,	$\frac{1}{2}$ in............	,,	1 Flem. ell.
7.	,,	3 oz. 5 drs. $1\frac{1}{4}$ sc...........................			,,	1 lb.
8.	,,	$\frac{5}{7}$ of $1\frac{1}{2}$ guineas.........................			,,	£5.
9.	,,	$\frac{3}{5}$ of $1\frac{8}{9}$ of an acre.......			,,	1 sq. yd.
10.	,,	$\frac{10}{11}$ of $1\frac{5}{6}$ of 3 miles..................			,,	5 miles.
11.	,,	1 hour................................			,,	$3\frac{4}{5}$ of $365\frac{1}{4}$ dys.
12.	,,	$2\frac{7}{10}$ of $\frac{3}{4}$ of 15 solid inches			,,	$3\frac{2}{3}$ cubic yd.

SIMPLIFICATION OF FRACTIONS; INCLUDING EXAMPLES IN THE COMPOUND RULES, WHICH INVOLVE FRACTIONS.

81. Under this head are arranged—1st. Exs. involving the expression of frac[l] quan[s] in positive terms, and requiring both Addition and Subtraction, as—

I. £$\frac{5}{8}$ + $\frac{3}{4}$ of 21*s*. − $\frac{7}{8}$ of 27*s*.

2ndly. Exs. in Compound Addition, Subtraction, Multiplication, and Division, in which fractional quantities are involved. I give one of each.

II. £3 6*s*. $8\frac{5}{8}$*d*. + £2 11*s*. $8\frac{7}{8}$*d*. + £4 13*s*. $9\frac{3}{8}$*d*.

III. £7 16*s*. $8\frac{5}{6}$*d*. − £3 11*s*. $10\frac{7}{8}$*d*.

IV. £8 14*s*. $2\frac{7}{12}$*d*. × $46\frac{8}{9}$.

V. £38 3*s*. $4\frac{3}{5}$*d*. ÷ $13\frac{7}{9}$.

82. In Ex. I. we may use either of the following methods, viz. reduce all the quantities to the same denomination, and then find the value of their sum as in (62); or, express in positive terms each fraction separately, and then find the value of the whole by Compound Addition and Subtraction.

By the first method, reducing the quantities to the fraction of £1, I have

$$\frac{5}{8}\text{£}+\frac{3}{4}\text{ of }21s.-\frac{7}{8}\text{ of }27s.$$

$$=\frac{5}{8}\text{£}+\frac{3}{4}\times\frac{21}{20}\text{£}-\frac{7}{8}\times\frac{27}{20}\text{£}$$

$$=\left(\frac{5}{8}+\frac{63}{80}-\frac{189}{160}\right)\text{£}$$

$$=\frac{100+126-189}{160}\text{£} \qquad \text{(since L. C. D.}=160)$$

$$=\frac{226-189}{160}\text{£}=\frac{37}{160}\text{£}$$

$$\text{and }\frac{37}{160}\text{£}=\frac{37\times\cancel{20}}{\cancel{160}_{8}}s.=\frac{37}{8}s.=4\tfrac{5}{8}s.=4s.\ 7\tfrac{1}{2}d.$$

By the second method—

$$\frac{5}{8}\text{£ }(64)=12s.\ 6d.$$

$$\text{also, }\frac{3}{4}\text{ of }21s.=\frac{63}{4}s.=15\tfrac{3}{4}s.=15s.\ 9d.$$

$$\text{and }\frac{7}{8}\text{ of }27s.=\frac{189}{8}s.=23\tfrac{5}{8}s.=\text{£}1\ 3s.\ 7\tfrac{1}{2}d.$$

$$\text{therefore }\frac{5}{8}\text{£}+\frac{3}{4}\text{ of }21s.-\frac{7}{8}\text{ of }27s.=12s.\ 6d.+15s.\ 9d.-\text{£}1\ 3s.\ 7\tfrac{1}{2}d.$$

$$=\text{£}1\ 8s.\ 3d.-\text{£}1\ 3s.\ 7\tfrac{1}{2}d.$$

$$=4s.\ 7\tfrac{1}{2}d.$$

83. Ex. II. £3 6*s*. $8\frac{5}{8}d.$ + £2 11*s*. $8\frac{7}{8}d.$ + £4 13*s*. $9\frac{2}{3}d.$

Commencing with the addition of the fractional parts, I have

$$\frac{5}{6}d.+\frac{7}{8}d.+\frac{2}{3}d.=\frac{20+21+16}{24}d.$$

$$=\frac{57}{24}d.=\frac{19}{8}d.=2\tfrac{3}{8}d.$$

£	s.	d.
3	6	$8\frac{5}{6}$
2	11	$8\frac{7}{8}$
4	13	$9\frac{2}{3}$
10	12	$3\frac{3}{8}$

nd carrying the 2*d*. to the row of pence, I complete the sum as in Compound Addition.

84. Ex. III. £7 16*s*. $8\frac{5}{6}d.$ − £3 11*s*. $10\frac{7}{8}d.$

Commencing with the subtraction of the fractional parts, I observe that $\frac{5}{6} < \frac{7}{8}$; therefore, borrowing 1*d.* from the 8*d.* in the former quantity, I have

$$1\tfrac{5}{6}d. - \tfrac{7}{8}d. = \left(\frac{11}{6} - \frac{7}{8}\right)d. = \frac{44-21}{24}d. = \frac{23}{24}d.$$

$$\begin{array}{rrr} £ & s. & d. \\ 7 & 16 & 8\frac{5}{6} \\ 3 & 11 & 10\frac{7}{8} \\ \hline 4 & 4 & 9\frac{23}{24} \\ \hline \end{array}$$

and complete the sum by Comp[d] Subtraction.

Ex. IV. £8 14*s.* $2\frac{7}{12}$*d.* × $46\frac{8}{9}$.

Now, since to multiply by $46\frac{8}{9}$ is to repeat the multiplicand 46 times and $\frac{8}{9}$ times, therefore, if I multiply the given comp[d] quan[y] by 46, and then take $\frac{8}{9}$ of the same, the sum of the two products will be the product required, viz. $46\frac{8}{9}$ times £8 14*s.* $2\frac{7}{12}$*d.* Proceeding as in Comp[d] Mult[n], and working in the margin those parts of the mult[n] which involve fractions, I have

$$\begin{array}{rrrl} £ & s. & d. & \\ 8 & 14 & 2\frac{7}{12} & \\ & & 9\times 5+1 & =46 \\ \hline 78 & 7 & 11\frac{1}{4} & \\ & & 5 & \\ \hline 391 & 19 & 8\frac{1}{4} & =45 \text{ times} \\ 8 & 14 & 2\frac{7}{12} & = 1 \;\;,, \\ 7 & 14 & 10\frac{8}{27} & = \frac{8}{9} \;,, \\ \hline 408 & 8 & 9\frac{7}{54} & =46\frac{8}{9} \;,, \\ \hline \end{array}$$

$$\begin{array}{rrr} £ & s. & d. \\ 8 & 14 & 2\frac{7}{12} \\ & & 8 \\ \hline 9)\,69 & 13 & 8\frac{2}{3} \\ \hline 7 & 14 & 10\frac{8}{27} \\ \hline \end{array}$$

The $\frac{8}{27}$*d.* is thus obtained: after dividing the pence by 9, there remain 2*d.*; therefore I have to divide $2\frac{2}{3}$*d.* by 9: the quotient

$$= \frac{2\frac{2}{3}d.}{9} = \frac{\frac{8}{3}}{9}d. = \frac{8}{27}d.$$

$$\text{Sum of the fractions} = \left(\frac{1}{4} + \frac{7}{12} + \frac{8}{27}\right)d. = \frac{27+63+32}{108}d.$$

$$= \frac{122}{108}d. = \frac{61}{54}d.$$

$$= 1\tfrac{7}{54}d.$$

Again $46\frac{8}{9} = \frac{422}{9}$; therefore I may multiply the given sum

by $\frac{422}{9}$; *i. e.* multiply by 422, and divide by 9. But as this second method involves more labour, and would therefore be not so generally used, it is not worth while to work the Ex. out, especially as the process of Compound Multiplication, where fractions are involved, is sufficiently shown in the former method.

85. Ex. v. £38 3*s.* $4\frac{3}{5}d. \div 13\frac{7}{9}$.

There is but one mode of working this Ex., viz. to reduce the divisor to an impʳ frⁿ—invert it, and proceed as in multiplication.

Now, $13\frac{7}{9} = \frac{124}{9}$; and this, when inverted, becomes $\frac{9}{124}$. Therefore the Ex. becomes £38 3*s.* $4\frac{3}{5}d. \times \frac{9}{124}$; *i. e.* I have to multiply the given sum by 9, and divide by 124. The work will be as follows:—

$$
\begin{array}{rrrrl}
 & \pounds & s. & d. & \\
 & 38 & 3 & 4\frac{3}{5} & \\
 & & & 9 & \qquad 9 \times \frac{3}{5}d. = \frac{27}{5}d. = 5\frac{2}{5}d. \\
\hline
124) & 343 & 10 & 5\frac{2}{5} & (\pounds 2 \\
 & 248 & & & \\
\hline
 & 95 & & & \\
 & 20 & & & \\
\hline
124) & 1910 & & & (15s. \\
 & 124 & & & \\
\hline
 & 670 & & & \\
 & 620 & & & \\
\hline
 & 50 & & & \\
 & 12 & & & \\
\hline
124) & 605 & & & (4d. \\
 & 496 & & & \\
\hline
 & 109 & & &
\end{array}
$$

There now remain 109*d.* and $\frac{2}{5}d.$, which have not yet been divided by 124; the quotient =

$$\frac{109\frac{2}{5}d.}{124} = \frac{\frac{547}{5}}{124}d. = \frac{547}{5 \times 124}d. = \frac{547}{620}d.$$

and the whole quotient is £2 15*s.* $4\frac{547}{620}d.$

3—5

Exs. 18. Find the value of

1. $\frac{3}{4}$ guinea $+\frac{3}{14}$ of 13*s*. 4*d*.$-\frac{4}{7}$ of 27*s*.
2. $\frac{3}{8}$ ton $+\frac{7}{8}$ lb. $+\frac{5}{12}$ cwt. (in lbs. and oz.)
3. $\frac{3}{5}$ mile$-\frac{4}{5}$ furlong $+\frac{1}{11}$ pole.
4. $1\frac{1}{3}$ acr. $+\frac{4}{7}$ rood$-\frac{2}{3}$ sq. yd.
5. £7 16*s*. $8\frac{1}{5}$*d*. + £138 4*s*. $10\frac{5}{9}$*d*. + £78 6*s*. $3\frac{5}{8}$*d*.
6. 27 yds. 1 ft. $8\frac{2}{3}$ in. + 5 yds. 0 ft. $9\frac{2}{5}$ in.$-$2 ft. $0\frac{3}{5}$ in.
7. 4 tons 3 cwts. $2\frac{5}{9}$ qrs. + 15 cwt. $57\frac{1}{2}$ lbs.$-$3 qrs. 13 lbs. $5\frac{1}{3}$ oz.
8. £463 12*s*. $4\frac{2}{3}$*d*. ~ £764 19*s*. $3\frac{1}{7}$*d*.
9. £114 8*s*. $0\frac{6}{7}$*d*. $\times 8\frac{5}{9}$.
10. £15 9*s*. $8\frac{11}{15}$*d*. $\times 23\frac{5}{7}$.
11. 17 yrs. 3 mo. 2 wks. 3 dys. $\times 375\frac{8}{11}$.
12. £135 0*s*. $11\frac{3}{16}$*d*. $\div 12\frac{11}{12}$.
13. 1684 lbs. 10 oz. 17 dwt. $11\frac{1}{3}$ grs. $\div 47\frac{2}{3}$.
14. £13 15*s*. $9\frac{3}{11}$*d*. $\times 17\frac{2}{3}$.
15. (£3 8*s*. $7\frac{1}{4}$*d*. + £2 17*s*. $5\frac{1}{3}$*d*.)$\div 15\frac{1}{3}$.

MISCELLANEOUS EXAMPLES.

86. These, of course, consist of questions involving one or more of the various processes which we have considered; and all that can be done towards guiding the pupil in working such Exs. is, merely to assist him in judging what operations are required in solving any proposed question.

What was said in (60) applies more particularly here, viz. that it is necessary most carefully to note the *signs*, or rather the words which express signs. As an instance of this caution, I will consider the subjoined Ex.; and in order to work it, I shall endeavour to see what operations are intended by the words employed in the question.

Ex. I. Multiply the sum of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{5}{6}$, and 4, by $7\frac{11}{12}$.

Now *the sum of* means addition, or (+), and *of* means (×); therefore, this Ex. expressed in signs is $\{\frac{2}{3}+(\frac{3}{4}\times\frac{5}{6})+4\}\times 7\frac{11}{12}$. This is now a mere Ex. of simplification as in (60); and the remaining work may be completed by a pupil.

87. Ex. II. How many persons may receive each $3\frac{5}{6}s.$ out of £$13\frac{4}{5}$?

This is only a Reduction sum, involving fractions; and, in plainer language, means—how often are $3\frac{5}{6}s.$ contained in £$13\frac{4}{5}$? or, if I divide £$13\frac{4}{5}$ by $3\frac{5}{6}s.$, what is the quotient?

The quotient required is $\dfrac{£13\frac{4}{5}}{3\frac{5}{6}s.}$

$$= \frac{13\frac{4}{5} \times 20s.}{3\frac{5}{6}s.} = \frac{\dfrac{69}{5} \times \dfrac{20s.}{1}}{\dfrac{23}{6}s.} \qquad \text{(W)}$$

$$= \frac{\overset{3}{\cancel{69}}}{\cancel{5}} \times \frac{\overset{4}{\cancel{20}}}{1} \times \frac{6}{\cancel{23}} = 72.$$

By (67) we know that the second fraction in line (W) will be an abstract number; and the result, 72, shows that $3\frac{5}{6}s.$ are contained 72 times in £$13\frac{4}{5}$.

88. Ex. III. Compare £$\frac{3}{7}$, $\frac{3}{8}$ of a guinea, and $\frac{2}{3}$ of $11s.\ 10\frac{1}{4}d.$

We cannot compare quantities without bringing them to some common name; *i. e.* in order to compare fractional quantities, we must reduce them to their L. C. D.

These quantities may therefore either be expressed in positive terms, as pounds, shillings, pence, &c., and then compared; and this is the best method when we wish to know the *difference* in value between the quantities; or they may be all expressed as fractions of one common quantity —say, of £1, or of 1 guinea, and then reduced to L. C. D. I use the latter method, because then I see more clearly the *ratio* between the quantities.

Reducing to fr^{ns} of £1, I have

$$\frac{3}{8}\text{ guinea} = \frac{3}{8} \times \frac{21}{20}£ = \frac{63}{160}£.$$

$$\text{Also, } \frac{2}{3} \text{ of } 11s.\ 10\tfrac{1}{4}d. = \frac{2}{3} \times \frac{142\frac{1}{4}}{240}£ = \frac{\cancel{2}}{3} \times \frac{569}{\underset{2}{\cancel{4}} \times 240}£ = \frac{569}{1440}£.$$

Hence the 3 given quan^s, when expressed in the same den^r, are

$$\frac{3}{7}£,\quad \frac{63}{160}£,\quad \text{and } \frac{569}{1440}£.$$

or, reduced to L. C. D. $\frac{4320}{10080}$, $\frac{3969}{10080}$, $\frac{3983}{10080}$.

or in the ratio of 4320 : 3969 : 3983.

89. I will work in full the following Ex., because upon the method of working it depends the solution of many questions in Arithmetic and Algebra.

Ex IV. If $\frac{1}{3}+\frac{1}{4}+\frac{1}{12}$ of a number amount to 36, what is the number?

Now, $\frac{1}{3}+\frac{1}{4}+\frac{1}{12}=\frac{4+3+1}{12}=\frac{8}{12}=\frac{2}{3}$.

And, by the question, this sum $=36$;

i. e. $\frac{2}{3}$ of the number $=36$;

therefore, dividing both sides by 2,

$\frac{1}{3}$ of the number $=18$:

and, multiplying by 3,

$\frac{3}{3}$ of the number, *i. e.* the whole number $=3\times 18=54$.

90. The following is an Ex. which is to be worked upon a principle similar to the last.

Ex. V. If A can do a piece of work in 3 hours, B in 5 hours, and C in 7 hours, in what time can they do it, all working together?

Now, A can do the work in 3 hours;

therefore A can do $\frac{1}{3}$ of the work in 1 hour;

so, B „ $\frac{1}{5}$ „ 1 hour;

and C „ $\frac{1}{7}$ „ 1 hour;

therefore the three, A, B, C, working together, can perform

$$\frac{1}{3}+\frac{1}{5}+\frac{1}{7},\ \text{or}\ \frac{35+21+15}{105},\ \text{or}\ \frac{71}{105}\ \text{in 1 hour;}$$

therefore, if they do $\frac{71}{105}$ in 1 hour,

they will do $\frac{1}{105}$ in $\frac{1}{71}$ hour,

and therefore $\frac{105}{105}$, or the whole in $\frac{105}{71}$ hours;

that is, in $1\frac{34}{71}$ hours.

By observing the fr^n $\frac{71}{105}$, expressing the amount done in 1 hour, and $\frac{105}{71}$, the number of hours required for the whole work, we find that they are the reverse of one another; and this we might have expected, for the quantity of work done in any given time bears an *inverse* ratio to the amount of time in which it is done (76).

Here I have been finding the time of doing the whole work. I give one more Ex., in which it is required to find how much of a given piece of work can be done in any fixed time.

Ex. VI. A cistern is filled by two spouts in 20 and 24 minutes respectively, and emptied by a tap in 30 minutes; what portion of it will be filled in 15 minutes, when they are all left open together, the influx and efflux being uniform?

Since the 1st tap would fill the whole in 20 min.
therefore it would fill $\frac{1}{20}$th in 1 min.
so also, the 2nd „ $\frac{1}{24}$th in 1 min.

therefore $\frac{1}{20}+\frac{1}{24}$ are poured in by both together in 1 minute; but $\frac{1}{30}$ is discharged in 1 minute by the third tap; therefore, subtracting the quantity discharged from that poured in, we have remaining in the cistern at the end of 1 minute, $\frac{1}{20}+\frac{1}{24}-\frac{1}{30}$;

$$\text{and this}=\frac{6+5-4}{120}=\frac{11-4}{120}$$

$$=\frac{7}{120}\text{ in 1 minute;}$$

therefore at the end of 15 minutes there remains $\cancel{15}\times\frac{7}{\underset{8}{\cancel{120}}}$ or $\frac{7}{8}$; hence in 15 minutes the cistern will be seven-eighths full.

Exs. 19. A.

1. Explain the terms *product, dividend, quotient.*

2. With 263 for quotient, 368,469 for dividend, and 6 for remainder, what is the divisor?

3. Find the difference between one million and ten, and ten thousand and nine.

4. Write an equation involving the signs of addition, subtraction, multiplication, and division.

5. If a man's yearly income be known, what rule must be used to find his daily income? Ex. If the income be £532 5*s.* 10*d.*, what is that per day?

6. What is the smallest number that can be exactly divided by the nine digits?

7. If light travels 192,000 miles per second, and the sun's light reaches us in $8\frac{1}{4}$ minutes, how far is the sun from the earth?

8. If 360 degrees be passed over in $365\frac{1}{4}$ days, how much is that per day?

9. To how many persons may £4 13*s.* 6*d.* each be given, out of a sum of £79 9*s.* 6*d.*?

10. What sum of money will be required to distribute to 10 poor men, 15 women, and 20 children, the respective sums of 2*s.*, 1*s.*, and 6*d.*?

11. Out of an income of 500 guineas, it is desired to lay by £150; what must be saved and spent daily?

12. Compare the product and quotient of $2\frac{1}{3}$ and $1\frac{1}{7}$.

13. Give the rules for multiplying and dividing fractions by whole numbers, and illustrate the processes by examples.

14. From unity subtract $\frac{1}{2}$, add $\frac{1}{3}$, subtract $\frac{1}{4}$, and add $\frac{1}{5}$; find the ratio of the result to the fraction $\frac{4}{5}$ of $1\frac{3}{8}$.

15. The sum realized by a bankrupt's estate is £7848, being $\frac{3}{16}$ of his debts, find the amount of the debts, and the dividend paid.

B.

1. How many dollars, each 4*s.* 6*d.*, are contained in 20 moidores?

2. How many times will a wheel, 7 ft. 4 in. in circumference, turn in $3\frac{3}{4}$ miles?

3. If a railway carriage move 42 feet per second, how many miles is that per hour?

4. Find the value of 1000 oz. of silver plate, at 13*s.* 9*d.* per oz.; and shew how many times more valuable the same weight would be in gold, at £13 8*s.* $1\frac{1}{2}$*d.* per oz.

5. A yard of Cambridge butter weighs 1 lb., what should be the length of one penny-worth, at $17\frac{1}{2}$*d.* per lb.?

6. What sum must be divided among 18 men, and 9 women, so that each man may have £1 3*s.* 6*d.*, and each woman $\frac{2}{3}$ of that sum?

7. A cask is required which can be filled by any one of the following measures, taken any number of times exactly, $\frac{1}{4}$ pint, $\frac{1}{4}$ gallon, 3 gals., 5 gals., and 9 gals.; find the smallest cask for this purpose.

8. Find the value of $\frac{7}{11} \sim \frac{9}{16}$ of a lb. Troy.

9. Out of a sum of £$18\frac{9}{16}$, how many persons may receive 2*s*. $7\frac{1}{2}$*d*. each?

10. Shew whether 3 : 19 is greater or less than $2\frac{1}{4}$: $14\frac{1}{4}$; and find the change in the last term, that the four may form a proportion.

11. Of a package of cloth, $\frac{1}{3}$ is sold at 2*s*. 6*d*. a yard; $\frac{1}{4}$ at 1*s*. 6*d*. a yard, and the remaining 25 yds. at 1*s*. 10*d*., gaining 2*d*. per yd.; find the whole gain on the package.

12. Find the product and quotient of $\frac{2}{1\frac{3}{4}}$ and $\frac{1\frac{3}{4}}{4}$.

13. What fraction of £2 15*s*. is £2 14*s*. 9*d*.?

14. If $\frac{1}{3}$ of a ship be worth £73 1*s*. 3*d*., what part of her is worth £250 10*s*.?

15. Find the G.C.M. of 324 and 720, explaining the truth of the process.

C.

1. Express in words (1) the sum, and (2) the difference of 4,000,309 and 70,002.

2. Find in lbs. the value of 3 tons 14 cwt. 57 lbs.—1 ton 17 cwt. 3 qrs.

3. In coining 40,000 penny pieces, each costing $\frac{3}{4}$*d*., how much profit was made?

4. What number of steps 2 ft. 11 in. long, will be taken in walking 45 yards?

5. The cylinder of a railway engine is 18 in. long; how far will the piston travel in 500 revolutions of the driving wheel?

6. In 56 guineas, as many pounds, moidores, and half-crowns, how many groats?

7. If 25 yds. of cloth cost £7 17*s*. 6*d*., shew without using any proportion statement what is the value of 36 yds.

8. If £1 sterling be worth 26 francs 50 cents, find the number of francs that can be obtained for 1000 guineas.

9. A clock gains $7\frac{1}{2}$ min. per day, find the gain per minute.

10. How many days would it take to count 800 million coins, at the rate of 125 per minute?

11. Find the L.C.M. of 6, 8, 12, 18, 24, 27, explaining the process that you use.

12. Express in the smallest integers the ratio of $\frac{3}{1\frac{1}{3}}$ to $\frac{6\frac{1}{2}}{5}$.

13. After using $\frac{2}{5}$ of a cheese, $\frac{2}{3}$ of the remainder sold for £1 2*s*. $5\frac{1}{2}d$.; what was the whole cheese worth?

14. From the sum of $\frac{2}{5}$ of 7*s*., and $\frac{4}{5}$ of half a guinea, take $\frac{3}{8}$ of a guinea, expressing the result as the fraction of a moidore.

15. What number added to $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{7}{9}$ of $\frac{5}{8}$, will make a sum total of 5?

D.

1. Find the value of $3645 \times 700705 \div 37$.

2. How many acres in a field 202 yds. long, and 167 yds. broad?

3. A bar of gold, valued at £3 17*s*. $10\frac{3}{4}d$. per oz., is sold for £1429 0*s*. $1\frac{1}{2}d$.; what was its weight?

4. A sheet of letter-press contains 24 pages, of which $\frac{5}{8}$ are large type, having 32 lines to the page, and 54 letters in a line; and the remainder is small type, with 45 lines to the page, and 64 letters in a line, how many letters are there in the sheet?

5. Out of a yearly income of £1000, £737 17*s*. 6*d*. is spent for 35 years, how much will have been saved in that time?

6. In a foot-race, where 50 yards start are given to A, the hindermost B gains 5 feet in every 50 yds.; where will the competitors be at the end of a mile?

7. The duty on tea was formerly 2*s*. $2\frac{1}{4}d$. per lb., how much must pay the duty, to make 5 millions sterling?

8. How many plots of land, each 50 square perches, can be made out of a square mile?

9. The pendulum of a church-clock vibrates 15 times in 4 minutes; how many vibrations in 24 hours?

10. Define proper, improper, and compound fractions, giving two examples of each.

11. Compare $\frac{11}{15}$, $\dfrac{2\frac{1}{3}}{7\frac{1}{5}}$, and $\frac{2}{5}$ of $\frac{5}{9}$.

12. A person receives £750 for $\frac{3}{16}$ of his share of a mine worth £10000; what fractional part of the whole was his share?

13. What is the ratio, expressed in integral numbers, between the sum and difference of $17\frac{2}{9}$ and $27\frac{1}{11}$?

14. If I gain $18\frac{2}{5}s$. in 15 guineas, how much is that in the pound?

15. Three men, A, B, C, can do a piece of work in 2, $2\frac{1}{2}$, and $3\frac{1}{2}$ hours respectively, how much of the work could be done in 20 minutes by them all working together?

DECIMAL FRACTIONS,

OR

DECIMALS.

91. It is known to all who understand *Numeration*, that where several figures are placed in a horizontal row, so as to form one number, the value of any figure depends upon its distance from the figure nearest to the right, or, as it is commonly called, from the units' place. Thus, if we take the number 6666, we know that the four figures, counting from left to right, have these values respectively—6000, 600, 60, 6; where we observe that the first 6 to the left has 10 times the value of the second—the second, 10 times the value of the third, and so on: or, going from right to left, each figure has one-tenth of the value of the one preceding it; in other words, any figure, when moved from right to left is multiplied by 10 every step, and when moved from left to right is divided by 10 every step. For example, in the number

$$\overset{\text{D}}{6}\;\overset{\text{C}}{6}\;\overset{\text{B}}{6}\;\overset{\text{A}}{6},$$

if I move 6 from A to D, or three places to the left, I in reality multiply it by $10 \times 10 \times 10$, or 1000, *i. e.* the 6 becomes 6000. Again, to change a 6 from C to A, or two places to the right, I divide it by 10×10, or 100; *i. e.* the figure which before represented 600 now represents 6.

92. Since, then, it has been shewn that successive figures to the right are found by dividing by 10, let this division be continued beyond the units' place; we ought,

therefore, to have, as the value of the first figure to the right of the units' place, $\frac{1}{10}$th of 6, or $\frac{6}{10}$; of the next, $\frac{1}{10}$th of $\frac{6}{10}$, or $\frac{6}{100}$; so, of the next, $\frac{6}{1000}$; of the next, $\frac{6}{10000}$, &c. And when these four additional figures are placed to the right of the units' place, the entire number will be $6666\cdot\overset{E}{6}\overset{F}{6}\overset{G}{6}\overset{H}{6}$, where a point has been set after the units' place, to show where the new figures commence. The entire number now consists of 6 thousands, 6 hundreds, 6 tens, 6 units (or 6), 6 tenths, 6 hundredths, 6 thousandths, 6 tenths of thousandths.

Also, observing $\cdot\overset{E}{6}\overset{F}{6}\overset{G}{6}\overset{H}{6}$, it may be seen that the same rule holds that was true of the whole numbers, viz. that to move any figure from right to left is to multiply by 10 every step, and from left to right, is to divide by 10. For example: if I change the second 6 from F to H, or move it *two* places to the right, I change it from $\frac{6}{100}$ to $\frac{6}{10000}$, which $= \frac{6}{100} \div 100$; *i. e.* I have divided it by 10×10, or by ten *twice*.

Again, if I change the fourth 6 from H to E, or move it *three* places to the left, I change it from $\frac{6}{10000}$ to $\frac{6}{10}$, which $= \frac{6}{10000} \times 1000$; *i. e.* I have multiplied $\overset{H}{6}$ by $10 \times 10 \times 10$, or by 10 three times; hence the same law holds both on the right and left of the point.

DEF. If a number be multiplied by itself any number of times, it is said to be raised to a power. Thus, the multiplication of $2 \times 2 \times 2$ is otherwise expressed by saying that 2 is raised to the power of 3; and a small figure 3 placed to the right of the 2 and above the line (thus, 2^3) indicates or explains how many factors, 2, have been multiplied together. In like manner, 10^4 expresses the multiplication of four factors, each 10, and is called the fourth power of 10.

This small figure is called the *Index*, or *Exponent*.

93. It appears that the figures on the right of the point in reality represent fractions, as $\frac{6}{10}$, $\frac{6}{100}$, $\frac{6}{1000}$, &c., all of which have as denominators either 10, or powers of 10; hence they are called *Decimal* Fractions, or Decimals. And the point which is placed to separate the whole numbers from the decimals is called the *decimal point*.

OBS. For the future, in speaking of Decimal Fractions, I shall use the single word *Decimals;* and for Vulgar Fractions, the word *Fractions*.

94. We have just seen (91) that in writing down the value which any single figure represents in the whole number 6666, viz. 6000, 600, 60, 6, we place as many ciphers at the right hand of each 6, as will keep it in the place which it had in the number 6666: so also, in the number ·6666, if we wish to write down the value of each one of these four figures separately, we shall have to place as many ciphers to the left of each 6 as will keep it in the place which it had in the number ·6666, or at its proper distance from the *point*. Hence these four figures, when placed singly, would be

$$\cdot 6, \quad \cdot 06, \quad \cdot 006, \quad \cdot 0006, \qquad (X)$$

and, as in whole numbers we might go farther to the *left*, and have as the next figure 60000; so, by going farther to the *right* in the decimals, I should have as the next figure ·00006.

Comparing the four quantities in line (X) with the value which we have shewn to be due to them, viz. $\frac{6}{10}$, $\frac{6}{100}$, $\frac{6}{1000}$, &c., we have this connection,

$$\frac{6}{10} = \cdot 6 \qquad \frac{6}{100} = \cdot 06 \qquad \frac{6}{1000} = \cdot 006 \qquad \frac{6}{10000} = \cdot 0006$$

where it will be observed, that the number of *ciphers* in the den^r of the left-hand side equals the entire number of figures after the decimal point, whether ciphers or not, on the right-hand side.

95. Taking this fact as proved when there is but one figure in the num^r of the decimal frac^n, I shall now shew that this is true, whatever be the number of figures in the num^r of the frac^n; for example, that $\frac{675}{10000} = \cdot 0675$, where there are four *ciphers* in the left-hand den^r, and four *figures* after the decimal point.

$$\text{For } \frac{675}{10000} = \frac{600}{10000} + \frac{70}{10000} + \frac{5}{10000}$$
$$= \frac{6}{100} + \frac{7}{1000} + \frac{5}{10000};$$

and these, by what has just been shewn, in last Article,

$= \cdot 06 + \cdot 007 + \cdot 0005$

$= 6$ hundredths $+ 7$ thousandths $+ 5$ tenths of thousandths

$= \cdot 0675$;

i. e. $\frac{675}{10000} = \cdot 0675$, which was the required result.

Hence any frac^n, having as a den^r any power of 10, can be immediately written as a decimal, by writing down the num^r only, and so placing the decimal point that it shall have as many figures on its right hand, as there are ciphers in the den^r of the given frac^n. And if the num^r does not contain as many figures as there are ciphers in the den^r,— that is, as many as it is necessary to have after the point, the amount must be made up by placing as many ciphers between the point and the figures taken from the num^r as shall complete the desired number.

$$\text{Thus, } \frac{3275}{1000} = 3 \cdot 275 \qquad \frac{743}{100000} = \cdot 00743.$$

In the latter Ex. I find it necessary to place two ciphers between the point and the 743, to make the number of

figures to the right of the point equal to the number of ciphers in the left-hand denʳ.

The pupil should now be able to read any decimal either in one sum, or express it in terms of the several parts of which it is composed. Thus, 3·275 may be read in terms of its several denⁿˢ, viz. 3, and 2 tenths, 7 hundredths, and 5 thousandths: or 3, and 275 thousandths; and here it will be observed that when a decimal is expressed in one denⁿ, that denⁿ will be the lowest contained. Thus in reading ·00743, since the 3 stands for hundredths of thousandths, therefore ·00743 stands for 743 hundredths of thousandths.

96. The position of the decimal point determines the value of every figure both on the right and left of it, that is, both of the whole numbers and the decimals. Therefore, to move the point to the right has the same effect as moving all the figures to the left; and to move the point to the left, is equal to moving the figures to the right.

Now, it has been shewn (92) that to move a figure one place to the left is to multiply it by 10; therefore, if in any number containing a decimal point, I move the point one place to the right, I in reality multiply every figure in the number, and therefore the entire number, by 10: similarly, if I move the point to the left one place, I divide it by 10. Hence, if I wish to multiply a number containing a decimal point by 10, 10^2, 10^3, &c., I move the point 1, 2, 3, &c. places to the right; and if I wish to divide the number by 10, 10^2, 10^3, &c., I move the point 1, 2, 3, &c. places to the left.

Ex. 3275·468. (Y)

Moving the point two places to the right, the number becomes 327546·8; and it will be found that any figure has

now 100 times the value that it had before. The 5 formerly stood for 5 *ones*, or 5, but now for 5 *hundreds*, or 500; the 6 for 6 hundredths, or $\frac{6}{100}$, but now for 6, or $\frac{6}{100} \times 100$; that is, each figure has been multiplied by 100, merely by moving the point two places to the right.

Next, let the point be moved three places to the left, and the number becomes 3·275468; and we then see that the 7, which in (Y) was 70, is now only $\frac{7}{100}$, or $\frac{70}{1000}$; that is, has one-thousandth of its former value: so, also, the 3, which before represented 3000, now represents only 3; hence it appears that the entire number has been divided by 1000, merely by moving the point three places to the left.

Exs. 20.

I. Multiply 8·034 by 10, 10,000, 100,000, and 1,000,000 successively.
II. Divide 175·04 by 100, 100,000, 1,000,000, and 10 successively.
III. Multiply ·005 by 10^3, 10^5; and divide it by 10, 10^2, 10^4.

CONVERSION OF VULGAR FRACTIONS INTO DECIMALS.

97. In the use of Decimals we shall find it necessary to know how to convert fractions into decimals. All examples of this kind will in reality be particular examples of Division of Decimals; but as the simpler ones can be worked mentally, or by Short Division, it is well to work them independently of the general rule of division. And it will be seen that there are two classes of fractions, one containing those which can be exactly expressed as decimals, and the other, such as cannot. We shall presently shew the ground upon which this variety rests; and shall first treat of the former class, which includes all fractions which, *when in their lowest terms*, have, as den^{rs}, numbers which contain the figures 2 and 5 as their only factors. These may be called *convertible* fractions.

98. We have seen that a frac^n whose den^r is a power of 10, can immediately, by inspection, be converted into a decimal. And if the den^r be any other than a power of 10, it must be got rid of by dividing the num^r by the den^r.

Any den^r which contains an equal number of both *twos* and *fives* will evidently contain as many factors 10, as 2 and 5, and therefore be of the form 10, 100, 1000, &c. Take for example as a den^r $2 \times 2 \times 5 \times 5 = 10 \times 10 = 100$: and when a fr^n with such den^r is converted into a decimal, there will plainly be as many decimal places as there are *twos* and *fives*.

Exs. 21. Express as decimal fractions

I. $\frac{3}{10}$, $\frac{11}{1000}$, $\frac{19}{100}$, $\frac{15}{100,000}$, $\frac{1001}{10}$

99. Next, let the den^r contain factors 2 only, or be a power of 2, as $\frac{3}{8}$ or $\frac{3}{2^3}$.

Now, since the 8 cannot be contained in the 3 units, I must therefore bring these units into tenths, and say, 8 in 30 tenths, which gives 3 *tenths* as quotient, and 6 tenths over; and since 6 tenths = 60 hundredths, 8 in 60 hundredths gives 7 *hundredths*, and 4 hundredths over, or 40 thousandths; 8 in 40 thousands gives 5 *thousandths;* and there being no remainder, the division terminates: and collecting the three quotients, viz. 3 tenths, 7 hundredths, and 5 thousandths, we plainly have $\frac{3}{8} = \cdot 375$.

It will be seen that like as there are *three* factors 2 in the den^r 8, so there are *three* places in the resulting decimal. The reason of this may be seen by representing the process thus, where we multiply num^r and den^r by 1000,

$$\frac{3}{8} = \frac{3}{2 \times 2 \times 2} \times \frac{10 \times 10 \times 10}{1000} = \frac{3 \times 5 \times 5 \times 5}{1000} = \frac{375}{1000} = \cdot 375.$$

It is here plain that the three factors 2 could not have disappeared in the cancelling, (i.e. in the division by the den^r 8,) had there not been also three factors 10 in the num^r, and therefore three in the den^r, making 1000, and giving three dec^l places in the result.

In Simple Short Division, as soon as I know where to put the *first* quotient, I can write down the others in order; so here, as soon as I find that the first quotient consists of tenths, I can place the other quotients without further consideration. But in determining this *first* quotient there is a liability to error; for if a pupil takes the fr^n $\frac{3}{80}$, and begins to divide by 80, he may incautiously cut off the ciphers from divisor and dividend, as in Simple Division: but, by placing the fr^n thus

$$\begin{array}{r|l} 80 & 3{\cdot}0000 \\ \hline & {\cdot}0375 \end{array}$$

it will be found that if I cut off a cipher at the end of the dividend, I remove a figure which has no value; whereas, to take 0 from the 80 diminishes it tenfold: I must therefore say, 80 in 3 units gives as quotient 0 units; so also, since 3 units = 30 tenths, 80 in 30 tenths gives 0 tenths. Again, 30 tenths = 300 hundredths, and 80 in 300 hundredths gives 3 hundredths; hence 0 must be put as quotient under the tenths, and 3 under the hundredths; the remaining quotients will then fall into their proper places by mere Simple Division.

But in examples like this, where the den^r contains a power of 10, it is best to divide by the other factors first, and then divide by that power of 10. In the Ex. $\frac{3}{80}$, we should divide both num^r and den^r by 8, and then divide by 10, by inspection. Thus

$$\frac{3}{80} = \frac{{\cdot}375}{10} = {\cdot}0375$$

$$\text{So, also, } \frac{3}{8000} = \frac{{\cdot}375}{1000} = {\cdot}000375 \quad \text{by (96)}$$

100. If the den^r^ of the fr^n^ be a composite number larger than 12, we may perform the division by breaking the den^r^ into factors, and dividing by them successively. For example

$$\frac{5}{32}=\frac{1\cdot25}{8}=\cdot15625, \qquad (A)$$

where the second fraction in (A) was obtained by dividing num^r^ and den^r^ by 4.

A fraction whose den^r^ has *fives* only as its factors, must be reduced to a decimal by a process precisely similar to the one exhibited in (98) and (99).

Ex. $\frac{3}{25}$. Here there are two factors *5* in the den^r^; therefore the div^n^ will terminate when I have used 2 ciphers in the num^r^.

$$\text{Thus, } \frac{3}{25}=\frac{\cdot6}{5}=\cdot12$$

$$\text{So, also, } \frac{3}{12500}=\frac{\cdot6}{2500}=\frac{\cdot12}{500}=\frac{\cdot024}{100} \qquad (B)$$

$$=\cdot00024 \qquad \text{by (96)}$$

where the three factors *5* in 125, (*i. e.* in that part of the den^r^ which is being got rid of,) produce three figures in the num^r^ of the last fraction in (B).

Exs. 21. Express as decimal fractions

II. $\frac{3}{5}$, $\frac{7}{8}$, $\frac{1}{2}$, $\frac{9}{4}$, $\frac{11}{8}$, $\frac{17}{16}$, $\frac{18}{25}$, $\frac{3}{500}$

III. $\frac{7}{125}$, $5\frac{3}{64}$, $11\frac{18}{625}$, $8\frac{1}{8}$ of $11\frac{3}{5}$, $7\frac{1}{80}$ of $5\frac{1}{16}$

The following equalities are worth remembering.

$$\frac{1}{8}=\cdot125 \qquad \frac{1}{4}=\cdot25$$

$$\frac{3}{8}=\cdot375$$

$$\frac{1}{2}=\cdot5$$

$$\frac{5}{8}=\cdot625$$

$$\frac{7}{8}=\cdot875 \qquad \frac{3}{4}=\cdot75$$

Their utility appears as follows:—if in converting a fraction to a decimal I have a divisor 8, and I come to the last figure in the dividend, one of the above equalities will enable me to write down the remaining quotients immediately.

Thus, if I have to reduce $\frac{75}{8}$ to a decimal, I find that after one quotient 9 has been obtained, 3 is my last dividend; hence, since $3 \div 8 = \cdot 375$, I can write $\frac{75}{8} = 9\cdot 375$, without *performing* the division for the last three quotients. Similarly, $\frac{11\cdot 7}{8} = 1\cdot 4625$, where the last three figures were obtained, as before, by remembering the value of $5 \div 8$. So also, $\frac{75}{4} = 18\cdot 75$,—the last two quotients being written without dividing.

101. Coming now to what we may call the *inconvertible* fractions, *i. e.* those which, when reduced to lowest terms, contain other factors than 2 and 5 in the den^{r}, as, for instance, 3, 7, 9, &c. we see at once, that the power of 10 which reduces the num^{r} to tenths, hundredths, &c. is not divisible by these factors; therefore they cannot be cancelled out, as the factors 2 and 5 were; hence the division will not terminate. But since the successive quotients diminish tenfold in value every step, so that the eighth to the right of the point represents hundredths of millionths, we generally pursue the division till we obtain 7 figures in the decimal, and then consider the remaining quotients as too small in value to be much appreciated.

Moreover, whatever be the factor which cannot be cancelled out of the den^{r}, we know that in dividing by any divisor, the remainder must always be at least one less than the divisor: therefore if any divisor, as 7, be the factor which causes the division not to terminate, there can be but six remainders; and hence, when 7 divisions have been performed, one of these six remainders must come over again, or recur:

and if the remainder, which, of course, influences the dividend, recurs, the quotients will also recur, and we shall then have the same set of figures *recurring*, as it is termed.

Ex. To convert $\frac{6}{7}$ into a decimal.

Working mentally, as in (99), I have

$$\tfrac{6}{7} = \cdot857142857142, \text{ \&c.} \qquad \text{(C)}$$

where it appears that the figures 857142 will recur, to whatever length the division may be carried: hence such decimals are called *Recurring Decimals*, and the set of quotients which recurs is called a *Period.** If the period consist of but one figure, that single figure is written with a dot (·) over it in place of the whole decimal; but if there be more figures than one, the period is written with a dot over the first and last of its figures.

$$\text{Thus, } \frac{6}{7} = \cdot857142857142\ldots\ldots \text{ or } \cdot\dot{8}5714\dot{2}$$

$$\frac{1}{3} = \cdot333\ldots\ldots \text{ or } = \cdot\dot{3}$$

$$\frac{1}{9} = \cdot1111\ldots\ldots \text{ or } \cdot\dot{1}$$

$$\frac{3}{22} = \frac{1\cdot5}{11} = \cdot13636\ldots\ldots \text{ or } \cdot1\dot{3}\dot{6}.$$

OBS. The places which are occupied by the figures to the right of the decimal point are often called *decimal places*. Thus, in the Ex. $\frac{6}{7}$, we say that the division was carried to

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* Observing the line (C) it may be noticed that whenever a fr^n has 7 for its den^r, the recurring decimal obtained will consist of the figures 857142; but this period will not always begin with the figure 8, but with some other of the above 6 figures; thus $\frac{3}{7} = \cdot428571$ &c., where it will be observed that the period is the same as in (C), except that we begin with the 4; and if these figures are retained in the memory, the whole may be written down at once, as soon as the first figure has been obtained by division.

If, however, when the den^r is 7, the num^r be already a recurring decimal, the above observation will not hold, because the additional figures at the end of the num^r are not ciphers, as in the above fr^{ns} $\frac{6}{7}$ and $\frac{3}{7}$.

six places before the figures began to recur.—(See Appendix, Art. Circulating Decimals).

Exs. 22. Express as decimal fractions

I. $\frac{1}{6}$, $\frac{1}{7}$, $\frac{3}{7}$, $\frac{2}{11}$, $\frac{8}{13}$, $\frac{7}{30}$, $\frac{1}{3}$ of $1\frac{3}{8}$

II. $\frac{1}{3}$, $\frac{5}{9}$, $\frac{11}{7}$, $\frac{13}{15}$, $\frac{17}{65}$, $\frac{4}{37}$, $\frac{2}{55}$, $\frac{3}{8}$ of $1\frac{1}{13}$

III. $8\frac{5}{12}$, $17\frac{4}{33}$, $375\frac{7}{66}$, $\frac{1215}{396}$

TO CONVERT TERMINATING DECIMALS INTO VULGAR FRACTIONS.

102. It will be easily seen that a number expressed as a decimal can be immediately exhibited as a vulgar fraction. For, since it has been shewn (95) that $\frac{675}{10000} = \cdot0675$, therefore we can reverse the process, and say that $\cdot0675$, when converted into a fr^n, becomes $\frac{675}{10000}$; and the correctness of this conversion may be proved thus:

$$\cdot0675 = \frac{6}{100} + \frac{7}{1000} + \frac{5}{10000}$$

$$= (\text{when reduced to L. C. D.}) \frac{600 + 70 + 5}{10000}$$

$$= \frac{675}{10000}. \qquad (D)$$

$$\text{So also, } 3\cdot0275 = 3 + \frac{0}{10} + \frac{2}{100} + \frac{7}{1000} + \frac{5}{10000}$$

$$= \frac{30000 + 200 + 70 + 5}{10000} = \frac{30275}{10000}, \qquad (E)$$

a fraction greater than 1, as might have been foreseen, because 3·0275 is partly a whole number and partly a decimal.

DEF. In a whole or mixed number, the part which is not fractional is called *Integral*, or an *Integer*.

103. In (D) we observe that the num^r of the vulgar fr^n into which we have changed the decimal, before it is reduced

to lowest terms, consists of the figures in the given decimal, excluding ciphers not preceded by other figures; and the den^r is 1, followed by as many ciphers as there are decimal places.

But in (E), where a cipher occurs in the given mixed decimal 3·0275, it remains in the num^r of the equivalent improper fr^n; and the den^r is formed as before.

The above fractions may, of course, be reduced to lowest terms; but I have left them in their present shape, in order to shew the connection between the given decimal and the fractional form into which it could be converted by inspection. Reducing them, and writing the Exs. as they should be exhibited in common use, we have

$$\cdot0675=\tfrac{675}{10000}=\tfrac{27}{400}; \text{ and } 3\cdot0275=\tfrac{30275}{10000}=\tfrac{1211}{400}=3\tfrac{11}{400}.$$

If the given number be partly a whole number and partly a decimal, we may leave the integral part unaltered, and then the resulting fr^n will appear at once as a mixed number.

$$\text{Thus, } 76\cdot0725=76\tfrac{725}{10000}=76\tfrac{145}{2000}=76\tfrac{29}{400}.$$

Exs. 23. Convert into vulgar fractions, in their lowest terms,

I. ·05, 10·73, 115·008, ·0001, 1·005, ·343.

II. 12·01, 10·008, ·00725, 135·55, ·10505, 9·99.

TO CONVERT RECURRING DECIMALS INTO VULGAR FRACTIONS.

104. Though at first it may be thought that a *non-terminating* decimal cannot be accurately represented by a fr^n; yet, since every recurring decimal is formed from one of that class of fractions which was discussed in (101), therefore every such decimal can of course be made to resume the shape from which it was derived: and the accuracy which

was lost in the change from a fraction to a decimal is restored by this reconversion to the original fraction.

105. Circulating Decimals are of two kinds; one in which the whole of the figures repeat, as

$$\cdot 363636 \ldots\ldots \text{or } \cdot\dot{3}\dot{6};$$

and the other, in which some of the figures to the right of the point are not repeated. These figures, of course, always stand to the left of the circulating part, because when a decimal once begins to circulate, it continues to do so. An Ex. of this kind is

$$\cdot 754365365 \ldots\ldots \text{or } \cdot 754\dot{3}6\dot{5}.$$

The former kind is called a *Pure*, and the latter a *Mixed* Circulating Decimal.

106. The best method of working Exs. under this case will not be intelligible without an acquaintance with one or two facts in algebra. A letter of the alphabet may be used to represent a quantity, the value of which we have to find, as in (73). Thus, I may let F stand for or represent the fraction which is equivalent to any circulating decimal; and it is for me to ascertain the value of this F in any particular example.

Also, a number placed just before this F, and with no sign connecting F and the number, is a multiplier of the F: thus, 10F, $\frac{2}{3}$F, mean ten times F, $\frac{2}{3}$ times F, or $\frac{2}{3}$ of F; and they have the same value as though they were written $10 \times F$, $\frac{2}{3} \times F$. F repeated once is not written 1F, as we should write 1s., but only F. These multipliers 10, $\frac{2}{3}$, and 1, are called *coefficients*. Moreover, since in the quantity 10F, F is the den^n, and the 10 tells us how many times F is taken, just as the 7 in 7s. tells us how many shillings are taken, therefore I can add to this 10F, or subtract from it, any

number of quantities of the same den^n, *i. e.* any number of times F.

$$\text{Thus, } 10F+3F=13F \qquad 10F-3F=7F$$
$$\text{and } 100F+F=101F \qquad 100F-F=99F.$$

107. I now proceed to find the fr^n which is equivalent to any circulating decimal, as

Ex. I. $\cdot\dot{3}\dot{6}$ or $\cdot 363636\ldots$

$$\text{Let } F=\cdot 363636\ldots \qquad \text{(G)}$$

Now if I multiply both sides of this equation by the same number, the equality will not be disturbed. My object is to move the point two places to the right in the decimal $\cdot 363636\ldots\ldots$, so that it shall become $36\cdot 3636\ldots\ldots$ This is done by multiplying both sides by 100; the left-hand side therefore becomes $100\times F$, or 100F; hence we have

$$100F=36\cdot 3636\ldots\ldots \qquad \text{(H)}$$
$$\text{also } F=\ \ \cdot 363636\ldots\ldots \qquad \text{(G)}$$

Subtracting (G) from (H), we observe that on the left-hand side, F or 1F taken from 100F leaves 99F as in (106); and on the right-hand side, since both decimal parts commence alike and go on for ever, their difference is 0; and after sub^n there remains only the integral number 36; hence we have

$$99F=36,$$

and dividing both sides by 99, we find

$$F=\frac{36}{99}=\frac{4}{11}. \qquad \text{(I)}$$

By observing (I) we learn that the fraction which represents the value of the circulating decimal $\cdot\dot{3}\dot{6}$ has for its num^r the figures in the period, viz. 36, and for the den^r, as many figures 9 as there are figures in that period.

In accordance with this rule we may therefore, by inspection, find the value of any circulating decimal which contains only such figures as do repeat.

$$\text{Thus, } \cdot\dot{6}=\frac{6}{9}=\frac{2}{3}$$

$$\text{and } \cdot\dot{1}4\dot{7}=\frac{147}{999}=\frac{49}{333};$$

but every pupil ought to work some Exs. out fully as in the beginning of this article.

108. Again, noticing (H) and (G) we see that the figures to the right of the decimal point are the same in both lines; and this result was produced by so multiplying both sides of the equation (G), that the point might be made to pass over one period; in this case it had to be moved two places to the right; that is, I had to multiply by 10 × 10, or 100. If the period had contained four figures, I should have multiplied by 10^4, or 10000. I will take as an Example

Ex. II. To express $\cdot\dot{2}21\dot{6}$ as a fraction.

$$\text{Let } F=\cdot22162216\ldots\ldots \quad \text{(K)}$$

$$\text{herefore, } 10000F=2216\cdot2216\ldots\ldots \quad \text{(L)}$$

Subtracting (K) from (L), we have

$$9999F=2216;$$

$$\text{therefore, } F=\frac{2216}{9999}:$$

and it is plain that this result might have been written down at once by taking as num^r of the required fr^n the period 2216, and as den^r a number consisting of as many figures 9 as there are figures in this period.

109. We now proceed to convert a mixed circulating 'nto a fraction. The nature of the process is the

same as in the last article; but one step more is required, the reason of which will appear in the operation.

Ex. III. To convert $\cdot 32\dot{7}1\dot{5}$ into a fraction.

$$\text{Let } F = 32715715\ldots\ldots \quad (M)$$

Then, as before, (107), carrying the decimal point over the period, or five places to the right—*i. e.* multiplying both sides by 100000, I have

$$100000F = 32715\cdot 715\ldots\ldots \quad (N)$$

It now appears that I cannot subtract (M) from (N), so as to get rid of the decimal parts, because the quantities to the right of the point are not the same: but I see that I can very readily obtain another equation which shall have the decimal part the same as in (N); for if in (M) I multiply both sides by 100, *i. e.* move the point two places to the right, or over the non-recurring part alone, I shall have

$$100F = 32\cdot 715715\ldots\ldots \quad (O)$$

therefore, subtracting (O) from (N), I have

$$[100000F - 100F = 32715\cdot 715\ldots - 32\cdot 715715\ldots] \quad (P)$$

$$\text{or } 99900F = 32715 - 32;$$

$$\text{therefore, } F = \frac{32715-32}{99900} \qquad (Q)$$

$$= \frac{32683}{99900}.$$

110. In the line (P) I learn that the larger coefficient of F has as many ciphers as there are figures from the point to the end of the first period; and the smaller coefficient has 2 ciphers, *i. e.* as many as there are figures in the non-recurring part: also, when the subtraction is *performed* in the next line, the coefficient of F has as many ciphers as there are figures in the non-recurring part: and the remaining figures to the left are all nines, and as many in number as there are figures

in the period. Hence, observing the right-hand side of (Q), I learn that in converting a mixed circulating decimal into a fraction I obtain as a num^r, "the figures of the given decimal to the end of the first period, *minus* the figures in the non-recurring part;" and as den^r, "as many figures 9 as there are figures in the period, followed by as many ciphers as there are decimal places in the non-recurring part." If there be an integer in the given recurring decimal, I may omit it while finding the value of the recurring part; and afterwards by inserting it I shall have the resulting fraction a mixed number; or I may retain it throughout, as in Ex. V. below, and the result will be an improper fraction.

I will write down one more Ex. of each kind of circulating decimals, in the form in which they ought to be worked.

Ex. IV. Find the fraction equivalent to $\cdot\dot{6}3\dot{9}$.

$$\text{Let } F = \cdot639639639\ldots\ldots \quad (\text{I.})$$

multiplying by 1000,

$$1000F = 639\cdot639639\ldots\ldots \quad (\text{II.})$$

Subtracting (I.) from (II.),

$$999F = 639,$$

$$\text{or } F = \frac{639}{999} = \frac{213}{333} = \frac{71}{111}.$$

Ex. V. Find the fraction equivalent to $2\cdot0\dot{3}4\dot{5}$.

$$\text{Let } F = 2\cdot0345345\ldots\ldots \quad (\text{III.})$$

multiplying by 10000,

$$10000F = 20345\cdot345\ldots\ldots \quad (\text{IV.})$$

Again, multiplying (III.) by 10, $10F = 20\cdot345345\ldots\ldots$ (V.)
and subtracting (V.) from (IV.)

$$9990F = 20345 - 20$$

$$\text{therefore, } F = \frac{20325}{9990} = \frac{6775}{3330} = \frac{1355}{666}$$

$$= 2\tfrac{23}{666};$$

or, omitting the integral part, as described above, I have

$$\text{the fractional part} = \frac{345}{9990} = \frac{69}{1998} = \frac{23}{666},$$

therefore, the entire quantity $= 2\frac{23}{666}$, as before; and this is the better method.

Exs. 24. Convert into vulgar fractions, in their lowest terms,

I. $\cdot\dot{1}$, $\cdot\dot{2}7\dot{3}$, $\cdot\dot{0}\dot{9}$, $\cdot\dot{0}10\dot{1}$, $15\cdot\dot{0}7\dot{5}$, $\cdot\dot{1}4285\dot{7}$.

II. $\cdot 1\dot{6}$, $\cdot 0\dot{2}\dot{7}$, $130\cdot\dot{2}8571\dot{4}$, $\cdot 000\dot{1}\dot{9}$, $\cdot 14285\dot{7}$, $35\cdot\dot{0}0\dot{9}$.

III. $\cdot 08451\dot{6}$, $60\cdot 0101\dot{4}$, $100\cdot 0\dot{3}\dot{6}$, $\cdot 16\dot{7}843\dot{2}$, $35\cdot 00\dot{9}$, $3500\cdot\dot{9}$.

ADDITION.

111. It has been stated in Part I. that the four elementary processes of Addition, Subtraction, &c. are applicable to quantities involving decimals; we have now to shew that the rules there given for their working are true.

Since we already know that quantities cannot be added together unless they be of the same den^n, so in the addition of decimals, we must take care to add tenths to tenths, hundredths to hundredths, and so on. And this will be readily done, if we so place all the numbers under one another, that the decimal points may be in a vertical row. By observing the Ex. worked below, we notice that the tenths are all placed under one another, as are also the hundredths, thousandths, &c.

Ex. I.

$$\begin{array}{r} 732\cdot 416 \\ \cdot 084 \\ \cdot 000007 \\ 93\cdot 268 \\ 2708\cdot 4153 \\ \hline 3534\cdot 183307 \\ \hline\hline \end{array}$$

Adding up the third row to the right of the point, which consists of thousandths, I find that it amounts to 23 thousandths, or $\frac{23}{1000}$, which $= \frac{20}{1000} + \frac{3}{1000} = \frac{2}{100} + \frac{3}{1000}$; I therefore put down the 3 thousandths, and carry the 2 hundredths to the next column, which consists of hundredths; and since this

step is just such as would be performed in Simple Addition, it is plain that all the rest of the work may be performed by that Rule.

112. But if in the quantities to be added there are circulating decimals, we may either, 1st, convert into a frⁿ each circulating decimal, and having found the sum of all these fractions, reduce the result to a circulating decimal: or, 2ndly, (and this method is the better,) write down the recurring decimals at length, to as many places as will include twice the longest period; observe where there are two vertical columns alike, though not necessarily close to each other; commence the addition three or four places to the right of the second of these two similar columns, and complete the addition, as before: the sum found will be seen to be carried far enough to enable a pupil to detect the period in the answer.

Ex. II. Find the sum of $\cdot\dot{7}1428\dot{5} + \cdot 9\dot{2}8571\dot{4} + 20\cdot 0\dot{9}2\dot{5} + 5\cdot 4\dot{0}4761\dot{9}$.

Writing these at length, and working according to the above directions, I have

$$\begin{array}{r}
\overset{A}{\cdot 7}14285\overset{B}{7}14285\ldots\ldots \\
\cdot 9285714285714\ldots\ldots \\
20\cdot 0925925925925\ldots\ldots \\
5\cdot 4047619047619\ldots\ldots \\
\hline
27\cdot 140211640\,20 \\
\hline\hline
\end{array}$$

Here I observe that the first two similar vertical columns are the second and the eighth, marked (A) and (B). Commencing the addition at the third column beyond (B), I find that I have figures enough in the result to shew that the sum of the given circulating decimals is $27\cdot 1\dot{4}0211\dot{6}$.

Exs. 25. Find the value of

1. $18\cdot 325 + \cdot 0007 + 70\cdot 1 + 358 + 3\cdot 04705 + 1000\cdot 06$.
2. $347\cdot 859 + \cdot 010101 + 639 + 2\cdot 573 + 11\cdot 01115 + \cdot 32784$.
3. $1\cdot 000009 + 45 + 3845\cdot 1 + 75\cdot 6832 + 10\cdot 01 + \cdot 04311$.
4. $7600 + 3\cdot 1009 + 473\cdot 842691 + \cdot 07 + \cdot 00001 + 1\cdot 1$.
5. $3458 + 5\cdot 143\dot{2}8\dot{5} + \cdot\dot{0}7\dot{5} + 145\cdot\dot{2}\dot{7} + \cdot\dot{8}7516\dot{9}$.
6. $\cdot\dot{3} + \cdot\dot{0}\dot{3} + 145\cdot 27\dot{3}4\dot{5} + 3\cdot\dot{0}0\dot{9} + 6\cdot\dot{1}4285\dot{7}$.
7. $\cdot\dot{6}8\dot{4} + 1\cdot\dot{5}8764\dot{9} + 3\cdot\dot{8}4100\dot{7} + \cdot\dot{0}\dot{7}$.
8. $1\cdot\dot{6} + 19\cdot 3485\dot{4} + \dot{0}\cdot 2\dot{7} + 5\cdot 34785\dot{6} + 111\cdot 1 + \cdot 0\dot{3}\dot{5}$.

SUBTRACTION.

113. As Addition of Decimals was shewn to be only an extension of Simple Addition, so Subtraction of Decimals is of the same nature with Simple Subtraction; but we must take care how we subtract, when the lower line contains more decimal places than the upper.

Ex. I. Find the value of $18{\cdot}0426 - 2{\cdot}005417$.

Placing the two quantities one under the other, so that the points are in a vertical row, we shall, as in Addition, have tenths under tenths, &c.

As there are no figures in the 5th and 6th places in the upper line, I may place ciphers there; and since there are no figures to the right of these ciphers, the value of the decimal will plainly remain unaltered.

$$\begin{array}{r} \text{A} \\ 18{\cdot}042600 \\ 2{\cdot}005417 \\ \hline 16{\cdot}037183 \\ \hline\hline \end{array} \qquad\qquad \begin{array}{r} 600 \\ 417 \\ \hline 183 \\ \hline\hline \end{array}$$

If we now take the last three figures in each row as whole numbers, so as to form a Simple Subtraction Sum, we find the difference to be 183: so also, since the 600 and 417 in (A) both represent millionths, their difference is 183 millionths: similarly the 42 and 5 are both thousandths, and their difference = 37 thousandths: hence we see that if the subtraction be performed in (A) as in a common Ex. in Subtraction, the result will be correct.

114. If an Ex. in Subtraction contain circulating decimals, we must, as in Addition, write down the periods sufficient to contain twice the longest period, and commence the subtraction about five places beyond the latter one of the two similar rows; the remainder which will recur will be found to contain figures enough to enable us to detect the period.

Ex. II. Find the value of $24{\cdot}\dot{4}0\dot{7} - 5{\cdot}98\dot{0}7697\dot{3}$.

Here, writing five of the periods in the upper line, and two in the lower line, we have

$$\begin{array}{r}
\overset{A}{}\qquad\quad\overset{B}{}\qquad\qquad\\
24{\cdot}407407407407407\ldots\\
5{\cdot}98076973076973\ldots\\
\hline
18{\cdot}4266376766377\\
\hline\hline
\end{array}$$

The columns marked (A) and (B) are the first pair which are similar; I therefore commence the subtraction at the fifth row beyond (B): and the remainder, which contains the circulating decimal, is evidently $18{\cdot}42\dot{6}6376\dot{7}$.

By observing the Ex. in Addition of circulating decimals, I find that the figures which should form the second period are somewhat below the value of those in the first period; but by commencing with the addition a little more to the right, I should have figures to *carry*, as it is called, which would have made the second period correct as well as the first. So in the Ex. just worked in Subtraction, the figures which should form the second period are too large; but by commencing the Subtraction a few more places to the right, the process of *borrowing* would make the figures of the second period quite correct.

Exs. 26. Find the value of

I. $3{\cdot}5-{\cdot}075$; $175{\cdot}8-1{\cdot}0024$; ${\cdot}715-{\cdot}70451$.

II. $1{\cdot}325-{\cdot}4736$; $10{\cdot}01-9{\cdot}875$; $1{\cdot}\dot{3}\dot{6}-{\cdot}75$; ${\cdot}\dot{3}-{\cdot}\dot{1}$.

III. ${\cdot}\dot{3}-{\cdot}07$; $3{\cdot}0758-1{\cdot}\dot{2}7\dot{3}$; $13{\cdot}4\dot{0}8\dot{9}-6{\cdot}85\dot{3}7492\dot{1}$.

MULTIPLICATION.

115. The rule for the multiplication of decimals is to be found by multiplying the vulgar fractions equivalent to any given decimals, and then observing the nature of the product so obtained.

Ex. I. To find the product of $3{\cdot}275$ and $18{\cdot}03$.

We have $3{\cdot}275\times18{\cdot}03=\dfrac{3275}{1000}\times\dfrac{1803}{100}$

$$=\frac{3275\times1803}{100000}=\frac{5904825}{100000}\qquad(\text{I.})$$

$$=59{\cdot}04825.$$

From (B) we learn that the product of the given decimals is a decimal fraction which has for its numerator the product of the given numbers treated as integers; and for its denominator, 1, followed by as many ciphers as there are decimal places in *both* the multiplier and multiplicand; and such a fraction is of course immediately (95) convertible into a decimal, which shall have as many decimal places as this fraction has ciphers; and this conversion is performed in the next line. Hence, as this Ex. differs not from any other which contains only terminating decimals, we conclude that the product of two such decimals is found by multiplying them together as in whole numbers, and pointing off in the product as many places as are found in both multiplier and multiplicand.

116. Also, if it be required to multiply together more than two numbers involving decimals, it will be found that the same rule must be observed. For suppose that four numbers were given, whose product was required: the product of the first pair might be found, as above; then this product and the third number might be multiplied: and, lastly, this second product and the fourth number: and so on for any number of quantities; hence we may find the product of any number of decimal quantities, as in the previous Example.

Ex. II. Find the value of $\cdot 0095 \times 2\cdot 07 \times 7\cdot 06 \times \cdot 0081$.

$$\text{The product} = \frac{95}{10000} \times \frac{207}{100} \times \frac{706}{100} \times \frac{81}{10000}$$

$$= \frac{95 \times 207 \times 706 \times 81}{1000\,000\,000\,000} = \frac{1124562690}{1000\,000\,000\,000}$$

$$= \cdot 001124562690$$

where the number of decimal places, viz. 12, = sum of the numbers of places in the four given quantities.

117. When a few Exs. have been worked illustrating the truth of the principles here stated, we may then work all similar Exs. as in Simple Mult^{n} of whole numbers, and point off as many decimal places in the product, as there are in all the numbers to be multiplied together.

Thus Ex. I. would have been commonly worked as follows;

```
     3·275
    18·03
     9825
  262000
  3275
  59·04825
```

Much instruction may also be derived from working an Ex. in the following manner:

Ex. III. To find the product of 271·405 and 93·6854.

```
  271·405
   93·6854
  814·215
  162·8430
   21·71240
    1·357025
     ·1085620
24426·45
25426·6859870
```

Placing the multiplier under the multiplicand in any position whatever, I commence multiplying by the figure in the units' place, viz. 3. Since then I am now merely repeating every figure in the multiplicand 3 times, therefore every figure when multiplied will give a product of the same *kind* as itself, and that product will occupy the same place with respect to the point as it did in the mult[d]; hence the product will clearly be 814·215.

If, now, I multiply by the 6, which is in the *tenths'* place, the product will be ten times less than it would have been, had the 6 been in the *units'* place; hence I place every figure in this product one place farther to the *right* than in the previous line; so, also, the product by the 8 will be *two* places to the right, by the 5, will be *three* places, &c.

Again, since the 9 in the multiplier represents 90, I place the product obtained by multiplying by the 9 one place more to the *left* than the first product; and if there were any figures in the places for hundreds, thousands, &c. of the multiplier, I should place the corresponding products two, three, &c. places to the left. The whole of the work will now be intelligible.

The above row of products may be written down in any order we please; provided that, in commencing the multiplication by any other figure than the one in the units' place, we use proper caution in placing the product in its proper situation with respect to the decimal point.

Exs. 27. Express as simple decimals

1.	1·5 × 1·5.	6.	1·5 × 3·15 × 2·17.
2.	2·375 × 3·48.	7.	·008 × 5·5 × 1·4.
3.	·006 × 78·928.	8.	3·75 × ·014 × ·875.
4.	1·0006 × 461·8.	9.	3·125 × 14·25 × ·01.
5.	10·375 × ·0074.	10.	35·01 × 7·98 × 1·0001.

118. Next, let the decimals to be multiplied together be either one or both of them circulating. These may be converted into equivalent fractions; and then, after having multiplied them, we may convert the product into a circulating decimal; or the product of the decimals themselves may be found by Simple Multiplication, provided that there be taken a sufficient number of figures to enable us to ascertain the period in the product. We will work two Exs. to illustrate both methods.

Ex. V. Find the value of $\cdot\dot{3}\dot{6} \times 75$.

By the first method I have

$$\cdot\dot{3}\dot{6} \times 75 = \frac{\overset{4}{\cancel{36}}}{\underset{11}{\cancel{99}}} \times 75 = \frac{300}{11} = 27\cdot2727\ldots\ldots = 27\cdot\dot{2}\dot{7}.$$

By the second method, I write down the period three times, so that there may be figures enough in the product to shew its period.

$$\begin{array}{r} \cdot363636\ldots \\ 75 \\ \hline 1818180 \\ 2545452 \\ \hline 27\cdot272700 \\ \hline\hline \end{array}$$

If a fourth period had been written in the multiplicand, the figures carried from the multiplication of it would have caused the period 27 to have been seen in the fifth and sixth decimal places in the product: and every additional period in the multiplicand would have produced one more period in the product: hence, since the number of periods, 36, in the multiplicand is unlimited, so, also, will be the number of periods, 27, in the product; *i. e.* the product is $27\cdot\dot{2}\dot{7}$, as before.

Ex. VI. Find the value of $37\cdot\dot{3} \times 9\cdot1\dot{6}$.

Upon working this Ex. by ordinary Multiplication, I find that even if I write down five periods in the multiplicand, and four in the multiplier, yet the product is such that the learner would hardly detect the period in it; and if the periods had contained several figures, the work would be exceedingly heavy; and since in such Exs. involving two or more recurring decimals, the former method is the better, I give it alone. I then have

$$37\cdot\dot{3} \times 9\cdot1\dot{6} = 37\tfrac{3}{9} \times 9\tfrac{15}{90} \qquad \text{by (110)}$$

$$= 37\tfrac{1}{3} \times 9\tfrac{1}{6}$$

$$=\frac{\overset{56}{\cancel{112}}}{3}\times\frac{55}{\underset{3}{\cancel{6}}}$$

$$=\frac{3080}{9}=342\cdot\dot{2}.$$

Exs. 28. Form the following products:

1. $\cdot\dot{3}\times 1\cdot 78$.
2. $1\cdot\dot{2}\dot{7}\times\cdot 0458$.
3. $13\cdot\dot{0}07\dot{9}\times 4\cdot\dot{5}$.
4. $27\cdot\dot{3}\times 4\cdot\dot{5}$.
5. $1\cdot\dot{6}\times\cdot\dot{3}48\dot{5}$.
6. $3\cdot 58\times\cdot\dot{2}\dot{1}\times 4\cdot 1\dot{6}$.

DIVISION.

119. In the Exs. under this head, either dividend or divisor, or both, may be a decimal. I will give one Ex. of each variety.

Ex. Find the value of $37\cdot 5\div 84$.

Where, as in this Ex., the divisor is a composite number containing no prime factor greater than 12, we can divide by successive factors, as in (100); thus, dividing first by 12, and then by 7,

$$\frac{37\cdot 5}{84}=\frac{3\cdot 125}{7}=\cdot 44642857142, \text{ \&c.}$$

$$=\cdot 446\dot{4}2857\dot{1}.$$

120. Where there are decimals in the divisor, we have, as in Part I., the following general rule:

RULE. Count the number of decimal places in the divisor; count also the same number in the dividend, and make a mark (\`) cutting off all the remaining figures of the dividend. If there be not as many places in the dividend, as in the divisor, add ciphers to make up the number, and then make the mark.

Divide as in whole numbers; all the figures in the quotient obtained by using the figures in the dividend to the left of the mark will be whole numbers. When these figures have been used, place the decimal point in the quotient; the remaining figures obtained by division will be decimals.

121. Obs. It may happen that the divisor is larger than that part of the dividend to the left of the mark; in this case there will be no whole numbers in the quotient. And if, when the mark is passed, the divisor is still too large, a cipher must be placed in the quotient after the point, for every figure that is used beyond the mark, until the dividend becomes large enough to contain the divisor, and therefore until a figure, other than 0, appears in the quotient.

And this will be seen at once to be correct, if we observe that the part of the dividend so cut off to the left, and the whole divisor, may be considered as thereby reduced to the *same* name, viz. the lowest in the divisor, as tenths, hundredths, as the case may be; hence the division up to the mark will give a whole number; and if continued beyond the mark, the dec^{l} point must be then placed, and the remaining figures of the quotient will be decimals.

In the following example I make the mark (') so as to retain *four* decimals to the left of it, because there are *four* in the divisor, and work precisely as with integers, placing the dec^{l} point in the quotient, as soon as the mark has been passed.

Ex. III. Find the value of 41·0632884 × ·0438.

```
·0438) 41·0632'884 (937·518
       3942
       ----
        1643
        1314
        ----
         3292
         3066
         ----
          2268
          2190
          ----
            788
            438
           ----
           3504
           3504
           ====
```

In the next example, since there are *two* places in the divisor, and the dividend is a whole number, I place the dec^{l} point after the dividend, and append *two* ciphers, and then place the mark.

Ex. IV. $9717 \div 1{\cdot}23$.

$$\begin{array}{r|l} 1{\cdot}23 & 9717{\cdot}00^{\prime}\ (7900 \\ & \underline{861} \\ & 1107 \\ & \underline{1107} \\ & 00 \end{array}$$

Sometimes, also, though there be a sufficient number of decimal places in the dividend to give one or two places in the quotient, yet if it is required that there should be 5 or 6 decimal places in the quotient, ciphers may be appended to the dividend, as before, and the division continued as far as we please.

Ex. V. $62{\cdot}5 \div {\cdot}025$.

For the sake of illustration, I work this Ex. in two ways, independently of any rule; first, by removing the point three places to the right in the numerator and denominator, and secondly, by converting the given decimals into fractions, and performing the division by the usual method of inverting the divisor.

$$\frac{62{\cdot}5}{{\cdot}025} = \frac{62500}{25} = \frac{12500}{5}$$

$$= 2500$$

$$\text{or, } \frac{62{\cdot}5}{{\cdot}025} = \frac{625}{10} \div \frac{25}{1000} = \frac{\overset{25}{\cancel{625}}}{\cancel{10}} \times \frac{\overset{100}{\cancel{1000}}}{\cancel{25}}$$

$$= 2500, \text{ as before.}$$

Exs. 30. Find the required quotients in the following examples:—

1. $13{\cdot}5 \div {\cdot}15$.	4. $345{\cdot}6 \div 1{\cdot}728$.	7. $576{\cdot}84325 \div 119{\cdot}3$.
2. $83{\cdot}75 \div {\cdot}128$.	5. $13{\cdot}358697 \div {\cdot}634$.	8. $3{\cdot}84765 \div 1536$.
3. $1080 \div {\cdot}008$.	6. ${\cdot}084007 \div 34{\cdot}3$.	9. $1{\cdot}0005 \div 106{\cdot}3$.

122. If either or both of the given decimals circulate, the circulator may be converted into a proper or improper fr^{n}, as the case may be, and the division proceeded with as in fractions; or the circulator may be left unaltered, if in the dividend, and only the divisor be converted into a fr^{n}, and the division then performed.

Ex. I. Find the value of $75 \div \cdot\dot{1}4\dot{8}$.

$$\text{By (108)} \qquad \cdot\dot{1}4\dot{8} = \frac{148}{999} = \frac{4}{27} \text{ (in lowest terms)}$$

$$\text{therefore } \frac{75}{\cdot\dot{1}4\dot{8}} = \frac{75}{\frac{4}{27}} = \frac{75 \times 27}{4} = \frac{2025}{4} = 506\cdot25.$$

Ex. II. Find the quotient of $\cdot96\dot{3}4\dot{5}$ when divided by $\cdot\dot{3}$.

$$\cdot\dot{3} = \frac{3}{9} = \frac{1}{3}$$

$$\text{therefore } \frac{\cdot96\dot{3}4\dot{5}}{\cdot\dot{3}} = \frac{\cdot96\dot{3}4\dot{5}}{\frac{1}{3}} = (\cdot96345345\ldots) \times 3$$

$$= 2\cdot89036035\ldots$$

$$= 2\cdot89\dot{0}3\dot{6}.$$

123. It has been stated in (113) and (121) that ciphers may be written after the last figure of a decimal without altering its value: similarly, they may be cut off without affecting it; and this is generally done, if the decimal resulting in any Ex. have ciphers at the end. Thus in Art. 116, Ex. II., I might have removed the cipher which is at the end of the final decimal; only that a pupil would have thought that there were pointed off only 11 decimal places, instead of 12: but, by referring to the last vulgar fraction used in that Ex., I find that it might have been reduced to lower terms, by dividing numerator and denominator by 10; and there would then have been but 11 ciphers in the den^r, and consequently 11 places in the decimal which follows.

Exs. 32. Find the required quotients in the following Examples:—

1. $\cdot\dot{3}\dot{6} \div \cdot072$.
2. $27\cdot\dot{5} \div \cdot0\dot{6}$.
3. $34\cdot\dot{7}\dot{5} \div 1\cdot\dot{5}\dot{6}$.
4. $1\cdot23123\ldots \div 3\cdot\dot{6}\dot{3}$.
5. $\cdot18 \times \cdot09 \div \cdot1\dot{6}$.
6. $13\cdot\dot{7}\dot{5} \div (1\cdot\dot{3} + 5\cdot\dot{6})$.

REDUCTION OF DECIMALS.

124. We have here to perform in Decimals the operation which in (62) was performed in Vulgar Fractions; and this merely requires that the decimal quantity should be reduced to successive lower denominations, until we have either no decimal part remaining, or until we reach the lowest denomination used.

Ex. I. Express in positive terms ·375 of £1.

$$\cdot375 \text{ of } £1 = \cdot375 \times 20s. = 7\cdot5\cancel{00}s.;$$
$$\text{and } \cdot5s. = \cdot5 \times 12d. = 6\cdot0d. = 6d.;$$
$$\text{therefore } \cdot375 \text{ of } £1 = 7s.\ 6d.$$

$$\begin{array}{r} £ \\ \cdot375 \\ \underline{20} \\ 7\cdot5\cancel{00}s. \\ \underline{12} \\ \underline{\underline{6\cdot0d.}} \end{array}$$

The work may be written out as annexed: where it is plain that at the end of each successive multiplication I point off as many decimals as there were in the preceding line, because there are no decimal places in the multiplier.

It will be seen that in both the above operations I might have omitted two ciphers at the end of the decimal part in the shillings, for the reason given at the close of the last article. I have therefore crossed them out.

By using the equalities mentioned at the end of (101), and referring to the aliquot parts of different denominations given in (64), we might have worked this Ex. very briefly.

$$\text{Thus, } \cdot375 \text{ of } £1 = \tfrac{3}{8} \text{ of } £1 = 7s.\ 6d.$$
$$\text{So, also, } \cdot875 \text{ of } £1 = \tfrac{7}{8} \text{ of } £1 = 17s.\ 6d.$$

Ex. II. Find the value of 7·14685 of 5s. $6\frac{3}{4}d.$

Here, since the concrete number is expressed in several denominations, I must either reduce the decimal to a fraction, or reduce the 5s. $6\frac{3}{4}d.$ to the fraction of a penny, and then perform the multiplication.

By the second method, giving only the principal steps of the work:

$$\begin{aligned} 7\cdot14685 \times 5s.\ 6\tfrac{3}{4}d. &= 7\cdot14685 \times 66\tfrac{3}{4}d. \\ &= 7\cdot14685 \times 66\cdot75d. \\ &= 477\cdot0522375d. \\ &= 39s.\ 9\cdot0522375d. \\ \textit{Answer.} &= £1\ 19s.\ 9\cdot0522375d. \end{aligned}$$

Just as in (68) I left the remainder as a fractional part of a penny, so here I leave it as a decimal fraction of a penny.

125. If any circulating decimals occur in Exs. under this head, we must reduce them to fractions, and proceed as before.

Ex. III. Find the value of $\cdot\dot{5}$ of £1 11*s.* 4*d.*

$$\cdot\dot{5} \text{ of £1 } 11s.\ 4d. = \frac{5}{9} \text{ of } 31\tfrac{1}{3}s.$$
$$= \frac{5}{9} \times \frac{94}{3}s. = \frac{470}{9 \times 3}s. = \frac{156\cdot666...}{9}s.$$
$$= 17\cdot4074074...s.$$

and
$$\cdot407407...s. = (\cdot407407...) \times 12d.$$
$$= 4\cdot888884...d.$$
$$= 4\cdot\dot{8}d.$$
$$= 4\tfrac{8}{9}d.$$

therefore $\cdot\dot{5}$ of £1 11*s.* 4*d.* = 17*s.* $4\frac{8}{9}$*d.*

Exs. 33. Express in positive terms

1. 1·375 of 1 guinea.
2. ·028 of a moidore.
3. 375·794 of £5.
4. 1·115 of a crown.
5. ·148325 of 7*s.* 6*d.*
6. 49·864 of £15 10*s.*
7. ·05 of $7\frac{1}{2}$*d.* + ·375 of 3*s.* 6*d.*
8. 11·05 of £2 — 17·5 of 6*s.* 8*d.*
9. ·375 of a mile + 7·5 of a yard.
10. 1·185 of a cwt. — ·0375 of a qr.
11. $\cdot\dot{5}$ of 6*s.* 8*d.* + $1\cdot\dot{2}\dot{7}$ of a guinea.
12. $\cdot\dot{0}3\dot{7}$ of 27*s.* — ·02 of £2.

126. The following operation is the converse of that performed in the last two Arts., and corresponds to that exhibited in (78) in Vulgar Fractions.

Ex. Reduce 4*s.* $6\frac{1}{2}$*d.* to the decimal of a sovereign.

In working this Ex. we shall first reduce the former of the given quantities to the fraction of the latter, and then convert into a decimal the vulgar fraction connecting the two quantities.

$$\frac{4s.\ 6\frac{1}{2}d.}{£1} = \frac{54\frac{1}{2}d.}{240d.} = \frac{54\cdot5}{240} = \frac{4\cdot54166...}{20} = \frac{2\cdot2708\dot{3}}{10} = \cdot22708\dot{3},$$

or 4*s.* $6\frac{1}{2}$*d.* = $\cdot22708\dot{3}$ of £1.

The mode most commonly adopted for obtaining this result is the converse of the operation exhibited in the margin of (124); thus,

$$\begin{array}{r|l} 2) & 1\cdot \\ \hline 12) & 6\cdot5 \\ \hline 20) & 4\cdot54166\ldots \\ \hline & \cdot2270833\ldots \end{array}$$

where, in the second line, after the division by 2 to bring the halfpenny into a decimal of 1*d.*, the 6 is placed before the point, giving a dividend 6·5*d.*; and in the third line, after the division by 12, the 4, denoting 4*s.*, is placed before the point.

This method is perhaps the readiest in practice; but I think it is seldom performed by pupils otherwise than mechanically; and it is not suitable to many of the Exs. given below; indeed only to such as merely involve a change from one simple denomination to another in the Tables of money, &c.

Exs. 34.

1.	Reduce	2*s.* 6*d.*	to the	decimal fraction of	15*s.*
2.	„	3*s.* 7*d.*	„	„	£5.
3.	„	25*s.*	„	„	3 guineas.
4.	„	£17 15*s.* ..	„	„	£100.
5.	„	354 yds.	„	„	1 league.
6.	„	11440 yds...	„	„	2 acres.
7.	„	$3\frac{1}{2}$ qrs.	„	„	15 tons.
8.	„	6 hours	„	„	135 days.
9.	„	1 leap year	„	„	3 weeks 4 days.
10.	„	$\frac{4}{5}$ of a mark	„	„	$\frac{8}{11}$ of a crown.

MISCELLANEOUS EXAMPLES.

127. The same remark applies to these Miscellaneous Examples in Decimals that applied to the corresponding Exs. in Fractions: and the only general assistance that can be given to a pupil is to shew him the neatest method of performing the operations required.

Ex. I. Multiply £8 17*s.* 6*d.* by 75·25, and reduce the result to the decimal of £100.

$$\text{Here } \frac{(\pounds8\ 17s.\ 6d.)\times(75\cdot25)}{\pounds100}=\frac{\pounds8\frac{7}{8}\times75\cdot25}{\pounds100}$$

$$=\frac{8\cdot875\times75\cdot25}{100}$$

$$=\frac{667\cdot84375}{100}$$

$$=6\cdot6784375.$$

Ex. II. Reduce $\frac{3\cdot275}{405}$ of $\frac{2\cdot5}{\cdot075} \times \frac{3\cdot125}{11} \times \frac{9}{9\cdot375}$ to a simple quantity.

Moving the point three places to the right in two numerators and in two denominators, and again one place to the right in the numerator and the denominator of the second fraction, the expression becomes

$$\frac{\overset{131}{\cancel{3275}}}{\underset{9}{\cancel{405}}} \times \frac{\cancel{25}}{\underset{6}{\cancel{750}}} \times \frac{\cancel{3125}}{11} \times \frac{\cancel{9}}{\underset{3}{\cancel{9375}}} = \frac{131}{9 \times 6 \times 11 \times 3}$$

$$= \frac{11\cdot90909...}{9 \times 6 \times 3}$$

$$= \frac{1\cdot32323232...}{6 \times 3}$$

$$= \frac{\cdot22053872053872...}{3}$$

$$= \cdot07351290684624..., \text{ \&c.}$$

Since a very large number of decimals is non-terminating, it might seem that vulgar fractions, which are always expressed in finite terms, would be preferable for every purpose. But this is not the case, for decimals have one advantage over fractions from the following consideration.

In ascertaining the comparative value of two or more fractional quantities, if they be expressed as vulgar fractions, it is necessary to reduce them to a common denominator; but if they are represented as decimals, mere inspection will detect their comparative value, as readily as can be done in whole numbers.

For example, if we have to compare $\frac{7}{15}$, $\frac{11}{25}$, $\frac{13}{28}$, we cannot see which is the greatest, and which the least, without reducing them to a common denominator: but if the quantities had been written in their decimal form, viz.

$$\cdot4666........;\quad \cdot44;\quad \cdot46\dot{4}2857\dot{1},$$

we could see *at once* that the first is the largest; the last one is the next; and the middle one is the smallest: therefore,

in order of magnitude they are $\frac{7}{15}$, $\frac{13}{28}$, $\frac{11}{23}$. The decimal form is more especially useful, when several fractional quantities are arranged in a table, and where it is requisite to be able to compare the different quantities at a glance.

We may take as an Ex. the accompanying table, in which the numbers represent the comparative weights of equal bulks of different substances; or, as they are generally termed, their Specific Gravities.

Sheet Glass	3·33
Plate Glass	2·5
Marble	2·716
Quartz	2·6
Rock Salt	1·92
Ivory	1·917
Ice at 0°	·926
Water at 60°	1·

Though the 5th and 6th numbers are very nearly equal, yet it can be seen at once that the 5th is larger than the 6th, by three thousandths: as fractions, these quantities would have been written, $1\frac{23}{25}$; and $1\frac{917}{1000}$; and the difference could not have been ascertained by inspection.

Exs. 35.

1. Express as a simple decimal the difference between $\frac{3}{5}$ of $\frac{11}{18}$ and $\frac{7}{8}$ of $\frac{9}{14}$.

2. Simplify the following expression:

$$\frac{1\cdot5}{\cdot075}\times\frac{3\cdot25}{1\frac{1}{2}}+\frac{1\cdot875}{2\cdot1}\times\frac{3\cdot5}{3\cdot75}.$$

3. How many times must ·35 of a groat be repeated, to produce $\frac{1}{2\frac{6}{7}}$ of a crown?

4. Form the following quantities into a decimal table, as in (127), arranging them in order of magnitude:

$$2\tfrac{7}{8},\qquad 1\tfrac{3}{4}\text{ of }1\cdot06,\qquad \tfrac{2}{7\cdot5},\qquad 1\cdot75\text{ of }3.$$

5. What is the average value of the above quantities, expressed as a vulgar fraction?

6. Find the simple decimal equivalent to $1\cdot\dot{3}\dot{1}\times(2\cdot\dot{4}+7\cdot\dot{5})$.

7. Express in positive terms the sum of $1\cdot0\dot{5}$ of a crown, $\cdot\dot{7}1428\dot{5}$ of a guinea, and $\cdot1\dot{6}$ of 6*s.* 8*d.*

8. Convert $\frac{5}{16}$ into a decimal, by multiplying both num^r and den^r by some common quantity.

9. Reduce $2\frac{2}{5}$ of $14\frac{1}{2}$ acres to the decimal of a square mile.

10. Find in a decimal form a fourth proportional to each of the following sets of numbers:

$$1,\ \cdot 2,\ \cdot 375;\qquad 3\cdot 25,\ \cdot 0175,\ 1\cdot 01;\qquad \frac{1\frac{1}{4}}{3},\ \tfrac{3}{7},\ 2\cdot 75.$$

Exs. 36. E.

1. The product is 154923000, and multiplicand 42375; what is the multiplier?

2. How many calendar months in 3¼ centuries?

3. If 90 degrees = 100 grades, find the number of degrees in 12½ grades.

4. A man enters into business with £20,000, and each year makes a profit of one-fourth of his investment, and adds that profit to his capital; how much will he be worth in 5 yrs.?

5. How many fathoms are there in a degree?

6. If there are 360^{0} in every circle, and in latitude 80^{0} a degree $= 34\cdot 75$ miles, what is the length of the circle which passes through latitude 80^{0}?

7. What is the average length of the calendar months, including leap year?

8. Find the abstract number which expresses the ratio of £12⅜ and 17⅗ shillings.

9. Assuming that the number of square feet in the area of an oblong surface is found by multiplying the number of feet in the length by the number in the breadth, find how many bricks, each 9 in. by 4½ in., are required to pave a floor 97½ ft. long, and 81 ft. broad.

10. Write an equation involving the signs of add^{n}, sub^{n}, $mult^{n}$, and div^{n}, and having the right-hand side = 237.

11. Explain the object and the process of reducing fractions to a C. D. Reduce to L. C. D. ⅔, ¾, ⅘, 5/9, explaining the work.

12. Convert into a vulgar fraction, in lowest terms, the sum of 9 tenths, 3 hundredths, and 5 thousandths.

13. Multiply the difference between 75 hundredths and 5 tenths, by 8 thousandths; expressing the result as a vulgar fraction.

14. Reduce the following expression to a simple decimal:—

$$\tfrac{1}{3}\,(6\tfrac{1}{2} + 2\tfrac{2}{3} - 3).$$

15. What decimal of £1 is ⅗ of 13*s*. 4*d*.?

F.

1. There are 100 links in a chain of 4 perches; how many links in a league?

5—2

2. The wheel of a locomotive which is 16½ ft. in circumference, turns round 25764 times, how many miles will it have run?

3. Find the cost of half an acre of building land, at 3*s*. 6*d*. per sq. yd.

4. What is the expense of painting the walls of a room, 17 feet long, 13 ft. broad, and 10 ft. high, at 7*d*. a square yard?

5. On a railway, from *A* to *B* there is a rise of 1 in 160 for $\frac{3}{4}$ mile, of 1 in 330 for 2 miles 50 yards, then a fall of 1 in 110 for 1 mile 360 yards, lastly, a rise of $\frac{1}{1320}$ for 1500 yards; what is the height of *A* compared with *B*?

6. A general sends away $\frac{13}{25}$ of his army, and then $\frac{3}{8}$ of the remainder; he has now 1350 men, what had he at first?

7. Find the exact value of $\cdot 1\dot{6}$ of a moidore + $\cdot\dot{5}7142\dot{8}$ of 2 guineas.

8. How many allotments, each 2r. 25p., are contained in 47½ acres?

9. If there are 100 links in a chain of 22 yards, find how many square links are contained in an acre.

10. A bankrupt owes £3840, and his property amounts to only £1656; how much will his creditors receive in the pound?

11. If a board be 23½ inches broad, how long must it be to contain 15 square feet?

12. If 500 slates would cover a surface 25 feet square, how many would be required for a roof 27 ft. by 36?

13. What number divided by $17\frac{1}{3}$ will give $13\frac{7}{8}$?

14. To $\frac{9}{10}$ of a gross add $\frac{3}{5}$ of a quarter of a hundred; from this sum subtract $\frac{7}{8}$ of a score, and divide the remainder by $17\frac{1}{3}$.

15. Simplify the expression $\frac{3}{5}$ of $\frac{10}{27}$, proving that the mode of working is correct; and explain what is called *cancelling*.

G.

1. If the interest of the national debt be £1 2*s*. 6*d*. each second of time, what is the amount per annum?

2. How many bottles in 103 casks, each containing 9¾ dozen?

3. A person pays a debt of £230 8*s*. in sovereigns, half sovereigns, crowns and shillings, of each an equal number; how many of each?

4. What is the amount of the following items, 54¼ lbs. at 5*s*. 4½*d*., 241 yds. at 7½*d*., and 512 pieces at 2*s*. 4¾*d*.?

5. Express as a Vulgar Fraction, in its lowest terms, the sum of 7 hundredths, 5 thousandths, and 375 tenths of thousandths.

6. How much paper 1¼ yard broad, will cover as much as 20 yards, of ¾ yd. broad?

7. If 4 lb. of silver be mixed with 4 lb. 5 oz. of gold, how much silver will there be to 6 oz. of gold?

8. If one whose rent is £430 pays a tax of £30 6*s*. $\dot{9}$*d*., what should be the rent of a man whose taxes come to £94 16*s*. $1\frac{1}{8}$*d*.?

9. Gunpowder being composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts; find how much of each is required in making 16 cwt. of powder.

10. I take successively $\frac{1}{3}$ and then $\frac{1}{4}$ of a sum of money, and find that I have left £15; what was the sum at first?

11. Explain the two methods of multiplying a fraction by a whole number, taking as an Ex. $\frac{5}{8} \times 3$.

12. In a bridge of 7 arches, the middle one is 75 feet span, and the others on each side are $\frac{1}{10}$th less in each succeeding arch; find the whole length of the bridge, allowing 15 feet for each pier.

13. Find the value of $\left(\frac{4}{5\frac{1}{3}} \text{ of } £5\right) \sim 3\cdot\dot{4}65\dot{3}$ of a guinea.

14. What is the price per lb. of an article, of which 1·5 cwts. cost 113·75 shillings?

15. Express in positive terms $\cdot 37\dot{6}$ of 27*s*. 6*d*. $\sim 1\cdot\dot{8}$ of 2 guineas.

H.

1. How many bottles of wine in 12 pipes, at the rate of 52 dozen 9 bottles each pipe?

2. Find the number of square yards in an area, $\frac{1}{2}$ a mile long, and $\frac{1}{4}$ of a mile broad.

3. If one man thrash 7 sheaves of corn in a day, and each sheaf yield $3\frac{1}{2}$ pecks, and each peck 15 lbs.; how much in quantity and weight will 15 men thrash in 6 weeks?

4. A general having an army of 24,000 men, increases it one-third by recruiting; afterwards he loses one-fourth by disease, and of the remainder one-fifth fell in battle; how many men has he left?

5. How many fathoms in a degree?

6. What is the ratio of a geographical mile to a British mile? How many geographical miles must I measure, so as to contain the least exact number of British miles?

7. If a person step at an average $2\cdot 1\dot{6}$ feet, how many steps must he take in ·325 miles?

8. Reduce $\frac{7}{256}$ to a decimal without using Long Division; and shew that $\frac{3}{125} = \cdot 024$, without dividing at all.

9. Find the value of $4\cdot 25 \div \cdot 10625$, proving the result by Vulgar Fractions.

10. Write in words ·75, 2·0324, 17·000001.

11. Find the value of (£25 16*s*. $7\frac{3}{5}$*d*.) × 8[illegible]$\frac{11}{12}$.

12. The circumferences of the fore and hind wheels of a carriage are respectively $9\frac{3}{7}$ ft. and $13\frac{8}{11}$ ft.; find how many more revolutions one makes than the other in $10\frac{3}{8}$ miles.

13. What fraction of a guinea and a half, together with £3 16*s*. 9*d*., will give £4?

14. Exhibit as a simple vulgar fraction the result of

$$\cdot 3\dot{7} \div 1\cdot 17 \times \cdot 052 \div 1\cdot\dot{8}.$$

15. Find the ratio between the product and quotient of 147 and $\cdot\dot{2}\dot{7}$.

PRACTICE.

128. **Practice** is a rule which endeavours to shew the readiest method of finding the cost of any number of articles at a certain price: and the work exhibited in any Ex. consists of a series of amounts such as a person would try to obtain, if he were working the question mentally.

Thus, if I had to find the value of 54lbs. at $1\frac{1}{2}d.$ each, I should say, 54 at $1d. = 54d. = 4s.\ 6d.$; and 54 at $\frac{1}{2}d. = 54$ halfpence $= 27d. = 2s.\ 3d.$: and therefore 54 at $1\frac{1}{2}d. = 4s.\ 6d. + 2s.\ 3d. = 6s.\ 9d.$

129. But if the number of articles had been much larger, or the price much greater, the value of them could not readily have been obtained mentally: we therefore, in Practice, use the above method of mental calculation, but we write down the successive results, and find their sum for the final result.

The Exs. to be worked under this Rule may be arranged as follows:—

When the price is under a shilling, as

Ex. I. 4108 at $7\frac{3}{4}d.$

When the price is between $1s.$ and £1, as

Ex. II. 4103 at $7s.\ 5\frac{1}{4}d.$

Ex. III. 6009 at $19s.\ 5\frac{1}{4}d.$

When the price consists of more than £1, as

Ex. IV. 7111 at £1 17*s*. $4\frac{1}{2}$*d*.

Ex. V. 4013 at £12 7*s*. $0\frac{1}{2}$*d*.

When there is a fraction in the given number of quantities, as

Ex. VI. $6583\frac{8}{15}$ at £1 19*s*. $11\frac{3}{4}$*d*.

When there is a fractional part of a penny other than farthings, as

Ex. VII. 4176 at £3 5*s*. $4\frac{9}{16}$*d*.

When the quantity, the price of which is required, consists of several denominations, as

Ex. VIII. 9lbs. 3oz. 14dwt. at £10 15*s*. 6*d*. per lb.

130. Mention was made in (64) of certain fractional parts of £1, 1*s*., &c., which were termed aliquot parts of £1, 1*s*., &c. It is advisable to have such parts of the denominations most in use familiarly in the mind; but a pupil will find in PRACTICE, that he has to take aliquot parts of many other quantities which are intermediate between such standard units as £1, 1cwt., &c.: and nothing will render him expert in taking such aliquot parts as he will require, but a readiness in the treatment of fractions.

The following are the most useful aliquot parts of £1 and 1*s*., and should therefore be remembered.

10*s*. 0*d*.	= $\frac{1}{2}$	£
6*s*. 8*d*.	= $\frac{1}{3}$	,,
5*s*. 0*d*.	= $\frac{1}{4}$	,,
4*s*. 0*d*.	= $\frac{1}{5}$	,,
3*s*. 4*d*.	= $\frac{1}{6}$	,,
2*s*. 6*d*.	= $\frac{1}{8}$	,,
1*s*. 8*d*.	= $\frac{1}{12}$	,,
1*s*. 4*d*.	= $\frac{1}{15}$	,,
1*s*. 3*d*.	= $\frac{1}{16}$	,,
1*s*. 0*d*.	= $\frac{1}{20}$	,,

6*d*.	= $\frac{1}{2}$	shilling.
4*d*.	= $\frac{1}{3}$	,,
3*d*.	= $\frac{1}{4}$	,,
2*d*.	= $\frac{1}{6}$	,,
$1\frac{1}{2}$*d*.	= $\frac{1}{8}$	,,
1*d*.	= $\frac{1}{12}$	,,

In all cases we shall find, that where the price given is not an aliquot part of the unit of the next higher denomination, it is necessary to split the price into two or more portions, of which the largest must be an aliquot part of this said unit, and the remaining portions are aliquot parts either of this same unit, or of some one of the portions already used in the Ex.

Ex. I. 4108 at $7\frac{3}{4}d.$

In this Ex. since the price $7\frac{3}{4}d.$ is not an aliquot part of $1s.$, I therefore break up this price into three parts, $6d.$, $1\frac{1}{2}d.$, and $\frac{1}{4}d.$, of which the largest, $6d. = \frac{1}{2}$ of $1s.$ the next higher denomination; the next, $1\frac{1}{2}d.$ is an aliquot part of $6d.$, viz. $\frac{1}{4}$th; and the last, $\frac{1}{4}d.$ is $\frac{1}{6}$th of $1\frac{1}{2}d.$ Now, I know that 4108 articles at $1s.$ each would cost $4108s.$; therefore, 4108 at $6d.$, *i. e.* at $\frac{1}{2}s.$, cost 4108 times $\frac{1}{2}s.$, or $\frac{1}{2}$ of $4108s.$: I therefore find the value of this quantity and write it down, viz. $2054s.$ So also, 4108 at $1\frac{1}{2}d. = 4108$ at $\frac{1}{4}$ of $6d.$, and therefore $= \frac{1}{4}$th of the value of 4108 at $6d.$, which was found just before Similarly, since $\frac{1}{4}d. = \frac{1}{6}$th of $1\frac{1}{2}d.$, therefore 4108 at $\frac{1}{4}d. = \frac{1}{6}$th of the previously found value of 4108 at $1\frac{1}{2}d.$

Ex. I. 4108 at $7\frac{3}{4}d.$

			4108	
A price $6d.$	$= \frac{1}{2}s.$	gives	2054	
$1\frac{1}{2}d.$	$= \frac{1}{4}$ of $6d.$	,,	513	6
$\frac{1}{4}d.$	$= \frac{1}{6}$ of $1\frac{1}{2}d.$	,,	85	7
$7\frac{3}{4}d.$		2,0	265,3	1
			£132 13	1

Hence the sum of my three amounts at $6d.$, $1\frac{1}{2}d.$, and $\frac{1}{4}d.$, will be the total value, at $7\frac{3}{4}d.$ The whole of the operations which have just been described are to be written out in the accompanying shape.

Had the price been below $6d.$, as for instance 3175 at $2\frac{3}{4}d.$, we should have taken as parts $2d. = \frac{1}{6}s.$; $\frac{1}{2}d. = \frac{1}{4}$ of $2d.$; and $\frac{1}{4}d. = \frac{1}{2}$ of $\frac{1}{2}d.$; and the remainder of the work as before.

In the second division in Ex. I. 4 being a divisor, I had to find the value of $\dfrac{2054s.}{4} = 513\frac{1}{2}s. = 513s.\ 6d.$; and generally, when, as in (64), a number of shillings, or pounds, is divided by a divisor, and a rem^r is left, the fractional quotient can be converted into positive terms *at once*, and more readily than by the usual method, in which the remainder is

reduced to lower denⁿ, and the division again performed. But in the next line, where the divisor is 6, we have to take a sixth part of the 6*d.* as well as of the 513*s.*, hence I have $\frac{513}{6}s. = 85\frac{3}{6}s. = 85s.\ 6d.$; and this, with the sixth part of 6*d.*, viz. 1*d.*, becomes 85*s.* 7*d.* Though it takes a long time to explain these processes, yet a pupil who is quick at working fractions will obtain the above results more readily than I can describe them, and much time will be saved by their use: but those who prefer the usual method of reducing the remainders, as in Compound Division, can of course adhere to it. The 2653 in the result evidently consists of shillings; and from this Ex. we see that the highest denomination in the sum of all the separate amounts is the same as that of which we took the first aliquot parts; and so also is the first remainder obtained in each of the divisions.

Exs. 37.

		d.			d.
1.	1857 at	¼.	7.	6329 at	10.
2.	7151 ,,	¾.	8.	8537 ,,	8¾.
3.	3982 ,,	1¾.	9.	11071 ,,	10½.
4.	6110 ,,	5¼.	10.	7705 ,,	10¾.
5.	3457 ,,	9¾.	11.	3956 ,,	11½.
6.	7403 ,,	9¼.	12.	8793 ,,	11¾.

Ex. II. 4103 at 7*s.* 5¼*d.*

				£	s.	d.
				4103		
A price	5*s.* 0*d.*	= ¼£	gives	1025	15	
	2*s.* 0*d.*	= $\frac{1}{10}$£	,,	410	6	
	4*d.*	= ⅙ of 2*s.*	,,	68	7	8
	1*d.*	= ¼ of 4*d.*	,,	17	1	11
	¼*d.*	= ¼ of 1*d.*	,,	4	5	5¾
	7*s.* 5¼*d.*			£1525	16	0¾

In breaking the price 7*s.* 5¼*d.* into parts, we find that 2*s.* is $\frac{1}{10}$th of £1, and not of the previous aliquot part 5*s.*; hence, in dividing by 10, we must take as dividend, not the line corresponding to 5*s.* but the top line, 4103, which is the value of 4103 at £1 each: and in taking aliquot parts, we must always be careful to take as dividend, that line which expresses the value given by that coin or denomination of which we are taking an aliquot part.

OBS. In this and the preceding Ex. I have written the denomination of which the aliquot part has been taken in every line: but for the future I shall generally omit the denomination when I am taking an aliquot part of the line preceding, but insert it when I am taking a part of some earlier dividend.

Exs. 38.

		s.	d.				s.	d.
1.	7992 at	1	7¼.		9.	10205 at	4	10½.
2.	8495 ,,	2	9¾.		10.	7248 ,,	9	1¼.
3.	3708 ,,	3	7¼.		11.	12826 ,,	8	11¼.
4.	8222 ,,	5	10¾.		12.	8793 ,,	9	7¾.
5.	1007 ,,	7	11¼.		13.	1250 ,,	9	11½.
6.	1173 ,,	3	8¼.		14.	7108 ,,	8	6¾.
7.	4351 ,,	8	0½.		15.	11489 ,,	9	10.
8.	3672 ,,	7	10.		16.	12146 ,,	9	8¾.

Ex. III. 6009 at 19*s.* 5¼*d.*

		6009		
A price 10*s.* 0*d.* = ½£	gives	3004	10	
5*s.* 0*d.* = ½	,,	1502	5	
4*s.* 0*d.* = ⅕£	,,	1201	16	
4*d.* = $\frac{1}{12}$	,,	100	3	
1*d.* = ¼	,,	25	0	9
¼*d.* = ¼	,,	6	5	2¼
19*s.* 5¼*d.*		£5839	19	11¼

This Ex. differs from the preceding, only in having the price above 10*s.* In such a case we always take 10*s.* as the first aliquot part; and the remaining shillings and pence as aliquot parts, either of 10*s.*, or of any amount which has been used in the course of the example. Sometimes it may happen that a second aliquot part of £1 is taken, as in 16*s.* 8*d.*, where I should take 10*s.* = ½£, and 6*s.* 8*d.* = ⅓£.

Exs. 39.

		s.	d.				s.	d.
1.	18147 at	10	6½.		9.	1948 at	19	0.
2.	3501 ,,	13	9.		10.	8218 ,,	15	10½.
3.	6234 ,,	11	4½.		11.	12327 ,,	17	10¾.
4.	7646 ,,	16	0¾.		12.	12497 ,,	19	10¼.
5.	5431 ,,	15	11¼.		13.	14839 ,,	18	0¼.
6.	11040 ,,	16	10.		14.	4103 ,,	14	10½.
7.	2067 ,,	17	8¾.		15.	12018 ,,	19	5¼.
8.	3459 ,,	18	6.		16.	8972 ,,	19	9¼.

Ex. IV. 7111 at £1 17*s.* 4½*d.*

A price £1 0*s.* 0*d.*	gives	7111		
10*s.* 0*d.* = ½ £	,,	3555	10	
5*s.* 0*d.* = ½	,,	1777	15	
2*s.* 0*d.* = $\frac{1}{10}$ £	,,	711	2	
4*d.* = $\frac{1}{6}$	,,	118	10	4
½*d.* = ⅛	,,	14	16	3½
£1 17*s.* 4½*d.*		£13288	13	7½

Here £7111 is the value of 7111 things at £1 each; therefore, if I find the value of 7111 at 17*s.* 4½*d.*, as in the last Ex., and add in the top line as £7111, I shall obtain a correct result.

Exs. 40.

			£	s.	d.
1.	1082	at	1	11	5½.
2.	8649	,,	1	0	6½.
3.	15432	,,	1	10	1.
4.	6666	,,	1	8	0½.
5.	5741	,,	1	17	6.
6.	7891	,,	1	14	11.
7.	11000	at	1	4	9¾.
8.	1572	,,	1	12	11½.
9.	7538	,,	1	11	8¼.
10.	19345	,,	1	13	5.
11.	1548	,,	1	19	7¼.
12.	8754	,,	1	19	10½.

Ex. V. 4013 at £12 7*s.* 0½*d.*

		4013		
		12		
A price £12 0*s.* 0*d.*	gives	48156		
5*s.* 0*d.* = ¼ £	,,	1003	5	
2*s.* 0*d.* = $\frac{1}{10}$ £	,,	401	6	
½*d.* = $\frac{1}{48}$	,,	8	7	2¼
£12 7*s.* 0½*d.*		£49568	18	2½

Here, since £4013 repeated twelve times gives the value of 4013 at £12; therefore I must multiply the 4013 by 12, and add the product as pounds to the other amounts obtained by proceeding with the 7*s.* 0½*d.*, as in Exs. III. and IV. The last division, by 48, cannot of course be performed mentally: a pupil may obtain the result by Long Division, and merely write down the amount.

Exs. 41.

			£	s.	d.
1.	6241	at	3	8	7½.
2.	999	,,	4	19	9¼.
3.	5683	,,	5	17	10.
4.	1429	,,	8	19	10¼.
5.	4108	,,	17	17	11½.
6.	10101	,,	9	9	10.
7.	864	,,	20	17	6.
8.	1875	,,	13	13	7.
9.	4278	,,	18	17	8½.
10.	5432	at	17	15	10¾.
11.	7701	,,	6	8	7¾.
12.	7032	,,	11	8	3¼.
13.	7702	,,	10	17	6½.
14.	3764	,,	18	14	7½.
15.	5505	,,	9	19	11½.
16.	2807	,,	29	0	7½.
17.	11078	,,	35	15	9.
18.	8970	,,	63	14	8.

Ex. VI. $6583\frac{8}{15}$ at £1 19*s*. $11\frac{3}{4}$*d*.

A price £1 0*s*. 0*d*.	gives	6583		
10*s*. 0*d*. $=\frac{1}{2}$£	„	3291	10	
5*s*. 0*d*. $=\frac{1}{2}$	„	1645	15	
4*s*. 0*d*. $=\frac{1}{5}$£	„	1316	12	
10*d*. $=\frac{1}{6}$ of 5*s*.		274	5	10
1*d*. $=\frac{1}{10}$	„	27	8	7
$\frac{1}{2}$*d*. $=\frac{1}{2}$	„	13	14	$3\frac{1}{2}$
$\frac{1}{4}$*d*. $=\frac{1}{2}$	„	6	17	$1\frac{3}{4}$
$\frac{8}{15}$ of £1 19*s*. $11\frac{3}{4}$*d*. = ...		1	1	$3\frac{13}{15}$
A price £1 19*s*. $11\frac{3}{4}$*d*.	gives	£13160	4	$2\frac{7}{60}$

This Ex. differs from Exs. IV. and V. only in the presence of the fraction $\frac{8}{15}$. I therefore, after having proceeded with the sum, as though the $\frac{8}{15}$ were not there, find the value of $\frac{8}{15}$ at (£1 19*s*. $11\frac{3}{4}$*d*.) i.e. of $\frac{8}{15}$ of (£1 19*s*. $11\frac{3}{4}$*d*.) which, according to the method of Ex. IV. in (84) = £1 1*s*. $3\frac{13}{15}$*d*.

The value of the $\frac{8}{15}$ might also have been thus obtained:

$$\frac{8}{15}=\frac{5}{15}+\frac{3}{15}=\frac{1}{3}+\frac{1}{5};$$

I might therefore have taken one-third and one-fifth of £1 19*s*. $11\frac{3}{4}$*d*. and their sum would have amounted to £1 1*s*. $3\frac{13}{15}$*d*., as before.

Exs. 42.

	£ *s*. *d*.		£ *s*. *d*.
1. $14287\frac{1}{5}$	at 10 13 $10\frac{3}{4}$.	5. $4538\frac{4}{5}$	at 0 16 6.
2. $3779\frac{15}{16}$	„ 14 4 $4\frac{3}{4}$.	6. $1008\frac{7}{12}$	„ 4 8 6.
3. $8976\frac{3}{7}$	„ 7 15 $11\frac{1}{2}$.	7. $2711\frac{9}{13}$	„ 7 3 3.
4. $4149\frac{3}{11}$	„ 0 8 7.	8. $3714\frac{5}{14}$	„ 2 9 $11\frac{1}{4}$.

Ex. VII. 4176 at £3 5*s*. $4\frac{9}{10}$*d*.

		4176		
		3		
A price £3 0*s*. 0*d*.	gives	12528		
5*s*. 0*d*. $=\frac{1}{4}$£	„	1044		
4*d*. $=\frac{1}{15}$	„	69	12	
$\frac{1}{2}$*d*. $=\frac{1}{8}$	„	8	14	
$\frac{1}{5}$*d*. $=\frac{1}{20}$ of 4*d*.		3	9	$7\frac{1}{5}$
$\frac{1}{5}$*d*. $=\frac{1}{20}$ of 4*d*.		3	9	$7\frac{1}{5}$
£3 5*s*. $4\frac{9}{10}$		£13657	5	$2\frac{2}{5}$

The presence of the fraction $\frac{9}{10}$*d*. is the only point in which this Ex. differs from III. and IV.; and just as we break up $\frac{3}{4}$*d*. into $\frac{1}{2}$*d*. and $\frac{1}{4}$*d*., so this $\frac{9}{10}$*d*. must be broken up into such portions as will be aliquot parts of 1*d*. viz. $(\frac{5}{10}+\frac{2}{10}+\frac{2}{10})$*d*., or $\frac{1}{2}$*d*. + $\frac{1}{5}$*d*. + $\frac{1}{5}$*d*.

If in this Ex. there had been a fraction at the end of the 4176, as in Ex. VI., we should have proceeded with it just as with the $\frac{8}{15}$ in that Ex.

Exs. 43.

	£ *s*. *d*.		£ *s*. *d*.
1. 5189	at 1 10 $2\frac{1}{7}$.	4. $2486\frac{1}{3}$	at 18 $7\frac{1}{5}$.
2. 7485	„ 4 5 $9\frac{3}{5}$.	5. 4321	„ 1 0 $6\frac{1}{5}$.
3. 1111	„ 14 6 $5\frac{3}{8}$.	6. 4231	„ 11 $8\frac{1}{5}$.

Ex. VIII. 9 lb. 3 oz. 14 dwts. at £10 15*s.* 6*d.* per lb.

Hitherto we have had the quantities whose value was required expressed all in one den^n; and we could therefore repeat the highest den^n of the price, as for instance, £1, as many times as there are units in the given quantity, and then take parts of this highest den^n for the remainder of the price. But in Ex. VIII. we cannot place 9 lb. 3 oz. 14 dwt. in the top line, and multiply it by the 10, because the result would not be £10 repeated an exact number of times: I therefore place the £10 15*s.* 6*d.* in the top line, and multiplying it by 9, I obtain the value of 9 lb. at £10 15*s.* 6*d.* per lb.; and the value of the 3 oz. 14 dwt. will be found, by taking the same parts of £10 15*s.* 6*d.* that 3 oz. 14 dwt. are of 1 lb. Thus working, we have

				£	s.	d.
				£10	15	6
						9
Value of 9 lb. 0 oz. 0 dwt.			is	96	19	6
3	0	$=\frac{1}{4}$ of 1 lb.	,,	2	13	$10\frac{1}{2}$
	10	$=\frac{1}{6}$ of 3 oz.	,,		8	$11\frac{3}{4}$
	2	$=\frac{1}{5}$ of 10 dwt.	,,		1	$9\frac{11}{20}$
	2	$=\frac{1}{5}$ of 10 dwt.	,,		1	$9\frac{11}{20}$
9 lb. 3 oz. 14 dwt. =			....	£100	5	$11\frac{7}{20}$

$$\left(\frac{11}{20}+\frac{11}{20}+\frac{3}{4}+\frac{1}{2}\right)d.=\frac{11+11+15+10}{20}d.$$
$$=\frac{47}{20}d.=2\tfrac{7}{20}d.$$

Also, since 3 oz. 14 dwt. $=3\frac{14}{20}$ oz. $=3\frac{7}{10}$ oz. $=\dfrac{3\frac{7}{10}}{12}$ lb. $=\dfrac{37}{120}$ lb., therefore the Ex. may be written $9\frac{37}{120}$ lbs. at £10 15*s.* 6*d.* per lb.; and it is then similar to Ex. VI.

Exs. 44.

		£	s.	d.	
1.	5 lbs. 10 oz. (Avoirdupois).....at	0	6	6	per lb.
2.	93 lbs. 3 oz. 1 dwt. 6 grs. ....,,	4	10	6	,,
3.	800 cwt. 0 qr. 16 lbs. ....,,	14	19	$6\frac{1}{4}$	per cwt.
4.	99 tuns 3 hhds. 16 gals. ....,,	128	18	9	per tun.
5.	155 gross and 75...................,,	1	10	6	per gross.

		£	s.	d.	
6.	135 quarters 7 bushels..........at	2	17	8	per quarter.
7.	6 tons 7 cwt. 2 qrs. 17 lbs. ......„	3	10	7	per ton.
8.	8 years 3 months 20 days ......„	5	7	6¾	per year.
9.	15 reams 9 quires 6 sheets......„	1	6	9	per ream.
10.	46 days 11 hours 35 minutes ...„	1	1	5¼	per day of 12 hours.

See APPENDIX, Decimal Coinage.

Exs. 45.

MISCELLANEOUS EXAMPLES.

1. What cost 1351 lbs. at 2*s*. 2¼*d*. per lb.?
2. A bankrupt pays 13*s*. 9*d*. in the pound upon £1575, how much money did he divide?
3. Find the value of 2078¼ yards at 8*s*. 7¾*d*. per yard.
4. What is the tax on £12345 15*s*. at 3*s*. 7½*d*. in the pound?
5. Find the worth of 24150 rupees at 1*s*. 11¾*d*. each.
6. A gold snuff-box weighed 7 oz. 15 dwts. 15 grs., find its value at £4 5*s*. 6*d*. per oz.
7. If 2*s*. 3*d*. in the pound is paid on an income of £1050; what is the net annual income?
8. My daily expenses are 10*s*. 11¾*d*.; how much can I save out of an income of £250?
9. Find the cost of a silver epergne weighing 175 oz. 14 dwts., at 45*s*. 9*d*. per oz.
10. What is the value of 3r. 17p. 25¼ yds., at £125 per acre?

131. In many Exs. similar in principle to those given above, a knowledge of fractions will enable a pupil to employ very brief methods of working; as, for instance, if I had 317 at 16*s*. 8*d*., I should say

$$317 \text{ at } 16s.\ 8d. = 317 \text{ at } \frac{5}{6}£ = \frac{1585}{6}£ = 264\tfrac{1}{6}£ = £264\ 3s.\ 4d.$$

Again, to find the value of 754 at 7*s*. 7*d*.

$$754 \text{ at } 7s.\ 6d. = 754 \text{ at } \frac{3}{8}£ = \frac{2262}{8}£ = £282\tfrac{3}{4};$$
$$= £282\ 15s.$$

and 754 at $1d. = 754d. = 62s.\ 10d.$

therefore, 754 at $7s.\ 7d. = £282\ 15s. + £3\ 2s.\ 10d.$
$$= £285\ 17s.\ 10d.$$

This method is especially worth notice in short Exs.

Thus, 97 at $7\frac{1}{2}d. = 97 \times \frac{5}{8}s. = \frac{485}{8}s. = 60\frac{5}{8}s. = £3\ 0s.\ 7\frac{1}{2}d.$

Again, when the price is an even number of shillings, under 20, as 327 at 16*s.*, we may work as follows:

$$327 \text{ at } 16s. = 327 \times \frac{8}{10}£ = \frac{2616}{10}£ = 261\tfrac{6}{10}£ = £261\ 12s.$$

and this mode of working is comprised in the following rule:

Multiply the given number by half the given price, doubling the first figure to the right-hand for shillings, and calling the rest pounds.

$$\text{Thus,}\quad \begin{array}{r} 327 \\ 8 \\ \hline £261\ \ 12s. \\ \hline\hline \end{array}$$

We may also observe, that since the cost of 12 things at 1*d.* each = 1*s.*, therefore, that of 12 things at $3\frac{1}{2}d. = 3\frac{1}{2}s.$ $= 3s.\ 6d.$, and of 12 things at $7\frac{3}{4}d. = 7\frac{3}{4}s. = 7s.\ 9d.$; *i. e.* if I have the price of one article in pence and a fractional part of a penny, the price of 12 articles will be expressed by the same figures as sh llings and parts of a shilling.

Hence, also, the value of any multiple of 12 things may be readily expressed as above, if the price of one be given in terms of pence.

Ex. Find the value of 96 lbs. at $10\frac{1}{4}d.$

Cost of 12 lbs. at $10\frac{1}{4}d. = 10\frac{1}{4}s. = 10s.\ 3d.$

therefore, cost of 8×12 lbs. at $10\frac{1}{4}d. = 8 \times (10s.\ 3d.) = 82s.$

$$= £4\ 2s.$$

With a little practice, such an Ex. as this might be worked mentally, more quickly than it could be written.

APPLICATIONS OF PROPORTION.

RULE OF THREE.

The Articles marked thus (*) may be omitted by those who have not read Fractions.

132*. In articles (65) to (76) the subject of Proportion has been fully discussed: and our business now is, to shew how to work Exs., which are in reality only different forms of the question asked in (73), viz.: If three numbers be given, what fourth number must be chosen, such that the four, when taken in order, shall be proportionals?

133. The Rule of Three is so called because in the questions given under this head there are three quantities proposed. These three numbers, when placed in order for working an Ex., are called *terms*. In using the expression first, second, and third *terms*, we intend to indicate the manner in which these three given quantities are arranged. Thus, if I were to write 9, 8, 24, as first, second, and third *terms* respectively, in the sense here intended, I should have

1st		2nd		3rd
9	:	8	::	24

where the dots placed between the terms are signs which express the relation existing among these terms, so that with them and the fourth term a Proportion is formed. When any three terms are properly arranged at the commencement of an Ex. in Rule of Three, they are said to

form a *Statement:* and the question is said to be stated. So long as only three terms are contained in a statement, the question is said to be one of *Simple Proportion:* but sometimes more than three quantities require to be so arranged: the question is then said to come under the head of *Compound Proportion*, or, as it is sometimes called, Double Rule of Three.

It is here evidently *assumed* that the three quantities, with the required fourth, will form a proportion. But to ascertain whether this is true, we must apply the tests given in (75) and (76); and if either of these is satisfied, the question may be worked according to the principles established there, and which are more fully drawn out in the next two Articles.

In the Appendix (Art. Proportion), three examples are produced, in two of which the existence of proportionality is shewn, which an ordinary learner could not assure himself to be true; and in the third, he might hastily assume that one of the usual forms of proportionality existed, where it is *not* true.

The pupil may work any ordinary question in the Rule of Three by the method given below; and it is to be observed that the process gives the solution without assuming the quantities to be proportionals; and, at the same time, by giving the same result as when they are so treated, shews that they may be rightly treated as proportionals, according to the common rule.

Ex. I. What is the coach fare for 130 miles, if it is £1 9*s*. 4*d*. for 85 miles?

The charge per mile is uniform, and the same in both parts of the question; hence, if

The fare for 85 miles be £1 9*s*. 4*d*.
" 1 " will be $\frac{1}{85}$ of £1 9*s*. 4*d*.
∴ " 130 " " $\frac{130}{85}$ of £1 9*s*. 4*d*.

the same result as would be obtained by the ordinary method.

Ex. II. If when a peck of flour is sold for 1*s*. 4*d*., the penny loaf weighs 12 oz., how much will it weigh when wheat is 2*s*.?

At a rate of 16*d*. per peck, 1*d*. buys 12 oz.; and since, when the flour is cheaper, any sum, as 1*d*., will buy a proportionately larger amount of it,

∴ at a rate of 1*d*. per peck, 1*d*. will buy 16 times as much as when it was at 16*d*. per peck,

i. e. at 1*d.* per peck, 1*d.* will buy 16×12 oz.:
since also when the flour is dearer, the 1*d.* will buy proportionally less;

$\therefore$ at 24*d.* per peck, 1*d.* will buy $\frac{16 \times 12}{24}$ oz. $= 8$ oz.

Though the method here shewn dispenses with the necessity of using a statement, and is very valuable as a discipline to the pupil's mind, yet it requires greater dexterity in applying principles and in employing fractions, than many pupils possess who have advanced as far as Rule of Three; and this applies still more strongly when we come to Compound Proportion, and other Applications of Proportion, as we shall presently see; so that I think teachers will generally find it necessary to employ the common mode.

134*. Observing (73) and (74), we learn the following relations between the four terms forming any Proportion.

1st. That the third and fourth terms are of the same kind, *i. e.* that the third term must always be of the same nature as the one required.

2ndly. That the first and second must always be of the same kind; and that they must be reduced to the same denomination, if they be not already so expressed.

3rdly. That the fourth term is obtained by multiplying the third term by the fraction $\frac{\text{2nd}}{\text{1st}}$; or, if this operation be performed at two steps, we multiply by the second term, and divide by the first.

4thly. That if the second term be greater than the first, the fourth term will be greater than the third; but if the second term be less than the first, then the fourth term will be less than the third.

5thly. That since the fraction $\frac{\text{2nd}}{\text{1st}}$ is an abstract number, therefore the fourth term, which $= \frac{\text{2nd}}{\text{1st}} \times$ 3rd, is of the same denomination as that in which the third was expressed.

Collecting together these facts, we deduce the following Rule.

135. RULE. Find out the nature of the quantity sought by the question; *i. e.* of the required fourth term.

Of the three quantities given in the question, find that which is of the same nature as the fourth term, and take that quantity as the third term.

In order to place the other two quantities in their proper situations, inquire whether, from the nature of the question, the fourth term will be more or less than this third term: if *more*, make the *larger* of the two remaining quantities the middle or second term; but if *less*, make the *smaller* the middle term: the only remaining quantity must of course fill the first place.

If the first and second terms be not expressed in the same denomination, reduce them till they become so: and if the third term consist of several denominations, reduce it to the lowest name mentioned.

Then multiply the second and third terms together, and divide the product by the first: the quotient will be the answer or fourth term, expressed in the same denomination as that in which the third term was left.

If this quotient be expressed in too low a denomination, [as, for example, 1257 farthings, or 1836 dwts.,] let it be reduced to a higher denomination: [as £1 6*s.* 2¼*d.* and 7lb. 7oz. 16dwts.]

136. I will now proceed to work some Exs. which will illustrate the different varieties that may be expected under the head of Simple Proportion: and it will be found that the principal difficulty consists in arranging the three terms according to the directions prescribed by the Rule. This is especially the case, when the question is given in such a

shape, that the three terms cannot be *immediately* obtained from the question as it stands. I will explain this more fully as I go through the various kinds of Exs. When the *Statement* is once obtained, the remainder of the work consists merely of Multiplication, Division, and Reduction.

I recommend a pupil to work every question that I have worked, so that he may the better see the correctness and ascertain the object of the successive operations in any Example.

Ex. I. If 12 yards of cloth cost £19, what will 8 yards cost?

Here I see that the required fourth term will be money; I therefore place the £19 in the third term. Also, the fourth term, which is to be the price of 8 yards, will be *less* than the third term, which is the price of 12 yards; therefore, according to the Rule, I place the *smaller* of the two remaining terms, *i. e.* the 8 yards, in the middle, and the 12 yards in the first place.

```
yds.   yds.     £
 12  :  8  ::  19
                8
           12)152
             £12 13s. 4d.
```

The first and second terms are already in the same name, and the third term contains but one denomination, therefore no reduction is required. I now multiply the second and third terms together, and divide by the first: the answer is £12 13*s*. 4*d*. And this fourth term and the other three terms form the following proportion:—

12 yds. : 8 yds. :: £19 : £12 13*s*. 4*d*.

or in words, 12 yards are to 8 yards, as £19 are to £12 13*s*. 4*d*.

Ex. II. If 17 cwt. 3 qrs. and 14 lbs. cost £8 18*s*. 9*d*., how much may be bought for £5 12*s*. 6*d*. at the same rate?

The fourth term will evidently be expressed in weight; therefore I put 17 cwt. 3 qrs. 14 lbs. in the third term. I now ask this question: "If the quantity in the third term can be obtained for £8 18*s*. 9*d*., will more or less be bought for the £5 12*s*. 6*d*.?" evidently *less*; therefore I place the *less* of the two prices in the middle, and the remaining one first. The first and second terms are not expressed in any single denomination; I therefore reduce them to threepences, which is the highest denomination to which they can both be reduced. Also, since the third term consists of more denominations than one, I reduce it to the lowest denomination

mentioned, viz. lbs. After I have multiplied the second and third terms together, and divided by the first, the quotient is 1260, which consists of lbs., because the third term was expressed in lbs. This quotient, when reduced to higher denominations, becomes 11 cwt. 1qr.

£ s. d.	£ s. d.	cwts. qrs. lbs.
8 18 9 :	5 12 6 ::	17 3 14
20	20	4
178	112	71
4	4	28
715	450	582
		142
		2002
		450
		100100
		8008

715) 900900 (1260 lbs.
715
1859
1430
4290
4290
0

28 { 4) 1260
7) 315
4) 45
11 cwt. 1 qr.

I will now give a few Exs. in which the difficulty consists in preparing the question for being stated; but I shall merely show how to overcome the difficulty, and leave the question to be worked out as in the former Exs.

Ex. III. A bankrupt's effects amounted to £980 10*s*., and he paid his creditors 13*s*. 4*d*. in the pound; what was the amount of his debts?

At first sight there appear to be only two terms in this question, but the £1 furnishes another term. Now all the three quantities are money; but by reading the question thus: "If 13*s*. 4*d*. be paid for a debt of £1, what debt will be paid by £980 10*s*.?" I learn that 13*s*. 4*d*. and £980 10*s*. are money *paid*, and the £1 is money *owed*, or *debt;* and since the fourth term is *debt*, I place the £1 in the third term. Also, the debts which are paid by £980 10*s*. are of course more than this third term; therefore I place the larger term, £980 10*s*., in the middle, and 13*s*. 4*d*. in the first place. The statement will then be 13*s*. 4*d*. : £980 10*s*. :: 1£. The fourth term will be found to be £1470 15*s*.

137*. The four terms, arranged as a proportion, will be

13*s*. 4*d*. : £980 10*s*. :: £1 : £1470 15*s*.

4. How many yards of paper, 27 inches wide, will hang a room 54 feet round and 10 feet high?

5. If an acre is 220 yards in length, and 22 in breadth, what must be the length when the breadth is $27\frac{1}{2}$ feet?

6. If I can buy $15\frac{3}{4}$ yds. of cloth for 10 guineas, how much can I buy at the same rate for £283 17*s.* 6*d.*?

7. An income of £150 pays a tax of £4 7*s.* 6*d.*; what will be the tax upon £586 1*s.*?

8. If the carriage of $15\frac{1}{2}$cwt. for 56 miles come to 10*s.* 6*d.*, how much can I have carried 72 miles for the same money?

9. The carriage of 4 cwt. for 72 miles cost 15*s.* 9*d.*; how many lbs. can be carried $13\frac{3}{4}$ miles for the same sum?

10. A field of 16 acres produces 440 bushels of wheat: how much is that upon every 22 square yards?

11. A creditor agreeing to receive £51 for a debt, finds that he has been paid at the rate of 12*s.* 9*d.* in the pound; how much was the debt?

12. Three men who weave at the rate of $5\frac{1}{4}$ yds. per day, finish 119 yds. in a certain time: at what rate per day must they weave, who finish 85 yds. in the same time?

13. If 15 men eat 35 shillings' worth of bread in a certain time, when wheat is 12*s.* per bushel; how much may they eat at the same cost when wheat is 7*s.* per bushel?

14. A rate of 2*s.* 9*d.* in the pound produces £352; what is the rental of the parish?

15. If an income-tax of 10*d.* in the pound amounts to $2\frac{1}{2}$ millions, what must be the poundage in order to produce £3,375,000?

16. A man buys 148 yds. at 2*s.* $7\frac{1}{2}$*d.* per yd.; at what price must he sell it to gain £12 12*s.* 10*d.* by the whole?

17. The penny loaf weighs 10 oz. when wheat is 5*s.* a bushel; how much will it weigh when wheat is 6*s.* 3*d.*?

18. The rental of a parish is £5626 10*s.*; and the assessment for the poor-rates is £405 13*s.* 4*d.*; how much will be the rate on £45 12*s.* 9*d.*?

19. If the net income of an estate after paying all taxes be £534 15*s.*, and the gross income be £570 8*s.*; how much in the pound did the taxes amount to?

20. The chain for measuring land is 4 perches in length, and is divided into 100 links; what is the length of a wall in feet, which measures 1550 links?

21. If 1 lb. of gold, and 1 oz. of alloy, can be coined into $44\frac{1}{2}$ guineas; find the value of 5 oz. of pure gold, considering the alloy of no value.

22. I spend 12 guineas in 35 days, and save £100 a year: what must I earn in the year?

23. A piece of gold at £3 17*s*. 10½*d*. per oz. is worth £150: what will be the worth of a piece of silver of equal weight, at 54*s*. 6*d*. per lb.?

24. Two floors are equal in size, one is 35 feet long, and 25 feet broad; the other is 40 feet long: what is its breadth?

25. The weights of gold and of water are as 19¼ and 1; find what number of solid inches of gold is equal in weight to 17½ cub. ft. of water?

26. A dollar is to a crown as 111 : 120; how many dollars are equal in value to £250?

27. A clock which gains 7½ minutes in 24 hours, is 14 minutes fast at Monday midnight; what time will it indicate at 6 o'clock in the evening of the following Thursday?

28. A person owes £1537 3*s*. 4*d*.; but can pay only £960 14*s*. 7*d*.: what will be the dividend, and how much shall I receive for a debt of £276 11*s*. 6*d*.?

29. The shadow of a stick 3ft. 6in. long is 2ft. 9in.: what is the height of a tree which at the same time throws a shadow of 154 feet? (See App. Art. Proportion.)

30. An income of £3827 12*s*. 6*d*. is taxed at the rate of 7*d*. in the pound; how much clear income will remain?

31. Bought 236 gallons of oil for £111 6*s*. 8*d*.; what profit will be made by selling it at the rate of 8*s*. 6*d*. for 3 quarts?

32. Paid £45 10*s*. for a hogshead of rum; how much water must be added, to be able to sell it without loss or gain at 11*s*. 6*d*. per gallon?

33. Out of an income of £312 10*s*. a year, the expenses are £55 in 146 days; in what time will 1000 guineas be saved?

34. If 90 English degrees correspond to 100 French degrees, how many French degrees are there in 36·45 English?

35. What must be the breadth of a piece of ground which is 14⅝ yds. long, so that it may be as large as a piece 40⅓ yds. long and 4⅞ broad?

36. The 6*d*. loaf weighs 3¼ lbs., when wheat is 50*s*. a quarter; what will it weigh when wheat is 40*s*. 3*d*. a quarter?

37. Given that the velocity of a falling body is proportional to the time during which it falls; find the time of descent of a body having acquired a velocity of 1000 feet, supposing that the velocity obtained in 2½ seconds is 80·5 feet.

38. The lengths of the arms of a lever are inversely proportional to the weights at the extremities of the arms; if the lengths of the arms be 3 feet and 2½ inches, what must be the weight at the longer arm to balance 20 lbs. at the shorter end?

39. A bankrupt paid £1520 to his creditors; £205 of his debts were paid in full; and his assets were to his remaining debts as 3 : 8; find the amount that he owed.

40. What can a man save per annum, who out of an income of £500 gives away $\frac{1}{11}$th, pays 7*d.* in the pound income-tax, and spends £16$\frac{3}{4}$ in 3 weeks?

COMPOUND PROPORTION.

140. It was observed in (133) that a question was classed under the head of Compound Proportion, when there were more than three quantities which required to appear in the statement. The following is an Example.

Ex. I. If 12 yards of cloth, 3 quarters wide, cost £19, what will be the cost of 8 yards, 5 quarters wide?

If the width of the two pieces of cloth were the same, we should take no account of this width, whatever it might be: and the question would then become Ex. I. in (136), or one of Simple Proportion. Referring to that Ex. we have the statement

$$12 \text{ yards} : 8 \text{ yards} :: £19 \qquad \text{(B)}$$

and the cost of the new piece $= \frac{8 \text{ yds.}}{12 \text{ yds.}} \times £19 = \frac{8}{12} \times 19£.$

We will now take into account the two breadths, 3 qrs., and 5 qrs., and put the following question:—"If a piece of cloth cost $\frac{8 \times 19}{12}$ £, when 3 quarters wide, what will it cost when the width is 5 quarters?" The statement would be

$$3 \text{ qrs.} : 5 \text{ qrs.} :: \frac{8 \times 19}{12} £ \qquad \text{(C)}$$

and the fourth term $= \frac{8 \times 19}{12} £ \times \frac{5 \text{ qrs.}}{3 \text{ qrs.}} = \frac{8 \times 19 \times 5}{12 \times 3} £.$

Now, if the whole of statement (B), and the first and second terms in (C), be converted into one statement, as follows:—

$$\begin{matrix} 12 \text{ yds.} \\ 3 \text{ qrs.} \end{matrix} : \begin{matrix} 8 \text{ yds.} \\ 5 \text{ qrs.} \end{matrix} :: £19 \qquad \text{(D)}$$

and we take the product of the two quantities which stand first, as our

first term, and the product of the two in the middle, as our second term, we shall have

$$\text{the fourth term} = \frac{8 \text{ yds.}}{12 \text{ yds.}} \times \frac{5 \text{ qrs.}}{3 \text{ qrs.}} \times 19£ = \frac{8 \times 5 \times 19}{12 \times 3} £,$$

which is precisely the same as was obtained from the two successive statements: hence such a statement as (D) will produce a correct result. Also, in forming this statement, independently of (B) and (C), I select for the third term that which is similar to the required fourth term, as in Simple Proportion: and in placing the remaining terms, I take each pair separately, and ask the usual question with the third term, as to whether the answer will be more or less than this term; and I arrange this pair precisely as though they were the only two terms which I had to consider. However many pairs of terms occur in the question, they must all be treated in like manner; for the same proof that has shewn how to combine the first and second statements, will shew how to combine the third with the result of the first pair.

I will work another Ex. and mention the mental operations which must be performed, in order to enable me to place each pair of terms correctly.

Ex. II. A field, 300 yards long and 280 broad, was ploughed by six horses in two days of eight hours each; how many horses will plough a piece of ground 500 yards long, and 315 broad, in three days of ten hours each?

The fourth term will be horses: I therefore place the six horses in the third term. Also, the new field is longer than the one which required six horses; hence, considering the effect of this pair of terms alone, it will require more horses, and I place the larger term, 500, in the middle: so, also, the second field is broader than the first, and therefore will take more horses, and I place 315 in the middle. Again, the first field was ploughed in two days, but the new one in three days; hence, since the time is longer, we shall, so far as this pair of terms is concerned, require fewer horses; and the smaller term, two, is to be in the middle. Also, in ploughing the first field, the days were eight hours, but for the second field they are ten hours; hence, with this

$$\left.\begin{array}{c} 300 \text{ yards} \\ 280 \text{ yards} \\ 3 \text{ days} \\ 10 \text{ hours} \end{array}\right. : \left.\begin{array}{c} 500 \text{ yards} \\ 315 \text{ yards} \\ 2 \text{ days} \\ 8 \text{ hours} \end{array}\right. :: 6 \text{ horses}$$

extra time, fewer horses will be required, and I place the eight in the second place. The whole statement is as annexed; and

$$\text{the fourth term} = \frac{500 \times 315 \times 2 \times 8 \times 6}{300 \times 280 \times 3 \times 10}\text{ horses}$$

$$= 3 \times 2 \text{ horses} = 6 \text{ horses};$$

precisely the same number as before: *i. e.* the increased size of the field, and the increased length of time allowed for the work, are so balanced, that the same number of horses as before is sufficient.

In arranging the several pairs of terms in the statement, I seem to be trying at one time to obtain a smaller term, and at another time a larger term than the third; and it is true that some conditions of the question tend to make the fourth term less than this third, and some to make it more. Each pair will produce its own effect in increasing or diminishing the required term; and we shall therefore find the result more or less than the third, according as the conditions in the question which would make it more, predominate, or not, over those which would make it less; *i. e.* according as the product of all the terms in the second place is more or less than the product of all in the first place.

It must be carefully observed, that if any pair of corresponding terms be not expressed in the same denomination, they must be so reduced, just as in Simple Proportion, before we commence forming the fraction which will give the fourth term.

According to the method of (133), without assuming the proportionality of the quantities involved, the work will be as follows.

In Ex. I. let A be the number of pounds sterling required, then

12×3 represents the area of cloth which cost £19

and 8×5 „ „ „ £A

$\therefore \dfrac{12 \times 3}{19}$ represents the area costing £1

and $\dfrac{8 \times 5}{A}$ „ „ „ £1

and since £1 will always buy the same quantity, the quality being supposed the same, therefore, inverting these equal fractions,

$$\frac{A}{8 \times 5} = \frac{19}{12 \times 3} \text{ or, } A = \frac{8 \times 5 \times 19}{12 \times 3}, \text{ as before.}$$

In Ex. II. let A be the number of horses;

Now, 6 horses can do 300×280 sq. yds. in 2×8 hrs.

or, $\dfrac{300 \times 280}{2 \times 8}$ sq. yds. in 1 hr.

$\therefore$ 1 horse can do $\dfrac{300 \times 280}{2 \times 8 \times 6}$ sq. yds. in 1 hr. (I)

So, also, A horses can do $\dfrac{500 \times 315}{3 \times 10}$ sq. yds. in 1 hr.

$\therefore$ 1 horse „ $\dfrac{500 \times 315}{3 \times 10 \times A}$ sq. yds. (II)

(I) and (II) are equal; $\therefore \dfrac{A \times 3 \times 10}{500 \times 315} = \dfrac{2 \times 8 \times 6}{300 \times 280}$

or, $A = \dfrac{500 \times 315 \times 2 \times 8 \times 6}{300 \times 280 \times 3 \times 10}$, as before.

Exs. 47.

1. If 10 men can dig 30 yds. of earth in 8 days, how many yards can be dug by 20 men in 4 days?

2. If £300 gain £10 in a year, in what time will £900 gain £175 10*s.*?

3. Three boats take 6000 herrings in 8 days; in how many days will 450 boats take 20,000 barrels, each containing 700 herrings?

4. I borrow £175 10*s.* for 10 months, when money is worth 5 per cent.; how much must I lend in return for 12 months, when money is worth $3\frac{1}{2}$ per cent.?

5. If five men can reap a field whose length is 800 feet and breadth 700, in $3\frac{1}{2}$ days of 14 hours each; in how many days of 12 hours each can seven men reap a field whose length is 1800 feet, and breadth 960 feet?

6. The papering of a room $10\frac{1}{2}$ feet high, and 20 yards round, cost £1 2*s.* 6*d.*; what will be the cost of papering another room 9 feet high, and 63 feet round?

7. If I pay 1*s.* 3*d.* for 6lb. 14oz. of bread, when wheat is 4*s.* 9*d.*

per bushel, what must I pay for 23lb. 12oz., when wheat is 5*s*. 5*d*. per bushel?

8. A printing machine turns out 37,260 sheets in a day, running $12\frac{1}{4}$ hours; if its speed be increased in the ratio of 4 to 3, how many sheets will be wrought in $7\frac{1}{2}$ hours?

9. A carriage wheel, the circumference of which is $16\frac{1}{2}$ feet, and which makes 45 revolutions per minute, goes 275 miles in a certain time; how many revolutions per minute must a wheel make, to perform 385 miles in the same time, the circumference of the latter wheel being $19\frac{1}{4}$ feet?

10. A field of 12 acres having 120 stalks to each square yard, and 70 grains to each stalk, produces wheat to the value of £96 16*s*.: what will be the worth of the produce of 800 square yards, having 175 stalks to the square yard, and 45 grains to each stalk?

11. An iron beam 16 ft. long, $2\frac{1}{4}$ ft. broad, and 8 in. thick, weighs 1280 lbs.: what must be the length of a beam whose breadth is $3\frac{1}{4}$ ft., thickness $7\frac{1}{2}$ in., and weight 2028 lbs.?

12. If a wheel which revolves at the rate of 470 times in 8 minutes, make 50 revolutions in a certain time; how many revolutions will another wheel make in the same period, at the rate of 360 revolutions in 7 minutes?

13. If 15 men eat 13*s*. worth of bread in 7 days, when wheat is 12*s*. per bushel; what should be the price so that 10 men should be furnished for $12\frac{1}{4}$ days at the same cost?

14. A hay-field which has $2\frac{1}{2}$ tons to the acre is mown by 20 men in 6 days working 8 hours a day; what number of hours per day must 13 men work for 8 days upon a field which has $3\frac{1}{4}$ tons to an acre?

15. The circumferences of the smaller and larger wheels of a carriage are in the ratio of 5 to 6. Let the carriage move in a ring, so that the circumferences of the circles described by the inner and outer wheels shall be as 7 : 8. Given that the inner large wheel makes 800 revolutions in describing $\frac{1}{7}$ of its path, find the number of revolutions made by the small outer wheel, while describing $\frac{1}{3}$ of its path.

16. If the price of 100 bricks, of which the length, breadth, and thickness are 16, 8, and 10 respectively, be 5*s*. 4*d*.; what will be the price of 9760 bricks, which are one-fourth greater in every dimension?

17. If 5 steam-engines of 9-horse power (when employed 3 days a week, and 10 hours a day) raise in one week through a certain altitude 25 three-bushel sacks of wheat, weighing 60lbs. a bushel; in what time will 9 engines of 8-horse power (when employed 5 days in the week, and

9 hours a day) raise through 15 times the former altitude, 75 two-bushel sacks of wheat, weighing 63 lbs. a bushel?

18. Three fire-engines, each having 4 pipes, 3 square inches in section, are worked at the rate of 20 strokes in 3 minutes, and discharge 4680 gallons of water in 16 minutes; how many engines, each having 3 pipes, 5 square inches in section, and worked at the rate of 17 strokes in $2\frac{1}{2}$ minutes, will discharge 20,000 gallons in half an hour?

INTEREST.

141. INTEREST is the payment made for the use of money for any time, and is generally reckoned at so many pounds a-year for £100 lent; or as it is commonly called, so many pounds per cent. For instance, if £5 be the interest of £100 lent for a year, we should say that the money is lent at the rate of five per cent. per annum.

The sum lent is called the *Principal;* the interest of £100 for one year the *Rate;* and the sum lent, together with its interest, for any length of time, is called the *Amount.*

When interest is paid only upon the sum originally lent it is called *Simple* Interest; but when at the end of any time agreed upon, as for instance a year, the interest is added to the principal, so that this amount forms the principal for the next year; and a similar addition is made at the end of every such period, then it is termed *Compound* Interest.

142. The questions which occur in this Rule are merely Examples of Proportion.

Ex. I. Find the Simple Interest of £382 10*s.* for one year at 5 per cent.; in other words—If the principal £100 give £5 interest, what interest will be derived from the principal £382 10*s.*? the statement will evidently be

£100 : £382 10*s.* :: £5.

Also, it will be found that every Ex. in Simple Interest

will furnish a similar statement, in which we observe that the first term is £100, the second is the principal, and the third is the rate.

We now multiply the second term by the 5, and divide by the 100; *i. e.* we multiply by the rate and divide by 100. And in these Exs. we do not, as usual in the Rule of Three, reduce the first and second terms to the lowest denomination expressed in either of them, but multiply by 5 and divide by 100, as in Compound Multiplication and Division. The sum, when worked in the usual form, stands thus:

$$
\begin{array}{rl}
\pounds 100 \;:\; \pounds 382\ 10s. \;::\; \pounds 5 & \\
5 & \\
\hline
1{,}00)\ 19{,}12\ 10 & \\
20 & \\
\hline
2{,}50 & \\
12 & \\
\hline
6{,}00 & \textit{Answer.}\ \pounds 19\ 2s.\ 6d.
\end{array}
$$

Here the division by 100 is performed by cutting off two ciphers at the end of the divisor, and two figures at the end of the dividend: and the remainder after each division is reduced, as in Compound Long Division. I have written the 100 as a divisor, but in practice it is omitted.

143. Since in (142) we multiplied by 5, and divided by 100, therefore we might at once have multiplied by the fraction $\frac{5}{100}$, or $\frac{1}{20}$: that is, the operation of finding the interest might have been performed mentally by taking $\frac{1}{20}$th part of the principal; similarly, $\frac{1}{40}$th, for $2\frac{1}{2}$ per cent.; $\frac{1}{10}$th, for 10 per cent. &c.; and this short method is generally used when the rate is an aliquot part of £100, but not otherwise.

144. If the interest for any number of years is required, multiply the interest for one year by the number of years. *If for any* number of months, aliquot parts of the interest

for one year may be taken, as in Practice: but if for any number of weeks or days, it should be generally found by Proportion.

Ex. II. Find the interest of £175 for three years and 135 days, at five per cent.

The interest of £175 for one year is £8 15*s*., and for three years is $3 \times$ (£8 15*s*.) = £26 5*s*. Now, to find what is the proportionate amount of interest for 135 days, we have this statement;

days.		days.		£ s.
365	:	135	::	8 15

and the answer is £3 4*s*. $8\frac{52}{73}$*d*.: therefore the whole interest = £26 5*s*. + £3 4*s*. $8\frac{52}{73}$*d*. = £29 9*s*. $8\frac{52}{73}$*d*.

Exs. of this kind, involving Simple Interest for years and days, may also be worked as follows:

Since we obtain Simple Interest for 1 year by multiplying by the rate per cent., and dividing by 100,

$$\therefore \text{ in this case, S. Int}^{t} \text{ for 1 year} = 175 \times \frac{5}{100} \text{£} \qquad \text{(E)}$$

and expressing the 135 days as a fractional part of a year, and multiplying the interest of 1 year by the number of years, viz. $3\frac{135}{365}$, or $3\frac{27}{73}$,

$$\therefore \text{ whole interest} = 175 \times \frac{5}{100} \times 3\frac{27}{73} = 175 \times \frac{5}{100} \times \frac{246}{73}$$

$$= \frac{215250}{7300} = \text{£}29\ 9s.\ 8\tfrac{52}{73}d.$$

OBS. I did not reduce the fr^{n} $\frac{5}{100}$ in (E) to lower terms, because the den^{r} is generally more simple, when the 100 is left uncancelled.

Under the head of Simple Interest may be included all questions generally classed under the heads of Commission, Brokerage, and Insurance; for all such quantities are calculated at a fixed rate for every £100.

Exs. 48.

Find the Interest of

	£	s.	d.		yr.		£	
1.	324	0	0	for	1	at	5	p. c.
2.	475	10	6	,,	1	,,	4	,,
3.	875	12	3	,,	1	,,	$3\frac{1}{4}$	,,

Find the Amount of

	£	s.	d.		yrs.		£	
4.	1025	0	0	for	3	at	$2\frac{3}{4}$	p. c.
5.	1750	9	0	,,	$1\frac{1}{2}$	,,	$3\frac{1}{2}$	,,
6.	1827	18	9	,,	$6\frac{1}{4}$	,,	$2\frac{1}{4}$	,,

	Interest of		Amount of
	£ s. d. y. m. £		£ s. d. y. m. £
7.	540 17 6 for 1 5 at 4 p. c.	9.	237 10 0 for 4 11 at $5\frac{1}{4}$ p.c.
8.	1845 guineas ,, 3 10 ,, 3 ,,	10.	11428 0 0 ,, 9 7 ,, $1\frac{3}{4}$,,

	Interest of		Amount of
	£ s. d. y. w. £		£ s. d. y. w. £
11.	755 6 8 for 1 15 at $2\frac{3}{4}$ p.c.	13.	1875 5 0 for 3 45 at $4\frac{1}{2}$ p.c.*
12.	985 13 4 ,, 3 39 ,, $4\frac{1}{2}$ p.c.	14.	2000 0 0 ,, 11 30 ,, $3\frac{1}{2}$,,

	Interest of		Amount of
	£ s. d. y. d. £		£ s. d. y. d. £
15.	1440 15 0 for 1 73 at 5 p.c.	17.	1175 2 6 for 4 300 at $3\frac{1}{4}$ p.c
16.	2500 0 0 ,, 3 90 ,, 4 ,,	18.	990 11 0 ,, 7 150 ,, 6 ,,

145. As an Ex. in Compound Interest we may take the following question :—

Ex. III. **What is the amount of £350 in three years at five per cent. Compound Interest?**

Putting down only the results of the three operations, we have

	£	s.	d.
Principal of first year	350	0	0
Interest of first year	17	10	0
Amount of first year, and principal of second year .	367	10	0
Interest of second year..	18	7	6
Amount of second year, and principal of third year	385	17	6
Interest of third year	19	5	$10\frac{1}{2}$
Amount at the end of third year.........................	405	3	$4\frac{1}{2}$

This is the amount; if the interest alone is required, we subtract the principal £350, from this amount; the remainder is £55 3*s*. $4\frac{1}{2}$*d*.

The above results can be obtained mentally when the per centage is an aliquot part of 100, as $2\frac{1}{2}$, 5, 10, 20, (143); or when it can be easily made up of such numbers, as 21 p. c. in (152), Ex. III.; but if any other be taken, the work must be performed at length, as in (142).

Exs. 49. Find, at Compound Interest,

	The Interest of		The Amount of
	£ s. d. yrs. £		£ s. d. £
1.	1500 0 0 for 4 at 5 p. c.	4.	750 0 0 for 6*y*. at..... $5\frac{1}{4}$ p. c.
2.	354 13 6 ,, $2\frac{1}{2}$,, $4\frac{1}{2}$,,	5.	2025 0 0 ,, 3*y*. 150*d*. at 5 ,,
3.	1820 15 0 ,, $3\frac{1}{4}$,, $2\frac{1}{2}$,,	6.	1825 11 6 ,, 4*y*. 7*mo*. ,, 3 ,,

* In this and the following Exs. the fractional parts of a penny have been neglected when finding the interest for the weeks.

In Compound Interest, when the int^t is paid half-yearly, or quarterly, the result is different from that obtained when it is paid yearly, because the principal is sooner increased by the addition of interest; and therefore Comp^d Int^t begins sooner to accrue. In all cases, the int^t must be added to the principal as soon as it is due.

146. Questions may be found in Interest which involve some little difficulty, because there do not appear at once three terms out of which to form a statement. And most pupils will find that in any difficult question involving the application of Proportion, as in Profit and Loss, Stocks, &c., they cannot succeed in thoroughly comprehending it, without placing it in a plain Rule of Three form.

Hitherto we have been finding the *Interest*, when the other three quantities, *Principal*, *Rate*, and *Time* were known; the following Exs. shew how to find *any one* out of the above four quantities, when the remaining three are known.

Ex. IV. In what time will £75 12*s*. 6*d*. amount to £99 16*s*. 6*d*. at four per cent. per annum?

Here the interest gained is found by subtracting the principal, £75 12*s*. 6*d*., from the amount, £99 16*s*. 6*d*., and it = £24 4*s*. Also, the interest of £75 12*s*. 6*d*. for one year at 4 per cent. is £3 0*s*. 6*d*. Hence I have this question:

If £75 12*s*. 6*d*. produce £3 0*s*. 6*d*. in one year, in what time will it produce £24 4*s*.? The statement is

£3 0*s*. 6*d*. : £24 4*s*. :: 1 year,

and the fourth term will be found to be 8 years.

Ex. V. What sum will amount to £104 2*s*. 6*d*. in four years, at $4\frac{1}{2}$ per cent. simple interest?

I must here inquire what £100 would amount to in four years, at $4\frac{1}{2}$ per cent.; the answer is £118. The question is now, therefore,

If £100 become £118, what sum will in the same time amount to £104 2*s*. 6*d*.? The statement is

£118 : £104 2*s*. 6*d*. :: £100

and the required fourth term is £88 4*s*. $9\frac{57}{59}$*d*.

Ex. VI. At what rate per cent. will £152 10*s*. amount to £191 7*s*. 9*d*. in six years?

I have in this Ex. to find the interest of £100 for one year. Now the interest gained by £152 10*s*. in six years is (£191 7*s*. 9*d*. — £152 10*s*.), or £38 17*s*. 9*d*., or in one year £6 9*s*. 7½*d*.: hence the question now is,

If £152 10*s*. gain £6 9*s*. 7½*d*., what will £100 obtain? The statement will be

£152 10*s*. : £100 :: £6 9*s*. 7½*d*.

OBS. These, and all future Exs., as in Profit and Loss, Partnership, &c., of which I put down only the steps, should be worked out fully by a pupil, and in all cases, where possible, fractionally. Thus in the last Ex., since £6 9*s*. 7½*d*. = 129*s*. 7½*d*. = 129⅝*s*.,

$$\text{the 4th term} = \frac{£100}{£152\frac{1}{2}} \times 129\tfrac{5}{8}s. = \frac{100}{\frac{305}{2}} \times \frac{1037}{8}s. = 100 \times \frac{2}{305} \times \frac{1037}{8}s.$$

$$= 85s. = £4\ 5s.$$

***i. e.* the rate per cent. is 4¼.**

Exs. 50.

1. In what time will £62 amount to £71 6*s*. at 5 per cent. per annum*?

2. In what time will £1215 15*s*. amount to £1291 14*s*. 8¼*d*. at the rate of 2½ per cent.?

3. For how many years must I put out £987 12*s*. to interest at 4¼ per cent., in order that I may receive £1197 9*s*. 3⅗*d*.?

4. What sum of money will in 1 year amount to £108 13*s*. 6*d*. at 3½ per cent.?

5. Required the principal which will in 3 yrs. amount to £136 2*s*. 1½*d*., at 2¼ per cent.

6. Find what sum will produce interest amounting to £330 15*s*. in 7 yrs. at 4½ per cent.

7. At what rate of interest will £95 15*s*. amount to £112 10*s*. 1½*d*. in 5 yrs.?

8. What must be the per centage in order that £1175 may become £1637 13*s*. 1½*d*. in 7½ yrs.?

9. Required the rate per cent. at which 1000 guineas will gain £590 12*s*. 6*d*. in 12½ yrs.

10. Find the rate per cent. at which any sum of money will double itself in 8 yrs.

11. What sum lent at 5 per cent. Compound Interest will in 3 years amount to £358 17*s*. $3\frac{3}{10}$*d*.?

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* Simple Interest is always implied, unless the contrary be expressed.

12. Find the difference between the Simple and Compound Interest of £416 13*s.* 4*d.* for 2 yrs. at $2\frac{1}{2}$ per cent.

13. What is the commission on £20500 at $3\frac{1}{8}$ per cent.?

14. Find the premium on a policy of life insurance for £2500 at £5 17*s.* 9*d.* per cent.

15. What annual premium must a farmer pay on stock valued at £370 10*s.*, at 2*s.* 3*d.* per cent.?

16. In what time will £818 18*s.* 4*d.* amount to £1064 11*s.* 10*d.* at $3\frac{3}{4}$ per cent.?

17. At what rate of interest will £732 10*s.* amount to £1245 5*s.* in 10 yrs.?

18. If the interest on £130 15*s.* 10*d.* for 10 days be 3*s.* 7*d.*, how much is that per cent. per annum?

147. Questions concerning Annuities, Leases, and Reversions involve applications of Interest; but they generally require for their solution either algebraical expressions or tables derived from these. The following is however a simple example of the kind.

Ex. VI. What is the amount of an annuity of £50 left unpaid for 5 yrs., allowing Compound Interest at 4 per cent. per annum?

I write down merely the outlines of the work, and neglect all sums below 1*d.*, as the fractions obtained after two or three divisions become exceedingly heavy.

	£	*s.*	*d.*
Amount due at the end of first year....................	50	0	0
Interest due at the end of second year	2	0	0
Add £50, due at the end of second year	50	0	0
Principal of third year	102	0	0
Interest at the end of third year	4	1	7
Add £50 due at the end of third year	50	0	0
Principal of fourth year	156	1	7
Interest at the end of fourth year	6	4	10
Add £50, due at the end of fourth year	50	0	0
Principal of fifth year..................................	212	6	5
Interest at the end of fifth year........................	8	9	10
Add £50, due at the end of fifth year	50	0	0
Amount due at the end of five years..................	270	16	3

If Simple Interest alone were allowed, I should write the interest for

the successive years by itself, and add its amount to the final amount;—by this means no interest would be allowed upon interest, *i. e.* there would be no Compound Interest.

The subjoined Examples are worth notice.

Ex. VII. What must I give for a freehold, let for £225 a year, so as to have $4\frac{1}{2}$ per cent. for my money? Or in other words,

If every £100 laid out bring £$4\frac{1}{2}$, what sum will produce £225? The statement will be

$$£4\tfrac{1}{2} \ : \ £225 \ :: \ £100$$

and the fourth term $= \dfrac{£225}{£4\frac{1}{2}} \times £100 = \overset{25}{\cancel{225}} \times 100 \times \dfrac{2}{\cancel{9}}\,£ = £5000.$

148. Sometimes in speaking of the price of a piece of property, it is said that a certain number of years' purchase is given for it: this is the same as so many years' rental. Thus, if a field, the rent of which is £4, be sold for £100, we say that 25 years' purchase was given for it, because the price is 25 times the rental.

Ex. VIII. How many years' purchase should be paid for freehold property to clear $4\frac{1}{2}$ per cent.?

I must here see how many times a rent of £$4\frac{1}{2}$ must be repeated to produce £100, the price of the land which gives £$4\frac{1}{2}$.

This number $= \dfrac{£100}{£4\frac{1}{2}} = 100 \times \dfrac{2}{9} = \dfrac{200}{9} = 22\frac{2}{9}$; the price paid is therefore said to be $22\frac{2}{9}$ years' purchase.

Exs. 51.

1. What will an annuity of £60, payable yearly, amount to in 6 yrs. at 5 per cent. Compound Interest?

7. Find the amount due from a pension of £100, payable half-yearly, which has been unpaid for $3\frac{1}{2}$ yrs., allowing 5 per cent. Comp. Interest?

3. What principal lent for $2\frac{1}{2}$ yrs. at 5 per cent. Compound Interest will amount to £700 12*s.* $9\frac{3}{10}$*d.*?

4. What should be the purchase money of an estate, of which the rental is £5200, so that the buyer may receive $3\frac{1}{4}$ per cent. for his money?

5. A purchaser invests £7500 in land, and receives $2\frac{3}{4}$ per cent. upon his investment; what is the rent?

6. What per centage is received upon the purchase money, when an estate whereof the rent is £367 10*s*., is bought for £10500?

7. How many years' purchase should be paid for freehold property, to produce $3\frac{3}{4}$ per cent.?

8. A freehold is sold at 33 years' purchase; what rate of interest is received on the investment?

9. What is the value of a perpetual annuity of £120 at the rate of $4\frac{1}{4}$ per cent.?

10. Property which brings 7 per cent. lets for £85 15*s*.; what was the purchase money?

DISCOUNT.

149. DISCOUNT is an allowance made by a creditor to a debtor who pays a debt before it is due. When this allowance is subtracted from the debt, the remainder, *i. e.* the sum that is paid, is called the present worth.

Discount is calculated at a certain rate per cent., and in common usage is treated just the same as Interest: we shall, however, show that this is not strictly correct, but that the person who pays the money has thereby more than the just allowance made to him.

For instance; if £50 were due to me at the end of one year, but I were willing to allow a discount of 5 per cent. for ready money, then, according to the common usage, I should throw off the interest of £50 for one year, viz. £2 10*s*., and receive only £47 10*s*. But if I make a debtor an allowance for paying ready money, I do so upon the supposition that I can place out to interest the ready money which I receive, and together with the interest can make up the £50 at the end of the year. Now, if I put out to interest £47 10*s*. at 5 per cent., I shall obtain as interest £2 7*s*. 6*d*., and therefore I shall in all receive £49 17*s*. 6*d*.: hence I lose 2*s*. 6*d*. by this arrangement. The real question

now is—What sum put out to interest for a year at 5 per cent. will amount to £50? Or—If £100 will amount to £105, what sum will amount to £50? The statement will be

Interest for 1 year	£ 5					
Principal	£100					
Amount	£105	:	£50	::	£100	(F)

and the fourth term will be found to be £47 12*s*. $4\frac{4}{7}$*d.*

Also, the interest of £47 12*s*. $4\frac{4}{7}$*d.*, for one year is £2 7*s*. $7\frac{3}{7}$*d.*; and this, together with the principal, = £50: and therefore I neither gain nor lose.

By observing (F), we notice that the third term, £100, is the present worth of the first term, £105; and the fourth term is the present worth of the second term, £50. Hence, in any question where the present worth of a sum is required, the third term is £100; the first term is the amount of £100 at interest for the given time; and the middle term is the sum due.

150. If the discount, and not the present worth, be required, we must place in the third term the discount of £105, viz. £5. But since the discount in the third term would generally require to be reduced to the lowest denomination expressed, and the work be thereby rendered heavy; it is therefore generally better to find the present worth, and then obtain the discount by subtracting the present worth from the bill due.

151. The most common form in which discount occurs is in the use of what are called *Bills*, which are stamped papers, bearing a written engagement to pay a sum of money at a certain future time. If such a bill be presented to a banker before it is due, *i.e.* before the time fixed for pay-

ment, and the persons who are responsible for this bill are considered able to meet it at the proper time, the banker will give ready money for it, retaining, however, the *discount* upon the sum, as his remuneration for the accommodation.

In practice, as was said, it is usual to charge interest, and not discount: therefore the banker gains by the transaction, and the amount of this gain will be found to be the interest upon the true discount. For if we refer to the Ex. in (149) we shall see that in discounting a bill of £50 due in twelve months, the banker would deduct £2 10*s.*, *i. e.* the interest on £50; whereas he ought to have deducted only £2 7*s.* 7$\frac{3}{7}$*d.*; which is the interest upon £47 12*s.* 4$\frac{4}{7}$*d.*; therefore the extra sum which he takes is the interest upon £2 7*s.* 7$\frac{3}{7}$*d.*, or the interest upon the true discount.

Though discount is not in practice correctly used, yet a pupil in working Exs. should always employ the true method.

The bills mentioned above are said to be *drawn* upon the person or persons who agree to pay the money, and those who allow any such bill to be drawn on them are said to *accept* it: hence they are called acceptors, and the bill itself is called an acceptance. These acceptances are generally for any number of calendar months: but in this country three days, called *Days of Grace*, are allowed after the bill is nominally due, before it is legally due; so that a bill drawn on March 30th, at three months, would not be legally due till July 3rd.*

Ex. II. What does a banker gain by discounting a bill of £403 4*s.*, drawn Oct. 13, at four months, and discounted, Dec. 5, at 4 per cent.?

Here the bill is legally due on Feb. 16; and from Dec. 5 to Feb. 16 are 73 days:

* A form of acceptance is given under the Art. *Book-keeping.*

	£	s.	d.
The interest for that time... =	3	4	$6\frac{18}{125}$
And the true discount ...... =	3	4	0
Therefore the banker's gain =			$6\frac{18}{125}d.$

Exs. 52.

Find the Discount on							Find the present worth of					
	£	s.	d.	*months*	£			£	s.	d.	*months*	£
1.	100	0	0	for 6	at 5 per ct.		5.	1000	0	0	due in 12	at 5 per ct.
2.	128	18	6	„ 3	„ 4 „		6.	875	10	0	„ 8	„ $4\frac{1}{2}$ „
3.	157	10	0	„ 9	„ 6 „		7.	119	6	0	„ 5	„ $3\frac{1}{2}$ „
4.	1128	17	6	„ 7	„ $3\frac{1}{2}$ „		8.	425	15	6	„ $2\frac{1}{2}y.$	„ 3 „

9. Find the difference between the interest and discount of £525 for $1\frac{1}{2}$ yrs. at 5 per cent.

What would a banker gain by discounting the following bills?—

		Drawn		*Discounted*	
10.	£325 8*s*. 4*d*.	March 15 at 4 months,		April 6th,	at 4 per cent.
11.	£90 7*s*. 6*d*.	Sept^r 1 „ 9 „		Jan. 15th,	„ 5 „

12. What is the present worth of £500, one-half of which is due in 4 months, and the remainder in 6 months, discount at $5\frac{1}{4}$ p. c. per ann.?

152. A very important application of Discount occurs in those branches of business, where it is the custom to take off 20, 30, &c., per cent. discount from the gross or invoice price of goods. And great errors may sometimes be made by tradesmen who do not know how much to add to the net value of an article, in order that they may, without loss, make the deduction agreed upon.

For instance, suppose a tradesman has an article of which the net price should be 50*s*., and it is usual to allow 20 per cent. discount, or deduct one-fifth from the invoice price. If he, thinking to allow for this discount, puts on $\frac{1}{5}$th of the 50*s*., and thus makes a gross price of 60*s*.; then, when he takes off 20 per cent. or $\frac{1}{5}$th, he will find his net price to be 48*s*., thereby losing 2*s*. But if, instead of putting on $\frac{1}{5}$th he had put on $\frac{1}{4}$th, he could then take off the required $\frac{1}{5}$th and be no loser.

Thus,			*s.*	*d.*
	Net price required	=	50	0
	Add $\frac{1}{4}$th of 50*s*..................	=	12	6
	Gross price..........	=	62	6
	Subtract $\frac{1}{5}$th of 62*s*. 6*d*.	=	12	6
	Net price as before	=	50	0

The general rule will be found as follows: Let the discount agreed upon be represented as a fractional part of £100: Thus, for 5, 10, 20, 30, 35, &c. per cent., the fractions would be

$$\frac{5}{100},\ \frac{10}{100},\ \frac{20}{100},\ \frac{30}{100},\ \frac{35}{100},\ \&c.$$

or, in their lowest terms,

$$\frac{1}{20},\ \frac{1}{10},\ \frac{1}{5},\ \frac{3}{10},\ \frac{7}{20},\ \&c. \qquad (G)$$

These are the fractional parts of the gross price to be *deducted* from it: and the corresponding frac[l] parts that must be first *added* to the net price will be

$$\frac{1}{20-1},\ \frac{1}{10-1},\ \frac{1}{5-1},\ \frac{3}{10-3},\ \frac{7}{20-7},\ \&c.$$

$$\text{or}\quad \frac{1}{19},\ \frac{1}{9},\ \frac{1}{4},\ \frac{3}{7},\ \frac{7}{13}, \qquad (H)$$

where it is to be observed that the fractions in (H) have numerators the same as in (G); but the den[rs] are the former den[rs], minus the respective numerators*.

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* The following demonstration of the above rule may be read by those acquainted with the elements of Algebra.

Let G = gross price; N = net price; also, let x represent the frac[l] part of N to be added thereto to reproduce G, so that $N+xN=G$; or $N(1+x)=G$. (1)

Let $\frac{m}{n}$ represent the frac[l] part of G to be taken off as discount so as to leave N; so that $G-\frac{m}{n}G=N$; or $G\left(1-\frac{m}{n}\right)=N$; substituting for N in (1), we have

$$G\left(1-\frac{m}{n}\right)(1+x)=G;\ \text{or}\ 1+x=\frac{1}{1-\frac{m}{n}}=\frac{n}{n-m};$$

$$\therefore\ x=\frac{n}{n-m}-1=\frac{n-n+m}{n-m}=\frac{m}{n-m};$$

i. e. if we wish to take off as discount $\frac{m}{n}$, we must put on $\frac{m}{n-m}$.

For example, if 30 per cent. discount is to be allowed upon an article of which the net price should be 21*s.*, we refer to the rows (G) and (H), and learn that if 30 per cent. or $\frac{3}{10}$ is to be taken off, $\frac{3}{7}$ must be put on: and we have as follows.

		s.
Net price	=	21
Add $\frac{3}{7}$ths of 21*s.*	=	9
Gross price charged in invoice	=	30
Subtract $\frac{3}{10}$ths of 30*s.*	=	9
Net price as before	=	21

PROFIT AND LOSS.

153. PROFIT AND LOSS is another application of Proportion. It is calculated at so much per cent., or at a certain *per centage*, and the general object of all Exs. under this head is, to find—(1) What per centage of profit or loss will result from selling an article at a certain price:—(2) At what price it must be sold, that there may arise a certain per centage of profit and loss; the prime cost of the article being in both cases known.

154. It will not be attempted to exhibit an Ex. of every kind of question that may arise; but a sufficient number will be given to show the principles upon which all the questions depend; and the particular method of applying the principles of Proportion in each case must be left to the judgment of the pupil.

Ex. I. If an article cost £2 7*s.* 3*d.*, and be sold for £3 3*s.* 0*d.*, what is the gain per cent.?—or, If £2 7*s.* 3*d.* become £3 3*s.* 0*d.*, what will £100 become? The statement is

£2 7*s.* 3*d.* : £100 :: £3 3*s.*

and the fourth term is £133 6*s.* 8*d.* Hence the gain upon £100 is £33 6*s.* 8*d.*; or the profit is at the rate of $33\frac{1}{3}$ per cent.

Ex. II. If, by selling tea at 6*s.* 4*d.* per lb., a grocer lose 6 per cent., what was the prime cost per lb.?

Now, to lose 6 per cent. is to obtain only £94 for every £100 laid out; hence the question is really this—If £100 be laid out, and £94 be received for it, what is laid out when 6*s.* 4*d.* is received?

Here, since £100 is prime cost, or buying price, and we want buying price in the fourth term, we have this statement:

£94 : 6*s.* 4*d.* :: £100

and the fourth term, or prime cost of 1 lb. is 6*s.* $8\frac{4}{47}$*d.*

Ex. III. At what price must I sell a commodity purchased at the rate of £14 5*s.* per cwt. so as to gain 21 per cent.?

In this Ex. it is required to receive £121 for £100 laid out: therefore £121 is the selling price of that which cost £100; and since the fourth term is to be the selling price of 1 cwt., we have

£100 : £14 5*s.* :: £121

and the required price is £17 4*s.* $10\frac{1}{4}$*d.*

Questions of this kind, wherein we require the price at a certain profit per cent., may often be worked more briefly by the method of Practice. Thus to gain 20 per cent on any sum of money invested is merely to add one-fifth of the sum to the previous amount; and Ex. III. may be worked as follows.

	£	s.	d.
Prime cost	14	5	0
Add 20 per cent. or $\frac{1}{5}$th	2	17	0
Add 1 ,, or $\frac{1}{20}$th of 20 p.c.		2	$10\frac{1}{5}$
∴ original sum + 21 per cent. profit =	£17	4	$10\frac{1}{5}$

The following Ex. involves the principles of Exs. II. and III.

Ex. IV. A person, by disposing of goods for £182, loses at the rate of 9 per cent.; what should have been the selling price, so as to make a profit of 7 per cent.?

We may work this question by two operations: first, find the prime cost, and then from it find that selling price which would give a profit of 7 per cent. Since to sell at 9 per cent. loss is to receive but £91 for that which cost £100, we have this statement:

£91 : £182 :: £100.

and the answer is the prime cost £200. Also, to obtain a profit of 7 per cent. is to receive £107 for that which cost £100; therefore, to find the selling price of that which cost £200, we have

£100 : £200 :: £107

and the answer is £214, the price at which the goods should be sold to make 7 per cent. profit.

In order to work the question by one statement, we may put it under this form—If goods sold for £182 bring £91 for every £100 laid out, what ought they to be sold for, so as to bring £107 for every £100?

Here we have the £100 laid out the same in both circumstances, and it will therefore not affect the question: £107 and £91 may be considered as the gaining and losing rates; also, £182 is the sum received for goods, and therefore of the same nature with the fourth term; hence the statement is

£91 : £107 :: £182

and the fourth term, or selling price, is £214, the same that was obtained from the former statements.

Questions under this head may be worked according to the principles described in the latter parts of (133) and (140).

Thus, to take a simple case, If a man buys for £4 10*s.*, and sells for £5 15*s.*, how much will he gain per cent.?

The actual gain is £5 15*s.*—£4 10*s.* = £1 5*s.*

$$\therefore \text{£}1\tfrac{1}{4} = \text{gain on £}4\tfrac{1}{2} = 4\tfrac{1}{2}\times\text{gain on £}1$$

$$\text{and £}\,\frac{1\frac{1}{4}}{4\frac{1}{2}} = \text{gain on £}1.$$

Let A = rate per cent. or gain on £100,

$$\text{then £}\,\frac{A}{100} = \text{gain on £}1;$$

$$\therefore \frac{A}{100} = \frac{1\frac{1}{4}}{4\frac{1}{2}},\ \text{or } A = \frac{100\times1\frac{1}{4}}{4\frac{1}{2}} = \frac{250}{9} = 27\tfrac{7}{9}.$$

Exs. 53.

1. Paid £137 12*s.* 6*d.* for goods, and sold them for £151 7*s.* 9*d.*; what was the profit per cent.?

2. By selling goods at 3*s.* 6*d.* I gain 12 per cent.; what shall I gain or lose by selling them at 4*s.* 9*d.*?

3. I give 3*s.* 9*d.* for goods; at what rate must they be sold to make a profit of 30 per cent.?

4. Bought cloth at 9*s.* 4½*d.* per English ell; it is required to find the selling price per yard so as to gain 17½ per cent.

5. Goods were bought for 2*s.* 9*d.*, being 17½ per cent. below their real value; what was that value?

6. Sold goods for £3 13*s.* 6*d.*, being 22½ per cent. profit; what was the prime cost?

7. I sell an article for £22 10*s.*, and by so doing lose 15 per cent.; what per centage would be lost or gained by selling it at £27?

8. At a selling price of 15*s.* I lose 10 per cent.; what must be the price to gain 10 per cent.?

9. I buy tobacco at 10 guineas per cwt.; at what price must I retail it per lb. so as to gain 12 per cent.?

10. When the price of a certain article is 12*s.* 6*d.* there is a gain of 25 per cent.; what would be the loss or gain if the price were 10*s.*?

11. I bought 145 quarters of wheat at 50*s.* per quarter, and in selling I make a profit of £36 5*s.*; how much per cent. was the profit?

12. A merchant sold a pipe of wine for £50, and by so doing lost 5 per cent.; at what price must he sell 3 other pipes so that he may gain 5 per cent. upon the prime cost of the 4 pipes?

13. A person having bought goods for £20, sells half of them so as to gain 10 per cent.; for how much must he sell the remainder so as to gain 20 per cent. upon the whole

14. I bought 56 gallons of brandy at 22*s.* 6*d.* per gallon, but 7 gallons were lost; at what price per gallon must I sell the remainder, to obtain 15 per cent. profit on the whole outlay?

PARTNERSHIP.

155. Partnership, or, as it is sometimes called, Fellowship, is the Rule by which we determine how to divide profits, which arise from different sums of money put into a business by two or more persons, either for the same or different periods of time.

Ex. I. Two persons enter into business as partners; one puts in £350, and the other £500; they gain £100. How is the profit to be divided?

Here the profit, £100, is made from the whole capital, £850; and each partner's share of the profit will be in proportion to his share of the capital: therefore the question divides itself into these two parts:—(1) If £850 produce a profit of £100, how much will £350 produce? (2) If £850 produce £100, what will £500 produce?

The statements for these two questions will plainly be

	£		£		£
(1)	850	:	350	::	100
(2)	850	:	500	::	100

	£	s.	d.
and the fourth term of (1) =	41	3	$6\frac{6}{17}$
„ „ „ (2) =	58	16	$5\frac{11}{17}$
and these together . . . . . =	100	0	0

Ex. II. A field of grass is rented by two persons for £27; the one keeps in it 15 oxen for ten days, and the other 21 oxen for seven days: find the rent to be paid by each, supposing the pasturage to remain equally good throughout?

Here it is plain that the keep of 15 oxen for ten days is the same as of ten times 15, or 150, oxen for one day: so also, of 21 oxen for seven days is the same as of 7 times 21, or 147, for one day; therefore the question is plainly this: If one man turn into a field 150 oxen, and another 147, for one day, and they together pay £27, how is that payment to be divided?

The whole number turned in would be 297, and the two statements would be similar to those in Ex. I., viz.

$$297 : 150 :: £27 \quad (1)$$
$$297 : 147 :: £27 \quad (2)$$

the fourth term of $(1) = \dfrac{150 \times \cancel{27}}{\underset{11}{\cancel{297}}} £ = £13\ 12s.\ 8\frac{8}{11}d.$

„ „ „ $(2) = \dfrac{147 \times \cancel{27}}{\underset{11}{\cancel{297}}} = £13\ 7s.\ 3\frac{3}{11}d.$

and their sum .... $= £27\ 0\ 0$

156. I will give one more Ex. which is the same in principle as Exs. I. and II., but is more complicated in its operations.

Ex. III. On the 1st of January *A* brought into a business £350, and on the 1st of April £500 more: on the 1st of June he takes out

£400; three months after this he brought in £600. *B* brought into the business £500: four months after this he takes out £150; and on the 1st of November he brought in £650. At the end of the year their clear gain is £1008. How much ought each to receive?

Here *A* put in £350 from January 1st to June 1st, or five months; also, £500 from April first to June 1st, or two months: he has now in the business £850, but he takes out £400, leaving £450. This £450 is in from June 1st to December 31st, or seven months. Also, he has £600 in from September 1st to December 31st, or four months. Hence he has in all

£				£			
350	for	5	months,	or 1750	for	1	month
500	,,	2	,,	or 1000	,,	1	,,
450	,,	7	,,	or 3150	,,	1	,,
600	,,	4	,,	or 2400	,,	1	,,
Therefore he has in all				8300	,,	1	,,

Again, *B* brought in £500 from January 1st to May 1st, or four months: he now takes away £150, and has in £350 from May 1st to December 31st, or eight months. Also, he brings in £650 from November 1st to December 31st, or two months. Hence he has

£				£			
500	for	4	months,	or 2000	for	1	month
350	,,	8	,,	or 2800	,,	1	,,
650	,,	2	,,	or 1300	,,	1	,,
Therefore *B*'s capital is				6100	,,	1	,,
and *A*'s capital was				8300	,,	1	,,
therefore the joint capital				= 14400	,,	1	,,

Hence the two statements will be

	£		£		£
For *A*'s share	14400	:	8300	::	1008
For *B*'s ,,	14400	:	6100	::	1008

and the fourth terms are £581 for *A*, and £427 for *B*.

157. In this place we may introduce an Ex. of the following kind.

Ex. IV. A wine merchant mixes together 20 gallons of wine at 12*s*. a gallon, 25 gallons at 14*s*., and 36 gallons at 16*s*.: what should be the price of a gallon of the mixture?

Here it is plain that 20 gallons at 12s. are worth 240s.

also	,,	**25**	,,	**14s.**	,, **350s.**
and	,,	**36**	,,	**16s.**	,, **576s.**
and therefore that		**81**	**of the mixture**		,, **1166s.**

hence the value of one gallon is plainly $\frac{1}{81}$th of the value of the whole;

or, price per gallon $=\frac{1166}{81}s. = 14s.\ 4\frac{20}{27}d.$

158. The following Ex. shows how to divide a given quantity into parts which shall have to each other given ratios. It is upon the same principle as the previous Exs., though not commonly recognized as such.

Ex. V. Divide 1065 into parts which shall be to each other in the ratio of 3, 5, 7; and also into parts which shall be in the ratio of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$.

Taking the former part of the Ex., we observe that $3+5+7=15$ is the smallest integer which can be divided in the ratio of 3, 5, 7; hence the required portions of 1065 will bear the same ratio to 1065 that 3, 5, 7 do respectively to 15. The first portion is obtained by the statement

$$15 \;:\; 3 \;::\; 1065;$$

and the fourth term $=213$. Also, if 5 and 7 be successively placed in the second term of the above statement, we shall find the remaining portions to be 355, and 497.

Similarly, in the latter part of the Ex., we see that $\frac{1}{3}+\frac{1}{5}+\frac{1}{7}$, or $\frac{35+21+15}{105}=\frac{71}{105}$ is a fraction which can be broken up into the fractions $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{7}$; hence the required portions of 1065 will bear the same ratio to the entire number 1065 that $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{7}$, respectively bear to their sum $\frac{71}{105}$; we have, therefore, as the statement for obtaining the first portion

$$\frac{71}{105} \;:\; \frac{1}{3} \;::\; 1065;$$

and the fourth term $=\frac{\overset{\overset{5}{\cancel{15}}}{\cancel{1065}}}{1}\times\frac{1}{\cancel{3}}\times\frac{105}{\cancel{71}}=525$. So also the second and

third statements will be

$$\frac{71}{105} : \frac{1}{5} :: 1065;$$

$$\text{and } \frac{71}{105} : \frac{1}{7} :: 1065;$$

and the corresponding fourth terms will be found to be 315 and 225.

Exs. 54.

1. Two persons invest in business £300 and £250 respectively; they gain £150: how is it to be divided?

2. *A* and *B* as partners lost £600; if *A*'s capital were £4000, and *B*'s £2100, how much of the loss must each bear?

3. *A*, *B*, and *C* were partners: *A* put in £1000 for 2 yrs., *B* £750 for 15 months, and *C* £1500 for 9 months; divide equitably a profit of £1000.

4. *A*, *B*, and *C* rent a field for £10; *A* put in 20 horses, *B* 15 oxen, and *C* 10 sheep; how should the expence be divided, if the eating of a horse, ox, and sheep be in the ratio of 3, 2, and 1?

5. *A* puts into a concern £500, and 6 months after puts in £300 more; *B* puts in £1000, and 3 months after puts in £1000 more; they trade for 2 yrs., and gain £650: what is the share of each?

6. Divide £20 amongst 3 persons, so that their shares shall be in the ratio of 3, 4, and 5.

7. Distribute £705 in the ratio of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

8. A mixture is made of 5lbs. of tea at 3*s.* 4*d.*, 10lbs. at 4*s.* 2*d.*, and 15lbs. at 5*s.*: what should be the price per lb. of the mixture?

9. A wine merchant mixes together 20 gallons at 12*s.*, 30 gallons at 15*s.*, 40 gallons at 20*s.*, and 10 gallons of water; what must be the retail price so as to gain 10 per cent. profit?

10. A testator bequeaths £1000 to one person, £500 to a second, and £300 to a third; but his property is found to realize only £1250: how much should each receive?

Exs. 55. I.

1. If 375 quarters are grown on 75 acres of land, how much land will be required to grow a peck?

2. Ten persons joined to buy three lottery tickets for £10, £25, and £40: the second gains a prize of £1000, how much does each man gain?

3. How many times does a clock tick in the month of January, if it ticks 15 times in 2 minutes?

7—2

4. A large bin contains 15 cub. ft. 267 cub. in.; and out of it 3 smaller bins are filled, each containing 4 cub. ft. and 375 cub. in.; how much will be left in the bin?

5. Out of a square plate of metal $15\frac{3}{4}$ feet long, how many circular pieces can be cut, $2\frac{1}{4}$ inches in diameter?

6. What is the smallest sum that a man must have in his pocket that he may be able to pay it away entirely either in moidores, guineas, marks, or 7 shilling pieces?

7. Assuming the method of multiplying and dividing fractions by whole numbers, shew that $\frac{3}{8} \div \frac{5}{9} = \frac{27}{40}$.

8. If 9 slabs, 12 in. long, and 12 inches broad, will cover a certain surface, how many slabs will be needed if they be 18 in. long, and 8 in. broad?

9. Find the cost of 8725lbs. at 1*s.* $10\frac{3}{4}$*d.*

10. What cost 25oz. 3dwts. 11grs. at £3 17*s.* $10\frac{1}{2}$*d.* per oz.?

11. If a tradesman gain 1*s.* 6*d.* on an article which he sells for 5*s.* 9*d.*, what is the profit per cent. on the prime cost?

12. Of two pieces of cloth, one is 42 in. wide, and costs 1*s.* 6*d.* per yd.; the other is 56 in. wide, and costs 2*s.* 4*d.* per yd.; what is the ratio of the qualities of the pieces, supposing the prices to be exactly proportionate to their real value?

13. Silver coinage has 37 parts pure silver, and 3 parts of copper; 1lb. Troy makes 66*s.*; what quantity of pure silver is there in 20*s.*?

14. Reduce 8 cwt. $3\frac{1}{2}$ qrs. $1\frac{1}{5}$ stones to the decimal of 15 tons.

15. What decimal of a square furlong is 1 perch?

16. If the net price of an article be found by taking off $\frac{3}{15}$ths of the gross price, what must be added to the net price to make the invoice price?

17. If I am required to throw off $\frac{1}{15}$th, $\frac{2}{31}$ths, $\frac{3}{17}$ths, from the invoice price of three parcels of goods, what must be the fractional parts to be added to the required net prices?

K.

1. How many strokes will a clock strike in the month of May?

2. Find the value of 4424 articles at 15*s.* $10\frac{1}{2}$*d.* each.

3. A block of stone 7 ft. long, $2\frac{1}{2}$ ft. broad, and 15 in. thick, weighs $3\frac{1}{4}$ tons; what must be the length of a block of the same kind, whereof the breadth is $3\frac{1}{4}$ ft., the thickness 10 inches, and weight 6500 lbs.?

4. What is the cost of 59 tons, 15 cwt., 3 qrs., 18 lbs., at £26 18*s.* 9*d.* per ton?

5. A person buys 68 yards of cloth for *£75*, and retails it at £1 18*s.* per English ell; what does he gain by the transaction?

6. What may a person spend per day out of an income of £1000 a year, if he lay by 20 guineas every calendar month?

7. Explain the reason of stating a Rule of Three sum; and shew why the answer results in the same name as the third term.

8. A tax of 3*d.* in the pound on a certain assessment produces £1080; how much will be produced by a tax of 7*d.* on an assessment of double the value?

9. What is the value of $(\frac{2}{5}+\frac{3}{8}-\frac{5}{12})$ of £360?

10. Explain the nature and advantages of Decimal Fractions, compared with Vulgar Fractions.

11. What fraction of a square mile are $2\frac{1}{4}$ perches?

12. I have to distribute 150 yds. to 10 men and 10 women, so that the men and women may have shares in the ratio of 2 : 3; how much will each have?

13. A boat is propelled by 8 oars, which take 10 strokes per minute; and it goes 9 miles an hour; find the rate of a boat propelled by 6 oars which take 8 strokes per minute, when 5 of its strokes are equal to 6 of the former.

14. Find the exact value of

$$\cdot 375 \text{ of } 6s.\ 8d. - \cdot 941875 \text{ of } 4s. + 1\cdot 9898\dot{3} \text{ of } 2s.$$

15. Simplify the following expression:

$$\frac{1}{2}+\frac{1}{3\times 2^3}+\frac{1}{5\times 2^5}+\frac{1}{7\times 2^7}.$$

16. If I throw off 17 per cent. from my invoice price, what fractional part of my net price must I put on?

L.

1. The quotient is 3276, the divisor £2 7*s.* 6*d.*, remainder 6*d.*; find the dividend.

2. On the 1st of March I borrow £10, to be repaid in a calendar month, and in return lend £15 on the 1st of April; when should it be repaid?

3. The girth of a tree at the surface of the ground is 6 feet, find the girths at 10 feet and 20 feet, if the height of the tree be 50 feet, and it tapers regularly.

4. Find the cost of $13\frac{7}{18}$ oz. at £3 7*s.* 6*d.* per oz.

5. At what distance from the end of a slab of 17 in. breadth must I cut, so as to have a rectangular piece, containing half a square yard?

6. An author pays 2*s*. 6*d*. for the printing &c. of a book: out of the publishing price 10 per cent. is allowed for advertising, 10 per cent for publisher's commission, 25 p. c. to the retail trade, 4 p. c. for damaged copies, and 5 p. c. loss of interest; what must be the publishing price, so that he may neither gain nor lose?

7. In the last question, what would he gain on 2000 copies, at a selling price of 5*s*. 6*d*.?

8. If 56 current shillings be worth £$2\frac{7}{18}$ in gold, how many current shillings are worth £65 in gold?

9. Two numbers are to one another as 8 : 11; and the greater one is 77; find the less.

10. Find the whole cost of a house, of which the rent is £27; the poor rate 3*s*. 4*d*. in the pound; gas rate two-thirds of the poor rate; and the paving rate three-fifths of the gas rate.

11. Reduce to a vulgar fraction

$$\frac{2{\cdot}3}{1{\cdot}\dot{7}} \times \frac{13{\cdot}\dot{8}\dot{5}}{1{\cdot}0\dot{2}} \times \frac{1{\cdot}21}{4{\cdot}9}.$$

12. Express the ratio of £$3{\cdot}\dot{7}$ to $4{\cdot}1\dot{5}$ guineas in the smallest integers.

13. A piece of work employs 15 men for 6 days, when the day is 12 hours long, and costs £$18\frac{5}{8}$; what will be the cost of a piece employing 25 men for 9 days of 10 hours each, the pay of the new workmen being $1\frac{1}{3}$ times that of the old?

14. A tank is filled by 3 pipes in 2, $8\frac{2}{7}$, and $7\frac{1}{2}$ hours respectively; in what time would they all fill it?

15. A discount of $\frac{2}{15}$ths is agreed upon; what must be the ratio between the net and gross prices, so that I may be able to make the deduction?

16. I add $\frac{7}{13}$ths to the net price of an article; what per centage of discount was agreed upon?

STOCKS.

159. The government of a country sometimes finds it necessary to borrow money; and it gives to the lender a bond acknowledging the debt, and agreeing to pay a certain rate of interest for the money. The amount owing to those who

hold these bonds is called the National Debt, or *The Funds*; and the interest paid is derived from the income of the country, arising principally from taxation.

The above-mentioned bonds are saleable, and are called *Stock*; and they of course vary in value, principally according to the plentifulness or scarcity of money.

Thus, suppose a person lend £100 to government, and receive an acknowledgment for it, with an agreement to pay £3 a year interest for the loan; then, if at any time he wishes to sell the bond, and money is scarcer than when he lent the £100, he will get less for it than he gave, perhaps £95; and if money be more plentiful, he will get more for it, perhaps £105. But still the bond represents an acknowledgment for £100, and £3 interest is paid to the holder of it, whatever may be the sum which he has paid for it. When, therefore, we say that 3 per cent. stock is selling at 80, we mean that the buyer of £100 bond, or, as it is called, £100 stock, has to give only £80 sterling for it; hence, as he gets £3 interest for it, it is not £3 *per cent.* to him, but £3 for £80. From this it is plain that the lower the stock is in price, the better interest the buyer obtains; and the higher the stock is, the less interest he obtains for his money.

When the market price of £100 stock is exactly £100, the stock is said to be at *par*; if the price be more than £100, it is said to be at a *premium*; if below £100, at a *discount*. The smallest variation in the price of stock is one-eighth of £1, or 2*s.* 6*d.*, for every £100 stock.

160. Since persons who wish to sell stock may not know any who wish to buy, therefore all sales and purchases are transacted through agents, who are called Stock-brokers. The broker of the buyer deals with the broker of the seller, and each charges his employer, or principal, as he is called, a

commission of $\frac{1}{8}$th of £1 for the transfer of every £100 stock; so that a buyer must always consider that he pays $\frac{1}{8}$th per cent. more, and the seller that he receives $\frac{1}{8}$th per cent. less than the selling price; but if in any Ex. the commission be not mentioned, no notice need be taken of it.

161. In working the various questions that occur in Stocks, a pupil must be careful not to confound stock and actual money. Also, in buying or selling stock, it is quite immaterial whether it be 3 per cent., 4 per cent., or any other kind of stock, unless we wish to know the *income* to be derived: for instance, if I have to sell out £100 stock, when the price is 95, it matters not whether it be in the 3, 4, or 5 per cents.: I have to receive £95 for every £100.

Ex. 1. What must be given for £5050 stock in the Three per Cents., at $85\frac{3}{8}$ per cent.?

Here the price of £100 stock is $85\frac{3}{8}$; and I have to find the price of £5050 stock: hence the statement will be

£100 : £5050 :: £$85\frac{3}{8}$.

Multiplying by $85\frac{3}{8}$, after the method of (84, Ex. IV.), and then dividing by 100, we have the annexed operation, in which we find the cost of £5050 stock at $85\frac{3}{8}$ = £4311 8*s*. 9*d*.

$$\begin{array}{lrr}
 & 5050 & \\
 & 85\frac{3}{8} & \\
\hline
 & 25250 & \\
 & 40400 & \\
\frac{2}{8}\text{, or }\frac{1}{4}\text{, of £5050} = & 1262 & 10 \\
\frac{1}{8}\text{ of £5050} \quad = & 631 & 5 \\
\hline
1,00) & 4311,43 & 15 \\
 & 20 & \\
\hline
 & 8,75 & \\
 & 12 & \\
\hline
 & 9,00 & \\
\end{array}$$

Next, let us find what amount of stock any given sum will buy, when the price of £100 stock is known.

Ex. II. How much stock can be bought for £1490, the price being $88\frac{5}{8}$, and commission $\frac{1}{8}$ per cent.?

Adding the commission to the market price, we have the cost to the buyer $88\frac{4}{8}$, or $88\frac{1}{2}$. The term sought is amount of stock; and in the proposed question £100 is the stock to be bought by £$88\frac{1}{2}$: we have therefore

$$£88\tfrac{1}{2} \ : \ £1490 \ :: \ £100$$

and the fourth term, or amount of stock bought by £1490 is £1683 12*s.* $3\frac{47}{59}$*d.*

Ex. III. If a person invest £2000 in the Three per Cents. when they are at $95\frac{1}{2}$, what is his annual income therefrom?

In this case the buyer gives £$95\frac{1}{2}$ for £100 stock, *i. e.* for the privilege of receiving £3 interest: hence the question is,—If £$95\frac{1}{2}$ produce £3 interest, what will £2000 produce? Our statement is

$$£95\tfrac{1}{2} \ : \ £2000 \ :: \ £3$$

and the fourth term, or interest of £2000, will be found to be £62 16*s.* $6\frac{102}{191}$*d.*

Ex. IV. Find what per centage will be obtained by investing in the Three and a half per Cents. at 91: or, in other words,—If £91 give £$3\frac{1}{2}$, what will £100 produce? The statement is

$$£91 \ : \ £100 \ :: \ £3\tfrac{1}{2}$$

$$\text{and the fourth term} = \frac{3\frac{1}{2} \times 100}{91}\,£ = \frac{\cancel{7}}{\cancel{2}} \times \frac{\overset{50}{\cancel{100}}}{\underset{13}{\cancel{91}}}\,£ = £3\tfrac{11}{13}.$$

Also, transferring the 100 from the left-hand numerator to the right-hand denominator, I have

$$\frac{3\frac{1}{2}}{91} = \frac{3\frac{11}{13}}{100}.$$

Observing this equation, I notice that the right-hand side gives the ratio of the interest of £100 to £100; and the left-hand side gives the ratio of the interest on £100 *stock* to the price of that stock. Now, in finding the per centage which any interest produces, we wish to know what is the ratio of the interest on £100 to £100; and since the right-hand fraction gives this ratio, therefore the former fraction also gives it: hence this fraction gives a standard, by which we can compare the value of the per centage derived from any two investments in different kinds of stock. Thus, if I wish to know whether it will be more advantageous to invest in the Four per Cents. at 95, or in the Three per Cents. at 85, I must compare the fractions $\frac{4}{95}$ and $\frac{3}{85}$.

By (46), the difference is $\frac{4\times85-3\times95}{95\times85}=\frac{340-285}{95\times85}$. Hence, since $340>285$, therefore $\frac{4}{95}>\frac{3}{85}$; or an investment in the Four per Cents at 95 will produce better interest than in Three per Cents. at 85.

If we wish to know how much better interest is obtained in the one case than in the other, we must observe that these fractions $\frac{4}{95}$ and $\frac{3}{85}$ express the respective portions of £100 which the two investments give as interest per cent.:

hence, the difference of the per centage obtained $=\frac{340-285}{95\times85}$ of £100

$$=\frac{\overset{11}{\cancel{55}}\times\overset{20}{\cancel{100}}}{\underset{19}{\cancel{95}}\times\underset{17}{\cancel{85}}}\text{£}=\frac{220}{323}\text{£}=13s.\ 7\tfrac{151}{323}d.$$

162. We will now give an Ex. combining two or more of the operations exhibited in the previous Exs.

Ex. V. A person transfers £1000 stock from the Four per Cents. at 90 to the Three per Cents. at 72; find how much of the latter stock he will hold, and the alteration made in his annual income.

The first part of the question may be thus expressed: "If a certain sum of money will buy £1000 stock at 90, how much can be bought when the stock is at 72?" The statement will be

$$\text{£}72 \ : \ \text{£}90 \ :: \ \text{£}1000$$

and the fourth term $=\frac{\overset{125}{\cancel{1000}}\times\overset{10}{\cancel{90}}}{\underset{\cancel{8}}{\cancel{72}}}\text{£}=1250\text{£}.$

To find the income derived from the £1000 stock and from the £1250 stock, two simple statements might be employed: but where, as in this case, the stock consists of £100 shares, we can work more briefly thus:—£1000 stock = 10 cents.; and since each cent. produces £4, the whole 10 produce £40. Also, £1250 stock = $12\frac{1}{2}$ cents.; and since each cent. of this stock produces £3, the whole produce $12\frac{1}{2}\times3\text{£}=\frac{25}{2}\times3\text{£}=\frac{75}{2}\text{£}$ = £37 10s.; hence the difference of the incomes from the two investments = £40 − £37 10s. = £2 10s.

Exs. 56.

1. What must be given for £2000 Stock when the funds are at 85?

2. When $3\frac{1}{2}$ per cent. Stock is at $93\frac{1}{2}$, what sum will buy £1250, allowing $\frac{1}{8}$ per cent. for brokerage?

3. How much Stock can be bought for £1176 10*s.* when the funds are at £$90\frac{3}{8}$, and broker's charge 2*s.* 6*d.* per cent.?

4. What sum must be invested in 3 per cent. Stock at $94\frac{1}{2}$, to yield an annual income of £500?

5. If a person invest in the 3 per cents. at 93, at what rate per cent. will he receive interest for his money?

6. A person lays out £1000 in $3\frac{1}{2}$ per cent. Stock, when the funds are at $92\frac{3}{8}$; what income does he derive from it?

7. A sum of £999 19*s.* $11\frac{1}{7}$*d.* in the $3\frac{1}{2}$ per cents. produces £44 0*s.* 6*d.*; what was the price of the Stock when the money was invested?

8. In what Stock must I purchase, so that I may derive an income of £75 from the investment of £1875 at par?

9. A capitalist invests for a short period £100,000 in 3 per cents. at $87\frac{1}{2}$: when he sells out, they have risen 2 per cent.; what does he gain, reckoning $\frac{1}{8}$ per cent. for brokerage, both in buying and selling?

10. What must be the price of a railway share, paying a 5 per cent. dividend, and of which the nominal value is £100, so that a purchaser may receive 7 per cent. for the money invested?

11. A railway share, originally costing £100, has paid a dividend of 8 per cent.: what must I give for such a share, so as to receive $4\frac{1}{2}$ per cent. for my money?

12. Railway shares which were purchased at a discount of $10\frac{3}{4}$ per cent., and sold at a premium of £$31\frac{1}{8}$, realised a profit of £357 2*s.* $10\frac{2}{7}$*d.*: how much was invested?

13. A person having an annual income which arises from £450 Stock invested in the 3 per cents., exchanges it for an annual income arising from £315 Stock in the 4 per cents.: what is his annual gain or loss by the exchange?

14. What is the interest for $5\frac{3}{4}$ yrs. of £293 invested in the 3 per cents., when they were at $87\frac{1}{2}$?

15. A person sells out of the $3\frac{1}{2}$ per cents. at $93\frac{1}{2}$, and realizes £18700: if he invest one-fifth of the produce in the 4 per cents. at 96, and the remainder in the 3 per cents at 90, find the alteration in his income.

16. Which is the better investment, to buy in the 3 per cents. at 85, or the 4 per cents. at 102; and by how much?

17. Find the difference of income arising from two investments of £5000: (1) in shares at $131\frac{1}{4}$, paying a 6 per cent. dividend; (2) in Bank Stock at $194\frac{1}{4}$, paying an 8 per cent. dividend.

18. A person wishes to bequeath an annuity of £100 a year; what sum must he devote to the purchase of $3\frac{1}{2}$ per cent. Stock at 97, so that the annuitant may receive the £100 free of 3 per cent. income tax?

EQUATION OF PAYMENTS.

163. If a person owe another several sums to be paid at different times, and it is required to know at what time it would be just to pay the whole at one payment, this would be a question to be solved by a Rule called Equation of Payments.

To ascertain the method of finding this time of payment, called the equated time, let us take a simple Example.

Ex. I. If £100 be due at the end of six months, and £200 at the end of twelve months; find when it is just to pay the whole in one sum.

It is quite clear that the time of payment will be at some period between the two fixed times, six months and twelve months; hence the former sum, £100, will be paid after it is due, and the latter sum, £200, before it is due.

Now, a person keeping the £100 beyond the appointed time ought, of course, if that were the only money to be paid, to pay interest for it; but, instead of paying interest, he is to make up for the privilege for keeping the £100 by paying the £200 before it is due; it is hence quite clear that he must pay this £200 such a time before it is due, that the interest of the £200 for that time shall just balance the interest he might obtain by keeping the £100 after it was due. The question then really is—How soon will the interest upon £200 produce the same as the interest upon £100? The answer evidently is, in half the time; *i.e.* the time of paying the £200 must be *earlier* than its original time of twelve months, by half as much as the time of paying the £100 is *later* than its original time of six months; therefore, if the payer keep the £100 *four* months beyond

the six months, and pay the £200 *two* months sooner than the twelve months, the interest gained in the one case and lost in the other will be just balanced; and the whole sum will have to be paid in 10 months.

164. Putting the question in another form, we may consider that when the sums were to be paid at different times, the payer had the £100 in his hands six months, *i. e.* he had the interest of £100 for six months, or of £600 for one month: also, he had the interest of £200 for twelve months, or of £2400 for one month; therefore he had in all the interest of £2400 + £600, or of £3000, for one month. If, then, the debtor has to pay the £300 in one sum, how long ought he to keep it, so that its interest shall equal the interest of £3000 for one month? The answer to this question will be obtained from the following statement:

£300 : £3000 :: 1 month;

and the fourth term will be ten months, the same result as before.

165. The truth of the above method has been disputed by arithmeticians upon this ground. Referring to the above Ex., it is said, that since the £100 is paid after it is due, the payer should pay the interest for the time that it is kept back; but since the £200 is paid before it is due, *discount* only should be allowed thereon: and since the discount is less than the interest, it is said that the payer, by the above method, receives as much more than his due as the interest of £200 exceeds the discount; hence, to make the payment perfectly correct according to this view, we ought to place the payment earlier than ten months. But since we have shown, by the working of the Ex. given above, that the interest gained in one case and lost in the other is equal, we shall, by placing the time for payment of the £300 earlier, rob the payer in two ways—1st, by depriving him of the

interest of the £100 during the latter part of the four months that he ought to be allowed to hold it: and 2ndly, by making him put the £200 into the hands of his creditor for a longer time than two months.

The fallacy of the reasoning which would place discount instead of interest in the question may be shown thus. It is here assumed that discount, not interest, is applicable to all cases where money is to be paid before it is due. Now, this is quite true where only one payment is to be made; for in that case we have laid it down, that the debtor is to pay such a sum as put out to interest shall just amount to the sum due: therefore, in all cases of discount the debtor pays less than he owes. But this is not the case in Equation of Payments: for here the creditor receives the whole of the latter portion of the debt, say £200, *though it be before it is due*, and he can put the *whole* out to interest, which he cannot, in real questions of discount; and the question now is,—not how much must be put out to interest to raise £200,—but how long must the whole £200 be put out, to raise an amount of interest equal to that which the creditor has lost, by allowing the first payment, say £100, to remain in the debtor's hands beyond its time. The question considered in this view has been satisfactorily answered in the Ex. worked above.

It should however be added that, strictly speaking, simple interest is almost a fiction; for the moment any sum of money is payable as interest, it matters not whether it be called principal or interest, it is the property of the lender; and if this be retained by the borrower, it ought, in justice to the lender, to be added to the principal, and be charged with interest afterwards*.

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* See Art. *Rebate* in the "Penny Cyclopædia."

Ex. II. A person owes £800; £200 to be paid in three months, £100 in four months, £300 in 5 months, and £200 in six months: if the whole were to be paid at once, what would the time of payment be?

Here, by the agreement, the debtor has the interest of

	£					£		
	200	for	3	months,	or	600	for 1	month
also of	100	,,	4	,,	or	400	,, 1	,,
also of	300	,,	5	,,	or	1500	,, 1	,,
lastly, of	200	,,	6	,,	or	1200	,, 1	,,
	800					3700		

therefore he has altogether the interest of £3700 for one month; and the whole sum to be paid is £800: hence, we have to find in how many months £800 will produce as much interest as £3700 in one month. The result is obtained from the following statement:

$$£800 \ : \ £3700 \ :: \ 1 \text{ month};$$

and the fourth term $=\dfrac{3700}{800}\times 1$ month $=4\frac{5}{8}$ months.

In the statements which are used in Exs. I. and II. we observe that the third term is one month; hence, since this will in every Ex. be the third term, we shall merely have to divide the second term by the first, in order to obtain the fourth term: also, the first term is the whole amount due; and the second is the amount of all the several sums, when each one is multiplied by its time of payment. Hence the equated time is found by the following Rule:

Multiply each sum into its specified time, and divide the total of the products so found by the whole amount to be paid.

Exs. 57.

1. Find the equated time of payment of £160, whereof £100 is due in 3 months, and £60 in 8 months.

2. I owe £100 in 2 yrs., £230 in $2\frac{1}{2}$ yrs., and £280 in 3 yrs. hence: when must I pay the whole in one sum, so as neither to gain or lose?

3. Of a debt, $\frac{1}{3}$ is due in 4 months, $\frac{1}{4}$ in 6 months, $\frac{1}{5}$ in 8 months, and the remainder in 12 months; find the equated time of payment.

4. Find the equated time of paying £200, which is due in monthly instalments of £20.

EXCHANGE.

166. EXCHANGE, in its simplest meaning, is merely the conversion of any sum of money from the coinage of one country to that of another. To perform this conversion, we must of course know for how much a certain coin of one country can be exchanged in coins of another: for instance, if I wish to exchange a sum of English money for French, I must know how many of the current coins of France, viz. *francs*, will be given me for £1 of English money. This rate of Exchange between two countries is called the *Course of Exchange*, but it is not always the same; and we shall presently show the cause of the variation.

Ex. I. Exchange £850 sterling for francs at 25 francs 15 centimes, or cents; or, in other words,—If £1 sterling can be exchanged for 25 francs 15 cents, what number of francs can be obtained for £850? To obtain this number I have the following statement:

£1 : £850 :: 25 francs 15 cents.

Also, since 100 cents make one franc, therefore cents may be expressed as decimal parts of a franc; *i. e.* 25 francs 15 cents = 25·15 francs;

$$\text{hence, the fourth term} = \frac{£850}{£1} \times 25{\cdot}15 \text{ francs}$$

$$= 21377{\cdot}5 \text{ francs}$$

or, 21377 francs 50 centimes.

Ex. II. How many pounds sterling can be obtained for 8457 marks $15\frac{1}{2}$ schillings, Hamburgh, at the rate of 13 marks 12 schillings for £1 sterling? The statement is

13 marks 12 sch. : 8457 mks. $15\frac{1}{2}$ sch. :: £1.

By Tables of Hamburgh Coinage, we find that 16 schillings = 1 mark: hence, reducing the first and second terms of the above statement to half schillings, we obtain by the usual process the fourth term = £615 2*s.* 6*d.*

167. The method of these two Exs. will enable us to convert any sums from the currency of one country to that of another, when the course of exchange between the two countries is known. But we must also be able to perform this conversion between two foreign countries.

Ex. III. Change 1932 florins at Amsterdam for ducats at Naples, the course of Exchange being $80\frac{1}{2}$ florins for 40 ducats. The statement is

$$80\tfrac{1}{2} \text{ florins} \;:\; 1932 \text{ florins} \;::\; 40 \text{ ducats};$$

and the fourth term will be found to be 960 ducats.

In the following Ex. the process is not so simple:

Ex. IV. Exchange 1000 American dollars for sterling money when Eng. money bears a premium of 10 p. c. in America.

Here, £110 worth of dollars in America would produce only £100 of English money—hence 1000 dollars must be reduced in the ratio of 110 to 100. Thus,

$$110 \;:\; 100 \;::\; \overset{\text{dols.}}{1000} \;:\; \frac{10000}{11} \text{ dols.}$$

$$\text{or } 909\tfrac{1}{11} \text{ dols.}$$

We have now to convert $909\frac{1}{11}$ dols., each 4*s*. 6*d*., into pounds sterling.

The required number of pounds sterling

$$= \frac{10000}{11} \times \frac{4\frac{1}{2}\text{sh.}}{20\text{sh.}} = \frac{9000}{44} = 204\tfrac{6}{11}$$

$$\text{or } £204\ 10s.\ 10\tfrac{10}{11}d.$$

If I had wished to change Eng. money for American, I should have *increased* the number of English pounds in the ratio of 100 to 110.

We now proceed to explain a further and more comprehensive meaning of the term Exchange.

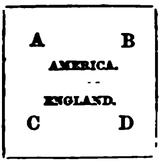
In commercial transactions between different countries it is not usual to pay for goods imported, in coin, or as it is sometimes called, in *specie* or *bullion:* and for two reasons—first, the quantity of foreign goods imported by a country like England is so great, that if paid for in coin, the payment would speedily drain all the coin out of the country, and business could not be carried on. Secondly, there would be

the probable loss of the coin by wreck or otherwise in the transmission; besides that there would arise a loss of interest on the money while it was being sent to its destination.

We shall now show how these difficulties may be avoided, when we are dealing with countries which send goods as well as receive them, *i. e.* which export as well as import.

168. The following is a simple Ex. of the manner of conducting these transactions.

Suppose *A* and *B* to represent two merchants in America, and *C* and *D* two others in England: let *C* buy of *A* a thousand pounds worth of goods, and therefore owe him £1000; so, also, let *B* owe *D* £1000 by a similar purchase; then if these sums be paid in coin, £1000 must cross the Atlantic twice. But, since *C* has to pay £1000 to *A*, he would as readily pay it to *D* in England, if by such payment he could get rid of his liability to *A*: so also *B* would pay *A*, if he could be rid of his debt to *D*. This simple transaction might, therefore, be completed thus:



Let *C* send to *A* a bill acknowledging the debt of £1000, and promising to pay £1000 to any one in England who may present the bill to him at the expiration of a certain time: *A* then sells to *B* this bill, and receiving £1000 for it, has no longer any claim upon *C*. *B* now sends this bill to *D*, and *D* uses it as a bill for £1000, until the expiration of the time named on the bill, when the money is paid by *C*. Thus these four merchants have been able to have commercial dealings to the amount of £1000 each, without any coin having left either country.

Of course the value of this bill for £1000 depends entirely upon the ability of *C* to *meet* it, that is, to pay the money at the expiration of the time agreed upon in the bill: and we

often find that *C*, who was considered able to pay at the time he gave the bill, has become a bankrupt before the time of payment; hence the loss falls on *D*. And this explains the reason why, in a commercial country like England, the failure of one merchant, or firm, causes others to fail: for, in the case above, *D* may also be liable for bills as well as *C*; and if not able to obtain the money which he expected from *C*, may himself become a bankrupt; and so in turn cause other merchants the same loss which he is himself suffering from the failure of *C*.

Now, there are thousands of merchants in the situation of *A* and *B* in America, and similarly of *C* and *D* in England. Hence there are, as a general rule, merchants wanting to buy bills, and others wanting to sell them, in both countries: and what has been said concerning England and America is true with respect to any two countries which export to and import from one another.

These purchases and sales of bills are, for the reason mentioned in (160), conducted through the medium of Bill-brokers.

169. We have just now been supposing that the two countries have imported and exported goods to an equal amount from one another. But suppose that goods had been sent from America to England to a greater amount than from England to America, for instance, to the extent of £1,000,000, then more bills to that amount would go from England to America than from America to England. The merchants in America have in this case plenty of bills, and of course want money for them: but as there are more persons wishing to sell bills than to buy, therefore the bills fetch a lower price: on the contrary, as bills in England are not so plentiful, there are more buyers than sellers, and the bills fetch a higher

price than usual. When this increased price exceeds the cost of insurance and loss of interest upon coin sent over to America, the English merchant prefers sending coin instead of paying the increased rate for a bill: and by successive exportations of bullion, the balance of £1,000,000, which was against us, will be paid off: but this increased price of a bill, (which swallows up the profits of business,) or the alternative of paying in coin, causes merchants to be slow in importing until our exports have increased and helped to restore the balance. Here, also, whatever has been said concerning England and America, is of course equally applicable to any two countries which have commercial transactions with each other. In those countries, as South America, where gold and silver are amongst their principal productions, bullion is as much a regular article of export, as woollen or cotton goods would be from England.

170. If the price of a bill in England, entitling the holder to receive gold in a foreign country, be less than the usual course of exchange, the exchange is said to be in favour of England, because there is then no need to export gold.

The following is an example of the quotation of the rate of exchange in the public prints. It is taken from the *Times* of July 18, 1860.

The course of exchange at New York on London for bills at 60 days' sight is $109\frac{3}{4}$ to $109\frac{7}{8}$ p. c.; which, when compared with the Mint par between the two countries [$109\frac{23}{40}$] shows that the exchange is slightly in favour of England; and after making allowance for charges of transport and difference of interest, the present rate leaves a small profit on the importation of gold from the United States.

DEF. The standard rate of exchange between any two countries is termed the *Par of Exchange*, or the *Arbitration Price:* but, as alluded to in (166), is not always the same as the *Course* of Exchange. Also, a *Bill on London* means a

paper entitling the holder to obtain gold in London, to the value of the amount mentioned in the bill.

Arbitration is called *Simple* or *Compound*, according as there are three or more places concerned.

Ex. V. Bills on Amsterdam, bought in London at 12 florins 15 cents per £ sterling, are sold in Paris at $57\frac{1}{2}$ florins for 120 francs: what is the rate of Exchange between London and Paris?

My object here is to express £1 in terms of francs; hence I express £1 in terms of florins, and florins in terms of francs, and thereby obtain the value of £1 in francs. Working fractionally, I have

$$\text{£}1 = 12 \text{ florins } 15 \text{ cents} = 12{\cdot}15 \text{ florins},$$

$$\text{also, } 57\tfrac{1}{2} \text{ flor.} = 120 \text{ francs},$$

$$\text{therefore } 1 \text{ flor.} = \frac{120}{57\frac{1}{2}} \text{ francs},$$

$$\text{and therefore } \text{£}1 = 12{\cdot}15 \text{ florins} = 12{\cdot}15 \times \frac{120}{57\frac{1}{2}} \text{ francs}$$

$$= 25 \text{ francs } 35\tfrac{15}{23} \text{ cents.}$$

The following Ex. involves three countries, and therefore three equations.

Ex. VI. A bill upon Hamburgh is bought at 13 marks $10\frac{1}{4}$ schillings per £ sterling, then sold at Amsterdam at $35\frac{3}{4}$ florins per 40 marks: if the proceeds are then remitted to Paris in French bills at $57\frac{1}{4}$ florins per 120 francs, what rate of exchange is there between London and Paris?

$$\text{£}1 = 13 \text{ m. } 10\tfrac{1}{4} \text{ sch.} = 13\frac{10\frac{1}{4}}{16} \text{ marks} = 13\tfrac{41}{64} \text{ m.},$$

$$40 \text{ mks.} = 35\tfrac{3}{4} \text{ flor.}, \qquad \text{or } 1 \text{ mk.} = \frac{35\frac{3}{4}}{40} \text{ flor.},$$

$$57\tfrac{1}{4} \text{ flor.} = 120 \text{ francs}, \qquad \text{or } 1 \text{ flor.} = \frac{120}{57\frac{1}{4}} \text{ francs};$$

$$\therefore \text{£}1 = 13\tfrac{41}{64} \times \frac{35\frac{3}{4}}{40} \times \frac{120}{57\frac{1}{4}} \text{ francs}$$

$$= 25 \text{ francs } 55\tfrac{1405}{3664} \text{ cents.*}$$

* The process by which a connection is established between the first and last terms, as for instance, between pounds sterling and francs, is sometimes termed the *Chain Rule*, because these extremities are joined, as it were, by successive links. And the results which we have just investigated, might have been written down at once, by taking as the left-hand side of the equation, the coin whose equivalent is desired,—and for the right-hand side, a fr^{n}, of which the num^{r} consists of the product of all the successive denominations contained in the question,—and the den^{r}, of all the remaining coins of the same kind as the num^{r}, with the exception of the last or required coin.

The subjoined is another example of the mode of reporting the exchanges, and is taken from the *Times* of March 22nd, 1848, under the head "Money Market and City Intelligence." It contains an exaggerated view of the general relation between the prices of gold at London and Paris,—because it refers to a time of revolution and panic; but the principle is the same.

"The last quotation of gold at Paris was about 30 per mille premium, which would give an exchange of 25·91. The last quotation of short bills on London being 26·50, the price of gold would appear to be about $2\frac{1}{4}$ per cent. higher in London than in Paris."

Here a comparison is instituted between the *nominal* rate of exchange of £1 for francs, and the *real* rate at the particular time mentioned. Thus, if 1000 represent the price of gold in Paris when £1 sterling is worth about 25·16 francs*, what will be the value of this £1 in francs when 1030 represents the price of gold in Paris? We have, of course, this statement:

$$1000 \ : \ 1030 \ :: \ 25{\cdot}16 \text{ francs},$$

$$\text{the fourth term} = 25{\cdot}91 \text{ francs},$$

that is, £1 will in Paris produce 25·91 francs. But a person in Paris wishing to buy a bill on London, entitling him to receive £1 in gold, must give 26·50 francs. The difference, about ·6 francs, is the amount by which £1 sterling in London is dearer than in Paris.

$$\text{Also, since } 26{\cdot}50 \text{ francs} = £1,$$

$$\text{therefore 1 franc} = \frac{1}{26{\cdot}5}£;$$

$$\text{and this extra price of } £1 = {\cdot}6 \text{ fr.} = {\cdot}6 \times \frac{1}{26{\cdot}5}£;$$

$$\text{and therefore the extra price of } £100 = 100 \times {\cdot}6 \times \frac{1}{26{\cdot}5}£$$

$$= \frac{60}{26{\cdot}5}£ = \frac{12}{5{\cdot}3}£ = 2{\cdot}26\ldots£$$

$$\text{or } 2\tfrac{1}{4}£ \text{ nearly},$$

that is, the price of gold in London is $2\frac{1}{4}$ per cent. greater than in Paris; or a bill which would entitle a person to receive £100 in gold in London would cost £$102\frac{1}{4}$ in Paris.

* See Appendix, Art. *Exchange*.

Exs. 58.

FRANCE AND ENGLAND.

The course of Exchange between France and England is generally about 25 fr. 30 cents for £1 sterling. 100 cents = 1 franc.

1.	Exchange £350 for francs, at 25 fr. 50 cts. per £ sterling.			
2.	,,	£75 10*s*. ,,	25·35½	,,
3.	,,	£425 15*s*. 6*d*.	26·5	,,
4.	,,	9349·90 francs for pounds, at 25·90 per £ sterling.		
5.	,,	21475·12 ,,	25·75	,,
6.	,,	1875·5 ,,	26·5	,,

HAMBURGH AND ENGLAND.

13 marks 8 schillings = £1 English. 16 schillings = 1 mark.

			m. *sch.*	
7.	Exchange £425 for marks, at		13 12	for £1.
8.	,,	£375 10*s*. ,,	13 9¼	,,
9.	,,	£87½ ,,	13 8	,,
			m. *sch.*	
10.	,,	1000 marks for pounds sterling, at	13 8	for £1.
11.	,,	8754 mks. 15 sch. ,,	13 10½	,,
12.	,,	3537·45 mks. ,,	13 11	,,

THE UNITED STATES AND BRITISH NORTH AMERICA.

1 Dollar = 4*s*. 6*d*.; also sterling money generally bears a premium of 9 per cent.; *i. e.* £100 sterling can be exchanged for as many dollars as will amount to £109; the exact par of exchange is $109\frac{28}{45}$.

13.	Exchange £100 for dollars when Eng. money has prem. of			8 p. c.
14.	,,	£425 10*s*.	,, ,,	10 ,,
15.	,,	£1256 5*s*.	,, ,,	9½ ,,
16.	,,	1000 dollars for Eng. money, when the prem. is		8 ,,
17.	,,	3225	,, ,,	7½ ,,

EAST INDIES. 1 Rupee about 23⅝*d*.

18. ,, £500 for rupees, at 23½*d*. per rupee.

19. ,, 7500 rupees for sterling money, at 24¼*d*.

20. Exchange 2420 rupees for francs, the course of exchange being 40 rupees for 94 francs.

21. Change 1750 marks of Hamburgh for florins at Amsterdam, at the rate of 135 marks for 120 florins.

22. Exchange 3700 francs for Hamburgh marks, at the rate of $187\frac{1}{2}$ francs for 100 marks.

ARBITRATIONS OF EXCHANGE.

23. If the Exchange between London and Amsterdam be $11\frac{3}{4}$ florins per £ sterling, and between Amsterdam and Paris be at the rate of $55\frac{1}{2}$ florins per 115 francs; find the rate of exchange between London and Paris.

24. Bills on Paris, bought at the rate of 25 francs 35 cents per £ sterling, are sold in Lisbon at 190 rees per franc; what rate of exchange is there between London and Lisbon?

25. The exchange between London and Hamburgh is 13 mks. 10 sch. per £ sterling; between Hamburgh and Paris is 150 marks for 275 francs; what exchange does that give between London and Paris?

26. If the exchange between Amsterdam and Hamburgh be at $11\frac{3}{4}$ flors. for $13\frac{1}{2}$ marks, between Amsterdam and Genoa be at $10\frac{3}{4}$ flors. for 25 lire, between Genoa and Portugal be 5 lire for 800 rees, between Lisbon and London be 1 milree or 1000 rees for *54d.*, what exchange does this give between London and Hamburgh? and what difference will there be, between remitting £500 from London to Hamburgh by this circular route, and sending it direct, at an exchange of 13 mrks. 11 schgs. per £ sterling?

27. The premium on gold at Paris is $5\frac{1}{2}$ per mille, which gives an exchange of 25·29; if the quoted exchange of Paris on London be $25{\cdot}22\frac{1}{2}$, shew that gold is 0·26 per cent. dearer in Paris than in London.

Exs. 59. M.

1. A debt of £144 *7s. 6d.* was paid in an equal number of guineas, half-guineas, and seven shilling pieces: required the number.

2. A peck of flour gives 20lbs. of bread; how much land would grow corn enough for $1\frac{1}{2}$ millions of people for a week, at $4\frac{1}{2}$ quarters to the acre, and $1\frac{1}{2}$lbs. of bread for each person per day?

3. The population of a town rises 1 per cent. for 3 years successively; if at the beginning of the 3 yrs. the population were 1 million, what would it be at the end?

4. Tea bought at *1s.* $10\frac{1}{2}$*d.* per lb. pays a duty of *2s.* $2\frac{1}{4}$*d.* per lb., what per centage of the whole cost is the taxation?

5. The $3\frac{1}{2}$ per cents. are reduced to $3\frac{1}{4}$ per cents.; 150 millions of stock are so converted: but the holders of 6 millions dissent; if they are paid off while the stock is at $97\frac{1}{2}$, how much will the nation gain or lose in the first year?

6. An estate of 270 acres is bequeathed to three tenants, to be divided in proportion to their rents, which are £180, £120, and £60; how must the land be divided?

7. Find a fourth proportional to $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{10}$; and to ·05, ·015, and ·0075.

8. Find the difference between the Simple and Compound Interest of £520 for 2 yrs. at 5 p. c., where the interest is paid half-yearly.

9. *A*, *B*, and *C* enter into partnership; *A* puts in £100 for 6 months, *B* puts in £200 for 4 months, and *C* £100 for 15 months; divide a profit of £150 equitably.

10. At what rate must I sell an article which cost 50*s*., so as to gain 15 per cent.?

11. Find the purchase money of £1500 stock in the 3 per cents., at $88\frac{1}{2}$, including $\frac{1}{8}$ per cent. commission.

12. At what rate per cent. will £100 double itself in 8 years, S. Int.?

13. Find the annual income from a legacy of £5000 Stock in the $3\frac{1}{2}$ per cents., after paying the legacy duty of 10 per cent.

14. Prove that the sum of the fractions $2\frac{1}{13}$ and $\dfrac{2}{1\frac{4}{9}}$ is equal to 5 times their difference.

N.

1. Show by a simple Ex., that Mult[n] is nothing more than a shortened mode of Addition.

2. What would be the length of an acre of ground, if its breadth were 40 yards?

3. Find the value of $4·05 \times ·000012$; also of $4·05 \div ·00012$; and prove both results by vulgar fractions.

4. What does £25 15*s*. amount to, when taken ·125 times?

5. If the means of a proportion be 9 and 16, and one of the extremes be 56, what is the other extreme?

6. A person bequeathed £5000 to be divided amongst three persons, in the proportions of 3, 5, and 7; find their respective shares.

7. Explain what fractions produce terminating, and what produce non-terminating decimals. Give Exs. of each.

8. Find the exact difference between ·127*s*. and $·12\dot{7}$*s*.; and express the result as the fraction of a crown.

9. What sum of money will in 5 years amount to £411 5*s*. at $3\frac{1}{2}$ p. c. Simple Interest?

10. Find the amount of insurance upon £12500, at 4*s*. 6*d*. per cent.

11. What is the present value of £463 10*s*. due in 5 months, allowing $4\frac{1}{2}$ per cent.?

12. How many yards at 6*s*. $7\frac{1}{2}d$. per yd. must be given in exchange for 105 yds. at 3*s*. 4*d*. and 375 at 4*s*. $10\frac{1}{2}d$.?

13. Explain the meaning of the terms *directly* and *inversely* proportional; and give Exs. of questions illustrating each expression.

14. What decimal multiplied by $\frac{3}{4}$ of $\frac{1}{6}$ of $3\cdot\dot{3}$ will become 17?

15. How many years' purchase should be paid for property, so as to receive $6\frac{1}{4}$ per cent.?

O.

1. Find the Simple Interest of £500 for 5 years at $3\frac{1}{2}$ per cent.

2. What is the number of cubic yds. in 1438790 cubic inches?

3. How often must the sum of 2*s*. $6\frac{1}{4}d$., 3*s*. $9\frac{1}{2}d$. and 18*s*. $8\frac{1}{4}d$. be repeated, to make £100?

4. If 10 francs be worth 6 florins, and 75 florins be equivalent to 4 moidores; how many francs must be given for 16 moidores?

5. Find the frac^n which being multiplied by $\frac{3}{4}$ of $\frac{1}{7}$ of $2\frac{3}{11}$ gives a product $=1$.

6. The interest on a railway share is $3\frac{3}{4}$ per cent.; what is the market value of each share and of the entire line, if money be worth 5 per cent., and the amount paid in dividends be £150,000?

7. A sum of money has doubled itself in 17 years at Simple Interest; what is the rate per cent.?

8. I sell an article for 15*s*. 9*d*., and by so doing gain 17 per cent.; what was its prime cost?

9. An estate which brings $3\frac{1}{2}$ per cent. lets for £546; what was the purchase money?

10. Divide £70 amongst three persons, whose shares shall be in the ratio of the numbers $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.

11. Shew whether it is better to invest in the 3 per cents. at 89, or the $3\frac{1}{2}$ per cents. at 94. What difference would there be, if £10000 stock were held?

12. Compare as vulgar fractions $\cdot025 \times \cdot07$, $11\cdot035 \times \cdot0008$, and $\cdot1\dot{9} \times \cdot00\dot{3}$.

13. A person mixes 25 bushels of wheat at 4*s*. 9*d*., 36 at 5*s*. 6*d*., and 15 at 6*s*. 6*d*.; what must be the selling price per bushel of the mixture, to gain 10 per cent. on the above prices?

14. Exchange £1050 for francs, at 25 fr. 50 cts. per £ sterling.

15. Exchange 9062 fr. $62\frac{1}{2}$ cts. for pounds sterling, at 25 fr. 35 cts. for £1.

P.

1. Find a fourth proportional to 1, ½, ¼; also a third proportional to 12 and 25.

2. Express $\frac{14\frac{3}{4}}{\cdot 025}$ as a simple ratio.

3. I lend £175 for 6 months when money is worth 7 per cent.; for what time ought I to be able to borrow £250, when money is worth but 4½ per cent.?

4. Explain the difference between Interest and Discount; and find the true discount of a bill of £65 13*s.* at 4 months, drawn Oct. 4th, and discounted Nov. 26th, at 5 per cent.

5. At what rate of interest would £350 amount to £389 7*s.* 6*d.* in 3 yrs., at Simple Interest?

6. Goods bought at £2 4*s.* 6*d.* are sold at £2 18*s.* 9*d.*; required the profit per cent.

7. A railway share, originally costing £50, has paid a half-yearly dividend of £1 10*s.*; what will be my rate of interest, if the share cost me £55½?

8. What will be the first year's expense of an insurance on £1500 at a premium of £2 13*s.* 10*d.* per cent., and a stamp duty of £3? Find the per centage, including the stamp.

9. If the price of 500 bricks, of which the length, breadth, and thickness are 12, 4½, and 3 inches respectively, be 12*s.* 6*d.*; how many shall I obtain for the same money, if the dimensions be 15, 6, and 4 inches?

10. Find the prices of investment in the 3, 3½, and 4 p. c. Stocks, when they each pay 3¼ per cent.

11. Distribute the sum of 1000 guineas in the ratio of 1, ½, ⅓.

12. The ratio of the invoice price to the net price is 11 : 8; what per centage has been thrown off as discount from the invoice price?

13. Exchange £415 10*s.* for dollars at 4*s.* 6*d.*, when English money bears a premium of 7½ p. c.

14. Exchange 3500 dollars for sterling money, when the premium on English money is 10½ per cent.

15. The exchange between London and Paris is 25·5 francs per £ sterling; between Paris and Amsterdam is 117 francs for 55 florins; between Amsterdam and Hamburgh is 11 florins for 13 marks; what is the exchange between London and Hamburgh?

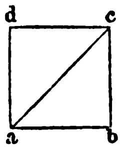
8—2

AREA AND VOLUME.

DUODECIMALS.

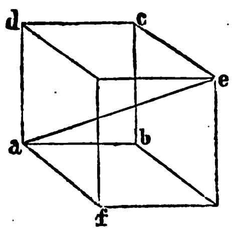
171. If any line be taken, as *ab*, and upon it a square, *abcd*, be described, this figure may be called the square of *ab*. And if we take *ab* as a unit of length, viz. 1 inch, 1 foot, &c., then *abcd* is called 1 square inch, 1 square foot, &c. We have here, therefore, inches, feet, &c. of length, or *linear* inches; and inches, &c. of surface, or *superficial* inches, or square inches.

Fig. 1.



172. In like manner, if upon *ab*, the annexed figure *abcdef* be described, having six sides, or surfaces, each equal to *abcd*, it is called a *cube:* and, as before, if *ab* be taken as representing 1 inch, 1 foot, &c., this figure will represent 1 cubic inch, 1 cubic foot, &c.

Fig. 2.



We have now, therefore, three kinds of units of measurement, viz. linear, or common inches; square, or superficial inches; and cubic, or solid inches. Also, these three units are said to contain 1, 2, and 3 dimensions respectively. For example, the floor of a room, having the dimensions of length and breadth, is of the same nature as a square; and any such surface is called an *Area:* but a cistern of water, having the three dimensions of length, breadth, and depth, is of the

same nature as a cube. The quantity contained by such a cistern or similar figure is termed its *Volume*, or solid content.

DEF. A surface which is so enclosed with lines, that any two which meet in a point are perpendicular to one another, is called *rectangular*.

OBS. Any surface bounded by straight lines may be denoted by two letters placed at its opposite corners; and any solid contained by such surfaces may be so denoted: but the two letters employed should not be joined together by a line. Thus, in Fig. 1, I should say the area *bd*, not *ac*; and in Fig. 2, I should say, the volume *fc*, and not *ae*.

DEF. The lines *ac*, *ae*, are called *diagonals*.

173. I have now to explain how to find the area of surfaces and the volume of solids: but as I do not propose to enter upon mensuration generally, I shall merely treat of rectangular surfaces, as squares and oblongs; and of solids, the surfaces of which are also rectangular.

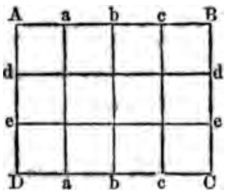
174. Quantities which can be accurately represented by numbers, whether whole or fractional, are called *commensurable;* and those which cannot be so represented, are called *incommensurable.* Thus it will be shown under the head of "Ratio," in the Appendix, that if in Fig. 1 the length of *ab* be represented by 1, *ac* cannot be accurately represented by any number whatever, whether whole or mixed. The quantity used for the value of *ac* is 1·4142...... and this being nearly the true value, is termed its *approximate* value.

175. We may here state as a fact, that the area of any rectangular surface is found by multiplying the numbers which represent its length and breadth; also that the volume of a solid, bounded by rectangular surfaces, is found by multiplying together the numbers representing the length, breadth, and depth or thickness. The above statements are

true, whether the lines bounding the area or volume be commensurable or not; but they cannot be proved to be universally true without the aid of geometry. We shall, however, give an Ex. illustrating the correctness in each case, choosing of course only commensurable numbers.

176. Let $ABCD$ be a rectangular figure whose sides, AB and AD, meeting in the point A, and called *adjacent* sides, contain an exact number of units,—viz. $AB = 4$ inches, $AD = 3$ inches; let the opposite sides, AB, DC, be divided into four equal parts in a, b, c, and the sides AD, BC, be divided into three equal parts, in d, e; let the lines aa, bb, cc, dd, ee, be drawn: the figure will be divided into equal squares, each of which has for its side 1 linear inch, as Aa, and is therefore a square inch. Also, counting vertically, there are three rows of squares, because $AD = 3$ inches; and each row contains four squares, because $AB = 4$ inches; so that there are in all 12 squares, *i. e.* 3×4 squares: hence we see that the number of square inches in the area $ABCD =$ the product of the number of linear inches in the two adjacent sides, AB, AD.

FIG. 3.



DEF. The lines aa, bb, which are drawn so that they are at an equal distance from one another, are called *parallel* lines; so also they are said to be *parallel* to AD or BC.

Hence, if I wished to draw lines, as aa, bb, or cc, in the above figure, I should say—through a, b, c, draw lines parallel to AD, or BC. So also, dd, ee, are drawn parallel to AB, or DC.

If AD had been $= AB$, then the figure would have been *a square*, and the number of square inches in it would be

4×4, or 4^2 (called 4 *squared*), $= 16$; *i. e.* the number of square inches in any square figure whose side is expressed in linear inches, is found by multiplying the number contained in the side by itself. Hence the second power of a number is called its square, because it represents the area of a square figure, the side of which is the number itself.

We can now show how the numbers mentioned in what is called "Square Measure," are obtained.

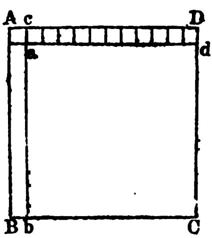
For if 12 linear inches = 1 linear foot;
therefore 12 in. × 12 in. = 144 square inches = 1 square foot.
So also, since 3 linear feet = 1 linear yard,
3 ft. × 3 ft. = 9 square feet = 1 square yard.

Again, $5\frac{1}{2}$ linear yards = 1 linear pole, or perch;
therefore $5\frac{1}{2}$ yds. × $5\frac{1}{2}$ yds. = $30\frac{1}{4}$ sq. yds. = 1 sq. perch;

and these square perches, yards, &c., are the quantities always made use of in measuring land; for no amount of perches in *length* could make up an acre, which consists of *surface*.

177. In measuring surfaces, such as square feet of timber, many arithmeticians have called the twelfth part of a square foot, as *Ad* or *Ab*, an *inch*, and a twelfth part of *Ad*, viz. *Aa*, a *part*: but since a pupil is taught in "Square Measure" that *Aa* is an inch, and that 144 such inches make up a square foot; it is clearly absurd to call *Ad* an *inch*, seeing that 12 such make up a square foot: we shall, therefore, confine the name *inch* to *Aa*, or any quantity equal to it. But the divisions of *AC* have been, and may be, called superficial *primes*, *seconds*, *thirds*, &c.; where it is to be remembered, that a prime is

FIG. 4.



$\frac{1}{12}$th of a square foot, and each succeeding denomination is $\frac{1}{12}$th of the one preceding it. So also, linear inches are sometimes called primes, and twelfths of an inch, seconds.

The square inch, *Aa*, might also be divided, exactly as we have divided *AC*; and its twelfth part would be called a *third*, and be similar in shape, though not in size, to *Ab* or *Ad*; and the twelfth part of this *third* would be called a *fourth*, and be similar in shape to *Aa*.

Also, observing Fig. 4, we learn that

1 foot × 1 foot = 1 sq. foot, as *AC*.

1 foot × 1 inch = 1 sup. prime, as *Ab*.

1 inch × 1 inch = 1 sq. inch, or sup. second, as *Aa*.

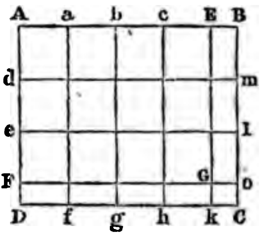
178. It must be distinctly noticed that we do not multiply together concrete quantities, as feet by feet, inches by inches, &c.; we merely multiply the *number* of feet or inches in the length by the *number* in the breadth; and then we observe that the number obtained in the product is equal to the number of superficial units, primes, &c., in the area contained by the above length and breadth. The same remark applies to the multiplication of three dimensions, in finding the volume of a solid.

Attention to this fact, that concrete quantities cannot be multiplied together, will save persons from the absurdity of attempting to multiply pounds, shillings, or pence, by pounds, shillings, &c. I can multiply *5s.* by the number 5, and the product is *25s.*; but if I attempt to multiply *5s.* by *5s.*, I know of no quantity which can correspond to such a product. And in considering (74) it must be noticed, that though in the practice of Rule of Three I appear to be multiplying the third term by the second, each of which is generally a concrete quantity, and then dividing by the first term, which is also generally a concrete quantity; yet, since the

concrete multiplier and divisor are of the same kind, the result is that I have merely multiplied by a fraction, *i. e.* by an abstract number, generally fractional; and the common process has been, that I have multiplied the third term by the numerator, and divided by the denominator of this fraction.

179. We can now find the area of any rectangular figure contained by commensurable lines.

FIG. 5.



Let *ADCB* be a rectangular figure, of which the side $AB = 4$ ft. 6 in., and the adjacent side $AD = 3$ ft. 3 in. Let *AB* be divided into feet in the points *a, b, c, E,* and *AD* so divided in the points *d, e, F.* Draw *af, bg, ch, Ek,* parallel to *AD,* or *BC*: and draw *dm, el, Fo* parallel to *AB* or *DC,* according to (176). Let *Ek, Fo,* intersect in *G.* Then the whole area $AC = AG + BG + DG + CG$.

Also, from (177) we have

		sq. ft.	sup. pr.	sq. in.
$AG = AE \times AF = 4$ ft. $\times 3$ ft. $= 12$ sq. ft.	=	12	0	0
$BG = EB \times EG = 6$ in. $\times 3$ ft. $= 18$ sup. pr.	=	1	6	0
$DG = FG \times FD = 4$ ft. $\times 3$ in. $= 12$ sup. pr.	=	1	0	0
$GC = Go \times Gk = 6$ in. $\times 3$ in. $= 18$ sq. in.	=		1	6
Area AC	=	14	7	6

180. Observing these four results, we may place the work in a condensed form, as annexed.

III.	II.	I.
	ft.	in.
	4	6
	3	3
1	1	6
13	6	
14	7	6

Since our proposal is to find the product of 4 ft. 6 in. by 3 ft. 3 in., we place one number under the other nearly as in Compound Multiplication, and commence multiplying the lowest denomination in the upper line by that in the lower line, and so on through the multiplier. I give the operations which are required to perform the work mentally, observing that each product as it is formed can be reduced to the next higher denomination by dividing it by 12. I commence at the right-hand and proceed thus: $3 \times 6 = 18$, which, divided by 12, gives 1 to carry to column (II.), and 6 to put down:

8—5

$3 \times 4 = 12$ for column (II.), which, with the 1 carried, is 13, and divided by 12 gives 1 to carry to (III.), and 1 to set down in (II.) Again, commencing with the multiplier 3 feet, I have $3 \times 6 = 18$ in (II.), which divided by 12 gives 1 to carry to (III.), and a remainder 6 for (II.): lastly, $3 \times 4 = 12$ for (III.), which, with the 1 to carry, becomes 13. Adding the two rows, I have the result 14 sq. ft. 7 sup. primes, 6 sq. in.; or, bringing the primes to square inches, I have 14 sq. feet 90 sq. inches, as in (179).

Comparing the operations of this article with those of the last, I notice that the steps which produced the first product of the multiplication sum in (180) are the same as the third and fourth in (179); and those which produced the second product are the same as the first and second in that article. Hence in Exs. similar to the one now worked, we need not draw any figure to insure the correctness of the work obtained by the multiplication in (180). This mode of working is called *Cross Multiplication*, and sometimes *Duodecimals*. The latter name is given in consequence of the work of such an Ex. being precisely the same as in Simple Multiplication; provided that, in working from right to left, we take every figure as having *twelve* times the value which it would have one place to the right, instead of *ten* times that value, as in common numbers.

That the area $= 4\frac{1}{2} \times 3\frac{1}{4}$ sq. ft. may also be thus shown; but to pupils unaccustomed to follow a demonstration, the above proof appealing to the eye is often more satisfactory.

Let the sides AD, AB, be divided into fourths of a foot, *i. e.* of the linear unit: the former ($4\frac{1}{2}$, or $\frac{18}{4}$) will contain 18 such parts, and the latter ($3\frac{1}{4}$ or $\frac{13}{4}$) will contain 13; and if parallels are drawn through these several points of division, the whole rectangle $ABCD$ will be cut up into a set of small squares, each of which is the 16th part of the square unit, (1 sq. ft.); and the number of the small squares will plainly be 18 taken 13 times, or 18×13. Hence the area, expressed in terms of the square unit, (1 sq. ft.), will be $\frac{18 \times 13}{16} = \frac{18}{4} \times \frac{13}{4} = 4\frac{1}{2} \times 3\frac{1}{4} =$ the product of the numbers representing the two adjacent sides.

$$\text{This area} = \frac{9}{2} \times \frac{13}{4} \text{ sq. ft.} = \frac{117}{8} \text{ sq. ft.}$$

$= 14\frac{5}{8}$ sq. ft. $= 14$ sq. ft. 90 sq. inches, as before.

181. Next, let one or both of the adjacent sides of any rectangular figure, whose area is required, consist of feet, inches, and twelfths of an inch; we must then further divide a square inch as we divided a square foot in (177): and we then learn that

1 inch $\times \frac{1}{12}$ inch $= \frac{1}{12}$th of a sq. inch, or 1 super. third;
$\frac{1}{12}$ inch $\times \frac{1}{12}$ inch $= \frac{1}{144}$ super. inch, or 1 superficial fourth.

We can now, without further explanation, follow the work of the accompanying Ex.

Find the area of a rectangular floor, whereof the length is 9 feet 4 inches 7 seconds, and the breadth 5 feet 6 inches 4 seconds.

		ft.	pr.	sec.
		9	4	7
		5	6	4
	3	1	6	4
4	8	3	6	
46	10	11		
51	10	4	0	4

The highest denomination, 51, in this product has principally been obtained from the multiplication of 5 feet and 9 feet, and is therefore square feet; and the remaining denominations are superficial primes, seconds, thirds, fourths: and the whole answer is written 51 sq. feet 10 sup. pr. 4 sec. (or sq. in.) 0 thirds, 4 fourths; or, neglecting the fourths, 51 sq. feet, 124 sq. inches.

Exs. 60.

1. Multiply 4 ft. 7 in. by 8 ft. 3 in.
2. „ 13 ft. 5 in. by 27 ft. 9 in.
3. „ 2 ft. 6 in. 4 sec. by 11 ft. 3 in.
4. „ 18 ft. 4 in. by 3 ft. 6 in. 9 sec.
5. „ 7 ft. 2 in. 5 sec. by 11 ft. 3 in. 4 sec.
6. „ 15 ft. 0 in. 7 sec. by 13 feet. 0 in. 11 sec.

7. What is the cost of paving a rectangular area, 20 ft. 6 in. by 4 ft. 3 in., at 30*s.* per square yard?

8. A room is 97 ft. 8 in. in circuit, and 9 ft. 10 in. high; what will the painting come to at $7\frac{1}{2}$*d.* per square yard?

9. How much will remain out of 393 square feet of carpeting, after covering a floor 23 ft. 6 in. long, and 16 ft. 7 in. broad?

10. Find the whole surface of a box, whereof the height, length, and breadth are respectively 3 ft. 2 in., 7 ft. $5\frac{7}{12}$ in., and 4 ft. 8 in. 5 sec.

11. What is the whole surface of a cubical box, the edge of which is 2 ft. 9 in.?

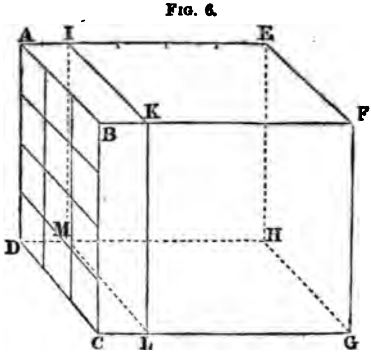
12. A house has 63 windows; 40 of them contain 12 panes, each 20 in. by 16 in.; the remainder contain 9 panes, 16 in. square; find the cost of glazing the whole at 2*s*. 3*d*. per square foot.

13. A Turkey carpet, measuring 11 ft. 6 in. by 9 ft. 8 in., is laid down on the floor of a room measuring 14 ft. by 12 ft. 6 in.; find the quantity of oil-cloth necessary to complete the covering of the floor.

182. We have stated that the length, breadth, and depth of a solid must be multiplied together to obtain its volume; we now proceed to give an Ex. of the truth of this, just as we have already worked one in a question of two dimensions.

First, let all the dimensions consist of whole numbers, viz. 3, 4, 5 inches; and let $ABCGH$ be the solid, where $AE=5$, $AB=3$, $AD=4$.

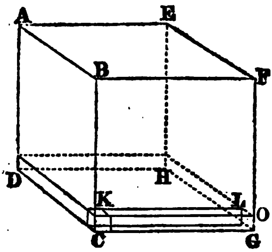
FIG. 6.



Let $IKLM$ be a section of the whole solid by a plane parallel to $ABCD$: then, since $AE=5$, the whole solid may be divided by similar planes into solid blocks, the base of each of which is $ABCD$, and the thickness 1 inch: hence there will be five times as many solid inches in the whole AG, as there are solid inches in AL.

Now, by (176) the area $ABCD = 4$ in. $\times$ 3 in. $=12$ sq. in: and if upon each of these square inches a solid inch be placed, there will be twelve solid inches in AL. Hence, in the whole AG, *i. e.* in five times AL, there will be 5×12 cubic inches; *i. e.* the volume of the solid $= 5 \times 4 \times 3$ solid inches. Of course, if the three dimensions had been expressed in feet ...ches, the result would have been $5 \times 4 \times 3$ solid feet.

FIG. 7.



Let now the solid have all its dimensions equal, and each = 1 foot, then the figure is a cubic foot, and its volume = 12 in. × 12 in. × 12 in. = 1728 cub. in., and 12 × 12 × 12, or 12^3 is called 12 *cubed*. Hence we can obtain the numbers exhibited in what is called "Solid Measure."

For 12 in. × 12 in. × 12 in., or 1728 solid inches = 1 solid foot;

3 ft. × 3 ft. × 3 ft., or 27 solid feet = 1 solid yard.

By a classification similar to that of (177), we term a twelfth part of a solid foot, a solid or cubic prime; a twelfth of a prime, a second; and so on: but we must always remember, that though we use but one set of names, we have three kinds of primes, seconds, &c., viz. linear, superficial, and solid; and the various sorts can never be added to or subtracted from each other—for it is evidently impossible to add an area either to a line or a volume.

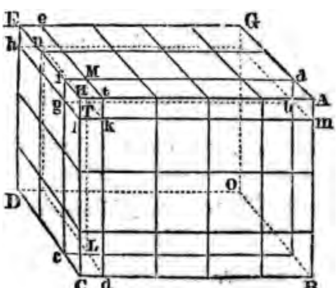
By observing Fig. 7 we learn that

1 sq. foot × 1 linear inch = 1 solid prime, as *DO*.

1 sup. prime × 1 linear inch = 1 solid second, as *CL*.

1 sq. inch or 1 sup. sec. × 1 linear inch = 1 solid third, or solid inch, as *CK*.

FIG. 8.



Let *ABCDEG* or *BE* be a solid contained by rectangular surfaces; let the three adjacent edges, *CB*, *CD*, *CH*, be respectively equal to 4ft. 5in., 2ft. 6in., and 3ft. 4in.; or $4\frac{5}{12}$, $2\frac{1}{2}$, $3\frac{1}{3}$ feet. According to the second method of (180), let each of these adjacent edges

be divided into twelfths (12 being the least common denominator of their fractional parts); then the number of twelfths in them will be 53, 30, 40 respectively; and through each of the points of division draw planes parallel to the several faces of the given solid; then the whole volume will evidently be divided into smaller cubes, each having for its edge one-*twelfth* of the linear unit, and the *volume* of which will therefore be $\frac{1}{1728}$th of the former solid unit. Also, the number of these small cubes will by (182) be $53 \times 30 \times 40$; and therefore the volume expressed in terms of the cubic unit, (1 cubic foot,) will be $\frac{53 \times 30 \times 40}{1728}$, or $\frac{53}{12} \times \frac{30}{12} \times \frac{40}{12}$, or $4\frac{5}{12} \times 2\frac{1}{2} \times 3\frac{1}{3}$, which equals the product of the numbers representing the three edges which meet in a point.

By Cross Multiplication, the work will be as follows:

$$\begin{array}{rrrr}
 & & 4 & 5 \\
 & & 2 & 6 \\
\cline{3-4}
 & 2 & 2 & 6 \\
 & 8 & 10 & \\
\cline{2-4}
 & 11 & 0 & 6 \\
 & & 3 & 4 \\
\hline
3 & 8 & 2 & 0 \\
33 & 1 & 6 & \\
\hline
36 & 9 & 8 & 0 \\
\hline\hline
\end{array} \text{ or } 36\tfrac{116}{144} \text{ sol. ft. or } 36\tfrac{29}{36} \text{ sol. ft.}$$

183. Working fractionally, the volume $= \frac{53}{12} \times \frac{5}{2} \times \frac{10}{3}$ sol. ft.

$$= \frac{1325}{36} \text{ sol. ft.} = 36\,\tfrac{29}{36} \text{ sol. ft.}$$

184. Since length × breadth × depth = volume (I.), therefore length × breadth $= \frac{\text{volume}}{\text{depth}}$; and since length × breadth is of two dimensions, therefore $\frac{\text{volume}}{\text{depth}}$ is of two dimensions,

i. e. $\frac{3 \text{ dimensions}}{1 \text{ dimension}}$ gives two dimensions.

So also, from (I.) length $= \dfrac{\text{volume}}{\text{breadth} \times \text{depth}}$; *i.e.* $\dfrac{3 \text{ dimen.}}{2 \text{ dimen.}}$ gives one dimension: and any fraction in which numerator and denominator are of the same dimension, is of no dimensions, or an abstract number. (See Art. 67.)

185. The following Exs., are amongst the most useful of those in which the measurement of surfaces and solids occurs.*

Ex. I. Find the number of acres in a rectangular field, of which the length is 35 chains 72 links, and the breadth 24 chains 8 links.

To understand this question, a pupil must know that large pieces of land are measured by means of a chain called *Gunter's Chain*, which is four poles, or 22 yards, in length, and is divided into 100 equal parts, called *Links*.

Also, an acre is equal to a rectangular surface of which the length is 40 poles, or 10 chains, and breadth 4 poles, or 1 chain: hence, the area of an acre which is 40 poles long, and 4 poles broad, $= 10$ chains $\times$ 1 chain $= 1000$ links $\times$ 100 links $= 100,000$ square links. Consequently, if the dimensions of a field be expressed in links, and its area thence be obtained in square links, this value, when divided by 100,000, will be expressed in acres; *i. e.* if five places be pointed off as a decimal, the result will be acres and decimal parts of an acre, which can be reduced to roods and poles. Returning now to the Ex., we have

$$
\begin{array}{rr}
\text{Length} = 35 \text{ chains } 72 \text{ links} = & 3572 \text{ links.} \\
\text{Breadth} = 24 \text{ chains } 8 \text{ links} = & \underline{2408} \text{ links} \\
& 28576 \\
& 142880 \\
& \underline{7144} \\
\text{Area} = \text{acres} & 86 \cdot 01376 \\
& \underline{4} \\
\text{Roods} & \cdot 05504 \\
& \underline{40} \\
\text{Perches} & \underline{\underline{2 \cdot 20160}}
\end{array}
$$

and therefore the field contains 86 a. 0 r. $2\frac{1}{5}$ perches, nearly.

Ex. II. Find how many gallons are contained in a cistern of which the length is 40 inches, breadth 36 inches, and depth 16 inches.

* For further information on Mensuration, see Lund's Geometry, Pt. III.

It must here be observed that an imperial gallon is equal to 277·274 cubic inches: hence, when the number of solid inches in any volume is known, the number of gallons which it will contain is found by dividing those solid inches by 277·274. In the above Ex. we therefore have

$$\text{the solid content} = (40 \times 36 \times 16) \text{ solid inches;}$$

$$\text{and therefore number of gallons contained} = \frac{23040}{277 \cdot 274} = 83, \text{ nearly.}$$

Ex. III. A roof of $27\frac{1}{2}$ feet by $18\frac{3}{4}$ is to be covered with lead weighing 8 lbs. per square foot: what would it cost at the rate of £5 4*s.* for 5 cwt.?

$$\begin{aligned}\text{The area of the roof} &= 27\tfrac{1}{2}\text{ ft.} \times 18\tfrac{3}{4}\text{ ft.}\\ &= \left(\frac{55}{2} \times \frac{75}{4}\right)\text{ sq. ft.}\\ &= \frac{4125}{8}\text{ sq. ft.}\end{aligned}$$

$$\text{hence, weight of lead} = \frac{4125}{8} \times 8 \text{ lbs.} = 4125 \text{ lbs.}$$

To find the cost of the lead, we have the following statement:

$$5 \times 112 \text{ lbs.} \quad : \quad 4125 \text{ lbs.} \quad :: \quad £5\ 4s.$$

The fourth term will be found to be £38 6*s.* $0\frac{6}{7}$*d.*

Exs. 61. Form the following products:—

1. 3 ft. 2 in. × 4 ft. 9 in. × 5 ft. 7 in.
2. 15 sq. ft. 73 sq. in. × 2 ft. 6 in.
3. 11 sq. ft. 9 sup. pr. 3 sec. × 9 ft. 7 in.
4. 2 ft. 4 in. × 3 in. × 11 in.

5. Find the capacity of a rectangular cistern, 12 ft. 3 in. long, 5 ft. 7 in. broad, and 2 ft. 11 in. deep.

6. How many bricks, of which the length, breadth, and thickness are 12, 9, 6 inches respectively, will be required to build a wall, whereof the length, height, and thickness are 64, 9, and $1\frac{1}{2}$ feet?

7. What is the price of a block of stone, of which the length, breadth, and thickness are 37 ft. 8 in.; 8 ft.; and 6 ft. 5 in., at 5*s.* 6*d.* per sol. foot?

8. How many square feet of board would be required to make a rectangular box, of which the length, breadth, and depth are respectively $3\frac{1}{2}$ ft., $2\frac{1}{4}$ ft., and 1 ft. $2\frac{5}{12}$ in.?

9. A cubic inch of water weighs 252·458 grains, required the weight (in lbs. Troy) of water in a full cistern $10\frac{1}{2}$ ft. long, $5\frac{1}{4}$ ft. wide, and 11 in. deep.

10. The bottom of a cistern is rectangular, and contains 15 sq. feet, 58 sq. in.; how deep must it be to hold 164 gallons, if a gallon contain $277\frac{1}{4}$ cubic inches?

11. The length of a rectangular field is 25 chains 37 links, and the breadth 17 chains 35 links; find the number of acres contained.

12. Find the number of chains and links in the breadth of a field, whereof the length is 35 chains 15 links, and the area is 45 a. 2 r. $31\frac{1}{4}$ p.

EXTRACTION OF ROOTS.

186. We have already seen (92) that a number, by being successively multiplied by itself, is said to be raised to a power; and the order of the power, whether it be second, third, fourth, &c., depends upon the number of times the original number is to be repeated. This process is termed *Involution;* and the reverse process of obtaining the original number from the power is called *Evolution.* The original number is called, with respect to its power, the *Root;* and this evolution is also termed *Extraction of Roots.*

The Square Root of any number or quantity is that number or quantity, which when squared, that is, multiplied by itself, will produce the original quantity.

The Cube Root is that which when cubed, or multiplied by itself *twice,* will produce the original quantity.

Thus, since 5×5, or 5^2, or 5 *squared* (176), as it is called, $= 25$; 5 is called the Square Root of 25. Similarly, since $6 \times 6 \times 6$, or 6^3, or 6 *cubed* $(182) = 216$, then 216 is termed the cube or third power of 6, and 6 the cube root or third root of 216.

The sign $\sqrt{}$ is used to express the operation of extracting a root; and a small figure, placed thus $\sqrt[3]{}$, shows what root [here the *third* root] is to be extracted: but the figure is generally omitted when the sign refers to the square root;

and the sign $\sqrt{\ }$ itself indicates extraction of the Square Root.

187. A number or quantity which is thus formed by the squaring, cubing, &c. of any number, is called a complete or *perfect* square, cube, &c.; and therefore the square, cube, &c. root, of such a power can be exactly extracted: but any number, as 20, which lies between two complete squares, 16 and 25, cannot have its root obtained exactly. But since 16 has a root 4, and 25 has a root 5, therefore the root of 20 will be between 4 and 5, or its value will be approximately expressed by the mixed decimal 4·472...; and similarly of every number which lies between two complete squares. So also every number which lies between two complete cubes must have its cube root expressed in a decimal form.

188. If a number be raised to a power, and then the same root of that power be extracted, we shall of course obtain the original quantity; therefore $\sqrt{3^2} = 3$; $\sqrt[3]{5^3} = 5$.

Also, since the square root of a given number has been defined to be that number which, when multiplied by itself, will produce the given number, therefore $\sqrt{2} \times \sqrt{2} = 2$: so also, $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$. Such quantities as $\sqrt{2}$, $\sqrt[3]{2}$, are called *Surds*, or *Irrational* quantities.

Obs. Since fourth, fifth, &c. powers of numbers can be obtained, of course fourth, fifth, &c. roots must also exist; but we shall here confine ourselves principally to the finding of square and cube roots.

What has been said of the raising of whole numbers to powers is also true of fractional quantities, whether vulgar, or commensurable decimals. Thus, since $\frac{2}{3} \times \frac{2}{3}$, or $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$; therefore $\sqrt{\frac{4}{9}} = \frac{2}{3}$, which equals $\frac{\sqrt{4}}{\sqrt{9}}$; *i. e.* the square root of

a fr^n must be obtained by taking the square root of numerator and denominator. In like manner, since $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$, or $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$; therefore $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$, which equals $\frac{\sqrt[3]{8}}{\sqrt[3]{27}}$; *i. e.* the cube root of a fraction will be found by taking the cube root of both numerator and denominator. If the root of a mixed number be required, it must be reduced to an improper fraction or a decimal. Thus:

$$\sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4} = 1\tfrac{1}{4}.$$

We shall presently show how to extract the square and cube roots of decimals as well as of whole numbers.

SQUARE ROOT.

189. Before proceeding to investigate a rule for the extraction of the Square Root, we must remember the following:

Digits,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Squares,	1,	4,	9,	16,	25,	36,	49,	64,	81.

From what has been said, it is plain that the square of 1, is 1; of 10, is 100; of 100, is 10,000, &c. &c.

Therefore the square root of		1	is	1
,,	,,	100	,,	10
,,	,,	10,000	,,	100
,,	,,	1,000,000	,,	1000 &c. &c.

Hence, if a number lie between 1 and 100, *i. e.* have not more than *two* figures, its square root is between 1 and 10, or has not more than *one* figure.

If between 100 and 10,000, *i. e.* have not more than *four* figures, the root is between 10 and 100, or has not more than *two* figures.

If between 10,000 and 1,000,000, *i. e.* have not more than *six* figures, the square root is between 100 and 1,000, or has not more than *three* figures: and so on.

So that if over every alternate figure of any number, beginning at the units' place, a point be placed, the number of points will show the number of figures in the square root. For example, 3757 has more than two figures, and not more than *four*, or lies between 100 and 10,000, and therefore its root lies between 10 and 100, *i. e.* has not more than *two* figures, and the number must therefore be thus pointed, $3\dot{7}5\dot{7}$; so also $\dot{1}3\dot{4}5\dot{9}$ is correctly pointed, for there will be three figures in the square root. The divisions which are formed by these points are called *periods*. The periods in $\dot{1}3\dot{4}5\dot{9}$ are 1, 34, and 59.

Also, since the square of ·1 is ·01, therefore
the square root of ·01 is ·1
So also, of ·0001 is ·01
" " ·000,001 is ·001 &c. &c.

It hence appears that in the square of any decimal an even number of decimal places will always be found; and that if there be an odd number in any proposed Ex., the number of places must be made even, by appending a cipher, which cannot alter the value of the decimal. We then point every alternate figure of a decimal, from left to right, beginning with the second figure from the units' place, so that the last figure will always be pointed, in decimals as well as in whole numbers.

190. The Rule for the Extraction of the Square Root is derived from an algebraical operation, wherein a complete square is taken, and a process is then contrived by which the root, which is already known, can be deduced from the complete square.

I shall therefore proceed, contrary to my usual method, to give a Rule for extracting the Square Root, without any more explanation than is necessary for the mere working of Exs.: and the arithmetical illustration of the above algebraical process, which I shall afterwards give, may be read or omitted at the reader's pleasure.

Rule. Divide the given number into periods. Find the greatest square number which is not greater than the first period; subtract it from that period, and place the root of ...ber in the place of the quotient in Long Division.

To the remainder, after subtraction, bring down two figures or one period, and consider the whole as a dividend.

For a divisor, double the root, and try how often it is contained in the dividend, except the last figure: the figure thus obtained by division place in the root, and annex it to the divisor.

Multiply this whole divisor by the last figure in the root, and subtract this *subtrahend* from the dividend.

Bring down another period; find a fresh divisor by adding the last figure of the former divisor to that divisor, and proceed exactly as before.

Obs. It will be found that on dividing by the *incomplete* divisor—especially when the early figures in the root are small, and the latter ones large—there will result a quotient larger than the one which must be taken. The reason of this will be explained hereafter.

I will work one Ex. which gives no remainder, *i. e.* where the given number is a perfect square, viz. 81796.

A pupil will find himself less liable to make mistakes in bringing down the periods, if in actual working he points off as I have done below.

```
      8'17'96' (286
      4
48)   417
      384
566)  3396
      3396
```

The greatest square number not greater than the first period, 8, is 4; I therefore subtract 4 from the 8, and place in the quotient, 2, the root of the 4: to the remainder, 4, I bring down the second period, 17, and consider 417 as my dividend. For a divisor, I double the 2 in the root: and on dividing 41 by this divisor 4, I obtain a quotient 10; but upon trial I find that 8 is the largest quotient that can be employed. I place the 8 in the root, as well as at the right of the 4 in the divisor, making a *complete* divisor 48: multiplying this 48 by the 8, I have a subtrahend 384; the remainder, after subtraction, is 33, and with the third period 96 gives a new dividend 3396: adding the last figure 8 in the divisor 48, to that 48, I have, as new partial divisor, 56, which goes six times in 339: placing the 6 in the root and in the divisor, and multiplying the whole divisor 566 by the 6, I have a subtrahend 3396, which, upon subtraction, leaves no remainder. The square root is therefore 286.

Since most of the numbers we meet with are not perfect squares, we can obtain only approximate roots of such numbers; and the operation is carried to about three places of decimals in the root, requiring of course, six places in the number. If the given number be an integer, or have not so many as six decimal places, the number must be made up by appending ciphers.

Ex. II. Find the square root of 876·535.

$$\begin{array}{rl}
 & 8'76'\cdot53'50'00'\ (29\cdot606\ldots \\
 & 4 \\
49) & 476 \\
 & 441 \\
586) & 3553 \\
 & 3516 \\
59206) & 375000 \\
 & 355236 \\
 & 19764
\end{array}$$

I append three ciphers, and point according to (189). When the third divisor, 592, is obtained, the quotient is 0; I therefore place the 0, as usual, in the root and in the divisor, then bring down another period, and proceed as before. Since there were two periods in the integral part of the given number, there will be two places of integers in the root, and the decimal point must be put after the 29.

191. I will now illustrate the algebraical process mentioned in (190), as far as can be done in arithmetic.

Let the square number 169 be taken, the root of which is 13. If this number and its root be expressed in the required algebraic form, they will be respectively written 100 + 60 + 9, and 10 + 3.

$$\begin{array}{rl}
 & 100+60+9\ (10+3 \\
 & 100 \\
20+3) & 60+9 \\
 & 60+9
\end{array}$$

Putting in the place of the quotient the 10 + 3, which is known to be the root, I observe that the first part of the root, viz. 10, is the square root of 100, the first part of the number: I may therefore consider that the square root of the 100 gives the 10, and the remainder 60 + 9 is to furnish the 3. Observing the former part of the remainder, viz. 60, I notice, that if it were divided by twice the 10 in the root, I should obtain the required number, 3: I therefore make 20, *i. e.* twice the first figure in the root, my divisor. Also, since there must be no remainder, I must have as subtrahend 60 + 9; and since the 9 is the square of 3, therefore, if I append the 3 to the 20 in the divisor by the sign (+), and multiply the whole divisor 20 + 3 by 3, I shall obtain a subtrahend equal to the dividend 60 + 9, and therefore shall have no remainder, as was *required.*

An algebraical process similar to the above would *prove* that this method of procuring a divisor and subtrahend would *always* succeed in bringing no remainder, *i. e.* in obtaining the exact root of a complete square. But this Ex. of course only *illustrates* the method of proof, and shows how the Rule is algebraically deduced. The operation, when condensed into an arithmetical form, stands thus:

$$\begin{array}{r|l} & 1'69'\ (13 \\ & 1 \\ \hline 23) & 69 \\ & 69 \\ \hline \end{array}$$

192. I will work one more Ex. which will show how an error in the quotient may arise in dividing by an incomplete divisor.

$(27)^2 = 729$, which may be put into the algebraic form $400 + 280 + 49$: and the root is $20 + 7$.

And this, when condensed, is

$$\begin{array}{r|l} & 400 + 280 + 49\ (20 + 7 \\ & 400 \\ \hline 40 + 7) & 280 + 49 \\ & 280 + 49 \\ \hline \end{array} \qquad \begin{array}{r|l} & 7'29'\ (27 \\ & 4 \\ \hline 47) & 329 \\ & 329 \\ \hline \end{array}$$

Obs. Since to divide 280 or 320 by 40 is the same as to divide 28 or 32 by 4, I may therefore call 4 the divisor, when 28 or 32 is the dividend, and consider 40 as the divisor, when 280 or 320 is the dividend.

If this Ex. be worked in the usual manner, it will be found that the incomplete divisor 4 will go eight times in 32, whereas the real quotient is found to be only 7. By observing the algebraic method, I learn, that when the dividend 329 is separated into its parts $280 + 49$, the true quotient is obtained by dividing only the former part, 280, by the 40, or 28 by 4: but in the arithmetical operation I cannot see how much of 329 is the former part which will give me a correct quotient; and therefore I have to divide

the whole 329 by 40, or 32 by 4, and so run the risk of an error. If the second part of the dividend 49 had been less than 40, the quotient would at once have appeared to be 7, and no error arisen. Hence the larger the second part of the dividend (as 49) is, when compared with the incomplete divisor (as 40), the greater will be the error: and since the 4 in the divisor was obtained from the *former* part of the quotient, viz. 2, and the 49 from the *latter* part, viz. 7, the error will be the greatest when the earlier figures of the root are small, and the latter are large.

This method of proof is not limited to numbers, the roots of which consist but of two figures: but any attempt to extend the illustration would be very cumbrous without the use of algebra.

Exs. 62.

Extract the Square Root of each of the following numbers:—

1. 729	4. 2832489	7. $1241\frac{152}{289}$	10. 197·4025
2. 11025	5. $27\frac{9}{16}$	8. $12122\frac{1}{100}$	11. 36·343
3. 8264446281	6. $371\frac{46}{49}$	9. 10·5625	12. ·002401

193. We have seen (176) that the area of a square is obtained by squaring any one of its sides; hence the number in the side of a square is found by extracting the square root of the number which represents the area of the square.

The number which represents the area of a square figure may not always be a complete square: for instance, if in Fig. 1, p. 171, a square be described with sides equal to ac or $\sqrt{2}$, then its area would be $(\sqrt{2})^2 = 2$, which is not a perfect square. When, therefore, the number representing the area of a square is not itself a perfect square, the number representing the side will be incommensurable or irrational.

Quantities which are not perfect squares, cubes, &c. may *be made so by* multiplying them by certain factors. For, in

order that a number may be a complete square, each of its factors must be contained 2, 4, 6, &c. times, *i.e.* some multiple of *two* times; to be a complete cube, each must be contained 3, 6, 9, &c. times, *i.e.* some multiple of *three* times; and so on for higher powers. For example, the number 144 will be found $= 2^4 \times 3^2$, where the indices 4 and 2 are multiples of 2, and therefore 144 is a perfect *square.* So also $1728 = 2^6 \times 3^3$, where the indices are multiples of 3, and therefore 1728 is a perfect *cube.*

Hence, if I resolve any number into its factors, I can tell by inspection whether it be a perfect square, cube, &c.; and if not, what additional factors must be introduced into it to make it so. Thus $20 = 2^2 \times 5$; and since the index of the 5 is not a multiple of 2, it must be made so, by introducing an additional 5; it then becomes $2^2 \times 5^2$, or 100, a perfect square. Again, to make 48 a perfect cube, I observe that $48 = 2^4 \times 3$; and in order that the indices may be multiples of 3, this must be changed into $2^6 \times 3^3$, and therefore be multiplied by $2^2 \times 3^2$, *i.e.* by 36; and we then shall have $48 \times 36 = 1728$, a perfect cube.

In obtaining the square root of a fractional quantity, we may extract the root of numerator and denominator, if the denominator be a complete square, as in (188).

$$\text{Thus, } \sqrt{\frac{729}{1225}} = \frac{27}{35};$$

or, taking the case of a mixed number,

$$\sqrt{4\frac{49}{144}} = \sqrt{\frac{625}{144}} = \frac{25}{12} = 2\tfrac{1}{12}.$$

But if the den^r be not a perfect square, it is better to reduce the fraction to a decimal, and then extract the root. For example, if I have to extract the square root of $27\frac{27}{128}$, then, since $27\frac{27}{128} = 27{\cdot}2109375$, a single extraction will

suffice to give the answer at once; whereas, trying the former method, I should have

$$\sqrt{27\tfrac{27}{128}}=\sqrt{\frac{3483}{128}}=\frac{59\cdot01\ldots\ldots}{11\cdot31\ldots\ldots},$$

and to obtain the result, I must extract *two* roots, and perform an operation in Long Division.

Again, we may make the den^r a complete square, as described in the preceding paragraph; and then, since the root of the den^r will be known by the very process of completing the square, it will be necessary to extract only the root of the num^r. Thus, taking the above example, and observing that a factor 2 will make the 128 become 256..., or 16^2, we have

$$\sqrt{27\tfrac{27}{128}}=\sqrt{\frac{3483}{128}}=\sqrt{\frac{6966}{256}}=\frac{83\cdot462\ldots}{16}=5\cdot216\ldots.$$

In finding the root of a circulating decimal, it will sometimes happen that the equivalent vulgar fraction will have both numerator and denominator complete squares, and we can then readily extract the root. Thus:

$$\sqrt{1\cdot\dot{7}}=\sqrt{1\tfrac{7}{9}}=\sqrt{\tfrac{16}{9}}=\tfrac{4}{3}=1\tfrac{1}{3},\text{ or }1\cdot\dot{3}.$$

But as this is rarely the case, we must generally find the approximate root by the usual method, though it will not be a recurring decimal.

194. Sometimes a fraction involving a surd may be in its lowest terms, but yet not in a form the most convenient for finding its value. Thus $\frac{1}{\sqrt{2}}$ is in the lowest terms; but multiplying numerator and denominator by $\sqrt{2}$, it becomes $\frac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{\sqrt{2}}{2}$, or $\frac{1}{2}\sqrt{2}$. And we shall find that this fraction is more simple than the former one. For, since $\sqrt{2}=1\cdot4142\ldots$

therefore $\dfrac{1}{\sqrt{2}} = \dfrac{1}{1\cdot4142\ldots}$, which will require a Long Division sum to bring out its value; but $\dfrac{1}{2}\sqrt{2} = \dfrac{1}{2}(1\cdot4142\ldots) = \cdot7071..$ by mere Short Division. Hence $\dfrac{1}{2}\sqrt{2}$ is more simple than $\dfrac{1}{\sqrt{2}}$.

And as a general rule, all quantities involving surds are in their simplest form, when the surds are in the numerator and not in the denominator.

The following Exs. illustrate the operations of the last few pages.

Ex. I. A square field contains 15 a. 2 r. 20 p. Find its side in chains.

15 a. 2 r. 20 p. = 2500 sq. poles; therefore the side of a field containing 2500 sq. poles = 50 linear poles

$$= \frac{50}{4} \text{ linear chains (since 4 poles = 1 chain)}$$

$$= 12\tfrac{1}{2} \text{ linear chains.}$$

Ex. II. Two acres of land are to be cut from a rectangular field, of which the breadth is 2 chains 50 links, by a line parallel to it. Find the length of the plot.

1 acre = 40 p. × 4 poles = 10 chains × 1 chain = 10 sq. chains; and length × breadth = 2 acres; or, since the breadth is $2\frac{1}{2}$ chains, therefore length × $2\frac{1}{2}$ chains = 2 acres = 2 × 10 sq. chains = 20 sq. chains.

$$\text{Hence length} = \frac{20 \text{ sq. chains}}{2\frac{1}{2} \text{ linear chains}}$$

$$= \overset{4}{\cancel{20}} \times \frac{2}{\underset{1}{\cancel{5}}} \text{ linear chains (184)}$$

$$= 8 \text{ linear chains.}$$

Exs. 63. Find the value (to 4 places of decimals) of

1. $\sqrt{3}$ 2. $\dfrac{1}{\sqrt{3}}$ 3. $\sqrt{6\cdot24\dot{9}}$ 4. $\sqrt{15\cdot\dot{3}}$

5. Find the side of a square field, whose area is equal to that of a rectangle 1860 yds. by 800 yds.

6. The sides of 2 squares are 15 ft. and 25 ft.; find the side of another which shall equal the sum of the two formers.

7. A rectangular field measures 64 ft. in length by 48 ft. in breadth; what is the length of the diagonal?*

8. A square field contains 3 a. 3 r. $2\frac{58}{121}$ p.; find the length of its side in yards.

9. The painting of a square area, at $7\frac{1}{2}d.$ per square yard, comes to £22 15*s.* $7\frac{1}{2}d.$; find the length of the side of the square.

10. Two sides of a triangle are respectively 236·25 and 243·75 ft., also the altitude is $2\frac{2}{5}$ ft.; find the length of the base*.

CUBE ROOT.

195. The following numbers must be first remembered:

Digits,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Cubes,	1,	8,	27,	64,	125,	216,	343,	512,	729:

and it will be observed that no two of these cubes end with the same digit: hence, if a given number be a perfect cube, the last figure in its root may be known. For instance, if a cube number end in 3, its cube root ends in 7: if the number end in 2, its root ends in 8. Recollection of this fact is often of service.

According to the method of (181), we learn that the cube root

of	1	is	1
,,	1,000	,,	10
,,	1,000,000	,,	100 &c. &c.

hence, if a number lie between 1 and 1000, *i. e.* have not more than three figures, its cube root is between 1 and 10, or has not more than one figure.

If the number be between 1000 and 1,000,000, *i. e.* have not more than six figures, its root has not more than two figures: and if the number have not more than nine figures, its root has not more than three figures.

So that if over every third figure, beginning at the units' place, a point be placed, the number of points will show the number of figures in the cube root.

Similarly, since the cube of	·1	is	·001
therefore the cube root of	·001	,,	·1
So also........................	·000,001	,,	·01
.. .. ..	·000,000,001	,,	·001 &c. &c.

It hence appears that the number of places in the cube of any decimal must always be some multiple of 3: and if the number of places in any

* See Appendix, Note to Art. Ratio.

decimal, of which we have to find the cube root, be not a multiple of 3, it must be made so by appending ciphers: we therefore commence pointing at the units' place, and point every third figure to the left over the integers; and to the right over the decimals, if there be any.

The following Rule, called Horner's method, though at first difficult for a learner, yet has the merit of exhibiting all the work in a very convenient form, especially when the given number is large. But the proof of it would be out of place here, as it involves some knowledge of the Theory of Equations.

RULE. I. Divide the given number into periods. Find the greatest cube number which is not greater than the first period. Subtract it from that period; place the root of the number in the quotient, and form a dividend as in Square Root.

II. To the left of the number, and at some distance from it, place two columns (A) and (B). Under (A) insert three times the root, and under (B) three times its square. Annex one cipher to (A), and two ciphers to (B), and with (B) as divisor, find the next figure in the quotient.

III. Add this figure to (A); also add to (B) the product of (A) by the last figure in (A); and multiply the sum by the last figure in the root, to form a subtrahend; subtract this from the dividend, and bring down one period.

IV. Under (A) place twice its units' figure, and under (B) the square of that figure. Find the sum of the last *two* lines in (A), and of the last *three* in (B), and again annex one cipher to (A) and two to (B). With (B) as divisor, find another figure in the root, and proceed to form another subtrahend, as in (III.) of this Rule.

OBS. Here, as in Square Root, the divisor often gives upon trial too large a quotient. But in working examples in Algebra by it, there is no such liability to error.

196. The following Example will be found to exemplify this Rule; and the lines have been written widely, to indicate the successive steps from (A) to (B), and from (B) to the subtrahends.

A		B		21'717'639' (279
60				8
7		1200 = total divisor)		13717
67 × 7	=	469		
14		1669 × 7	=	11683
810		49		
9		218700		) 2034639
819 × 9	=	7371		
		226071 × 9	=	2034639

Since the first figure in the root is 2, I place 3 × 2, or 6, under (A), and 3×2^2, or 12, under (B), annex one cipher to (A) and two ciphers to (B). Using this 1200 as a divisor, I have 7 as the second figure in the root, which I add to (A), making 67, and to (B) I add the product of this 67 by 7, viz. 469, making 1669: I multiply this 1669 by the same 7, making a subtrahend 11683, and obtain a dividend 2034639. I now place under (A) twice its last figure 7, and under (B) I place 7^2 or 49; I now add the last *two* lines in (A), making 81, and the last *three* lines in (B), viz. 49, 1669, and 469, obtaining 2187; then annex one cipher to (A), two ciphers to (B), and making this 218700 in (B) as a divisor, I obtain a figure 9 in the root. I now add this 9 to the 810 in (A), and multiply the whole of it, or 819, by 9, making 7371, then add it to (B), making 226071; lastly, multiply this by the same 9, placing the product as a subtrahend. If there had been any more periods, I should have commenced forming a fresh divisor, as in (IV.).

197. The observations which were made upon the extraction of the Square Root of Fractions in (193) apply also to a similar use of the Cube Root.

Just as the number in the side of an area of square form is found by extracting the square root of the number representing the area: so the number in the edge of a volume of cubic form is found by extracting the cube root of the number which represents the volume.

I will work one or two Exs. to illustrate the application of Cube Root.

Ex. IV. A beam is 6 ft. 9 in. long, 2 ft. 3 in. broad, and 9 in. thick; required the side of a cube of equal capacity.

Working fractionally, we have

$$\text{solid content of beam} = 6\tfrac{3}{4}\text{ ft.} \times 2\tfrac{1}{4}\text{ ft.} \times \tfrac{3}{4}\text{ ft.}$$

$$= \left(\frac{27}{4} \times \frac{9}{4} \times \frac{3}{4}\right)\text{ sol. ft.} = \frac{729}{64}\text{ sol. ft.};$$

$$\text{therefore edge of the cube} = \sqrt[3]{\frac{729}{64}}\text{ sol. ft.} = \frac{9}{4}\text{ lin. feet} = 2\tfrac{1}{4}\text{ linear ft.}$$

Ex. V. Find the area of any one of the six surfaces of a cube containing 11 cubic feet, 675 cubic inches.

To find the area of this surface, I must find the edge of the cube, and square it. Reducing the cubic feet to inches, I have

$$\text{volume of cube} = 19683\text{ cubic inches};$$

$$\text{therefore edge of the cube} = \sqrt[3]{19683} = 27\text{ linear inches} = 2\text{ ft. } 3\text{ in.};$$

$$\text{and area of the side in sq. ft.} = (2\tfrac{1}{4})^2 = \left(\frac{9}{4}\right)^2 = \frac{81}{16} = 5\tfrac{1}{16}$$

$$= 5\text{ sq. ft. } 0\text{ pr. } 9\text{ sq. in.}$$

Ex. VI. Assuming that the volume of any cylinder equals its length × area of its base, find the value of this area in a cylindrical wire 50 feet long, and made out of a square inch plate of metal ·05 inches in thickness.

Here, since area × length = whole volume of metal

$$= 1\text{ sq. inch} \times \text{thickness of the plate};$$

$$\text{or, since area} \times 50\text{ ft.} = 1\text{ sq. inch} \times {\cdot}05\text{ inches};$$

$$\text{therefore area} = \frac{1\text{ sq. in.} \times {\cdot}05\text{ in.}}{50 \times 12\text{ in.}}$$

$$= \frac{{\cdot}05}{50 \times 12}\text{ sq. inches}$$

$$\left(\begin{matrix}\text{multiplying numerator}\\ \text{and denominator by } 100\end{matrix}\right) = \frac{5}{5000 \times 12}\text{ sq. inches}$$

$$= \frac{1}{12000}\text{ sq. inches.}$$

Exs. 64. Extract the Cube Root of each of the following numbers:—

1.	42875	5.	$42\frac{7}{8}$	9.	77·854483
2.	970299	6.	$1367\frac{631}{1000}$	10.	284890·312
3.	6539203	7.	$2345\frac{22}{343}$	11.	1334·633301
4.	32798729601	8.	$423987\frac{457}{512}$	12.	$\frac{\cdot000405224}{\cdot064}$.

Find the value (to 2 places of decimals) of

13. $\sqrt[3]{15}$ 14. $\sqrt[3]{155}$ 15. $\sqrt[3]{342\cdot\dot{9}}$ 16. $\sqrt[3]{\frac{1}{15}}$

17. Find the edge of a cubical box containing 13824 solid inches.

18. A cistern is 72 ft. long, 24 ft. broad, and 27 ft. deep: find the edge of a cubical cistern of the same content.

19. Find the length of the edge of a cube which contains 94 yds. 14 ft. 1088 inches.

20. Find the whole surface of a cube which contains 15 solid feet and 1080 solid inches.

TO EXTRACT ANY ROOT WHATEVER.

198. Upon the same principle as the Cube Root Rule given above, is the following Rule for the extraction of any root whatever.

I. Divide the given number into periods, each containing as many figures as the index of the root to be extracted. Find the root of the first period, and place it as a quotient.

II. Form, at equal distances from each other, columns, called A, B, C, &c. equal in number to the above index; and consider the first period in the given number as the head of the last column. Under that to the left, place the figure in the root; under the next, the square of that figure; under the next, the cube, and so on. The highest power of this figure will fall under the first period; subtract it from that period, and form a new dividend as usual.

III. To find a trial divisor:—Under (A) again place the figure in the root, multiply (mentally) the sum by this root; *but place* the product under (B), not under (A); add the two

lines in (B), multiply the sum by the root, and place the product under (C); proceed in like manner to the *last column* which stands before the proposed number.

Commence at (A) precisely the same process, and continue it to the *last column but one;* again continue the same process, dropping a column each time, till only the first one is employed. Now annex *one* cipher to (A), *two* to (B), *three* to (C), and so on; and make the result of the column, which precedes the proposed number, a trial divisor of the dividend, thus obtaining another figure in the root. The subtrahend will be obtained presently.

IV. The next trial divisor is obtained by forming a series of products with the new figure, of precisely the same kind as those obtained from the former figure. Also, in forming the first row of these new products, the last one to the right will be the subtrahend from the lately formed dividend, and will after subtraction furnish a new dividend as usual.

Ex. To extract the fifth root of 60466176.

A	B	C	D	E
				60466176 (36
3	9	27	81	243
3	18	81	324	
6	27	108	4050000	) 36166176
3	27	162	1977696	36166176
9	54	270000	6027696	
3	36	59616		
12	9000	329616		
3	936			
150	9936			
6				
156				

In the above Ex. it may be seen that the first figure in the root, viz. 3, must be found by trial, as in Cube Root. Under the five columns, A, B, C, D, E, there have been placed, 3, 3^2 or 9, 3^3 or 27, 3^4 or 81, 3^5 or 243, this last power being the subtrahend from the first period. Also, this figure 3 has been added to (A) four times; each successive

result has been multiplied by this figure 3, and the products added to (B). In like manner, of the three results similarly formed in (B), the first two have been multiplied by the root figure, and the products carried to (C). The result of the first addition alone in (C) has been multiplied and carried to (D). The results of the columns are now 15, 90, 270, 405, and with the ciphers added become 150, 9000, 270000, 4050000, which last, taken as a trial divisor, gives 6 as the next figure in the root.

To find a subtrahend, we proceed through the first steps which would be necessary if we had a third period, and were obtaining another trial divisor.

Shew that 25 is the fifth root of 9765625, and 27 the sixth root of 387420489.

Exs. 65. Q.

1. Explain the process of finding the area of an oblong, the sides whereof contain any number of feet and inches.

2. Explain by figures the nature of the products when feet are multiplied by feet, feet by inches, inches by inches, &c. down to twelfths of an inch.

3. Find how much money must be paid for £10800 stock at $84\frac{1}{8}$, and 2*s*. 6*d*. per cent. brokerage.

4. A mixture is made of 40 lbs. at 3*s*. 6*d*., 44 lbs. at 3*s*. $10\frac{1}{2}$*d*., and 55 lbs. at 4*s*. 6*d*.; what will be the gain per cent. if the mixture be sold for £39 4*s*.?

5. What is the interest of money invested in the $3\frac{1}{2}$ per cents. at $89\frac{3}{4}$?

6. An inclined plane 3 miles long has a total rise of 212·15 feet; find the rise per yard in decimal parts of an inch.

7. Reduce to a simple fraction the ratio between the sum and difference of these expressions:—

$$\left(\tfrac{3}{2}+\tfrac{2}{3}\right)\times\left(\tfrac{5}{4}+\tfrac{4}{5}\right) \text{ and } \left(\tfrac{3}{2}-\tfrac{2}{3}\right)\times\left(\tfrac{5}{4}-\tfrac{4}{5}\right).$$

8. Form the following product, (17 ft. 4 in.) × (18 ft. 7 in.), by Duodecimals, and by Fractions.

9. Shew that $\dfrac{1}{\sqrt{3}} = \dfrac{1}{3}\sqrt{3}$; which is the simpler of the two, and why?

10. Find the square roots of 6424·0225 and of $\frac{729}{1296}$.

11. What is the solid content of a box, of which the height, length, and breadth are respectively 3 ft. 2 in., 5 ft. 4 in., and 2 ft. 7 in.?

12. What is the true discount on a 4 months' bill for £151 7*s*., drawn on January 1st, and discounted on the 20th of February, at $4\frac{1}{2}$ per cent.?

13. An article which sold for 50 guineas caused a loss of $5\frac{1}{2}$ per cent., what should it have fetched, to produce $7\frac{1}{2}$ per cent. profit?

14. If the rate of exchange between Amsterdam and Paris be 21·1 francs for 10 florins, and between London and Amsterdam be 12·15 florins for £1 sterling, what is the rate of exchange between London and Paris?

R.

1. State two numbers between 1 and 100, such that the first has exact square and cube roots, and the second has exact square and fourth roots.

2. If a cubic foot of air weighs $1\frac{1}{4}$ oz., what is the weight of air contained in a room 25 ft. 6 in. long, 16 ft. broad, and 10 ft. 9 in. high?

3. A cubical block contains 24 ft. 1403 in.; find its edge, and the area of its 6 sides.

4. Find the value of $\sqrt{23\frac{29}{49}}$; and of $\sqrt[3]{163\frac{5}{16}}$ to two places of decimals.

5. What is the extent of surface of a covered box, of which the dimensions are 5 ft. 10 in., 3 ft. 6 in., and 7 ft. 2 in.?

6. If the painting of a room 9 ft. 6 in. high, 15 ft. 3 in. long, and 10 ft. broad, come to £5 7*s.* 6*d.*; what will be the expense of painting another of which the length, breadth, and height are 18 ft. 2 in., 11 ft. 7 in., and 12 ft. 8 in.?

7. If 5000 rupees produce £505 4*s.* 2*d.*, what is the rate of exchange between England and the East Indies?

8. If by selling goods at £2 5*s.* I lose 15 per cent., what would be the loss or gain per cent. if the price were £2 18*s.* 6*d.*?

9. The sides of two square pieces of ground are 360 yds. and 160 yds.; find the value of the latter, if the former be worth £300.

10. Railway shares which were purchased at a premium of 50 per cent. and sold at a discount of $25\frac{1}{2}$ per cent., produce a loss of £7550; how much money was invested?

11. The length of a rectangular field is 12 chains 35 links, and the breadth is 10 chains 75 links; find the number of acres contained.

12. A square field contains 9 a. 0 r. 4 p.; find the length of its side in chains.

13. If $\frac{1}{3}$ of a debt be due in 3 months, $\frac{1}{4}$ in 4 months, $\frac{1}{5}$ in 6 months, and the remainder in 10 months; what is the equated time of payment?

14. Explain the meaning of the term "Fractional Quotient."

S.

1. A person employs 6 men for 5 days at 8 hours each, and 5 women for 6 days at 10 hours, at the respective wages of 4*d.*, and $2\frac{1}{2}d.$ per hour; how much must he pay them?

2. Find the number of men in a side of a square, if when drawn up in rank and file they number 81 by 36.

3. Reduce to the simplest form ($\frac{3}{8}$ of $\frac{7}{9}$ of $1\frac{1}{11}$) $\div$ ($2\frac{1}{3}$ of $3\frac{4}{7}$).

4. The discount on £153 due half a year hence is £3, what is the rate of interest?

5. In what time will £750 amount to £918 15s. at $4\frac{1}{2}$ per cent. Simple Interest?

6. How many years' purchase should be paid for freehold property to produce $5\frac{1}{2}$ per cent.?

7. What is the value of a perpetual annuity of £60, reckoning money worth 4 per cent.?

8. A testator bequeaths £500 to *A*, £300 to *B*, and £450 to *C*, but his estate only produces £600; find each man's share.

9. A cubic foot of water weighs 1000 oz.; what is the weight of water in a full cistern, the dimensions of which are 7 ft. 6 in., 5 ft. 2 in., and 3 ft.?

10. The mercury in a barometer rises uniformly from 29·15 to 30·73 in 12 days; find the ratio of the daily rise to the average height.

11. A square field has a diagonal 160 yards long; find the area of the field*.

12. If a solid foot of gold weigh 1260 lb. 8 oz., and a solid foot of cork weigh 15 lb. Troy; how much cork will weigh as much as 1 inch of gold?

13. A cubical mass of metal, of which the edge is 3·35 inches in length, is drawn out into a wire, of which the area of a section is ·561125 square inches; find the length of the wire.

14. Shew how to make any number a complete 2nd, 3rd, 4th, &c. power. Ex. Find a multiplier which shall make 75 a perfect cube.

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* See Appendix, Note to Art. *Ratio*.

SCALES OF NOTATION.

199. It will often have been observed that each figure in any number has *ten* times the value that it would have had, if it had been one place to the right. The selection of this number *ten* has arisen from its being the number of fingers on the hands; and hence has arisen the use of the term *digits*, for the Arabic figures, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Hence 10 is called the Radix of the common Scale* of Notation, and the scale itself is called the *denary* scale, from the Latin distributive numeral DENI, *in tens*.

If any other number be chosen as the radix of a scale, the new scale derives its name from the corresponding Latin term.

Thus, if the radix be	2,	the scale is called the	*Binary*.
,,	3	,,	*Ternary*.
,,	4	,,	*Quaternary*.
,,	5	,,	*Quinary*.
,,	6	,,	*Senary*.
,,	7	,,	*Septenary*.
,,	8	,,	*Octenary*.
,,	9	,,	*Nonary*.
,,	10	,,	*Denary*.
,,	11	,,	*Undenary*.
,,	12	,,	*Duodenary*.

In the two latter scales, since 11 and 12 digits are required, therefore the letters *t* and *e* are used to indicate *ten* and *eleven*. Multiplication in the duodenary scale is really *Duodecimals*.

200. We have to shew how to convert any number in the common scale into another scale; and conversely.

Now it will be easily seen that any number, as 5372, in the common scale, can be written in the following algebraical form,

$$5000+300+70+2, \text{ or } 5\times10^3+3\times10^2+7\times10+2. \qquad \text{(I.)}$$

* The term Scale has here the meaning of the word SCALA, a ladder, whereby we, as it were, *ascend* from lower to higher numbers by steps of 10.

And if 8 were the radix of the scale in which this said number 5372 was arranged, the digits of which it is composed would be different; and if a, b, c, d represented these new digits, which I do not yet know, I should have

$$a \times 8^3 + b \times 8^2 + c \times 8 + d = 5372. \qquad \text{(II.)}$$

Now, in series (I.), we may observe that if we divide by the radix 10, we shall have as quotient $5 \times 10^2 + 3 \times 10 + 7 + \frac{2}{10}$; *i. e.* $\frac{2}{10}$ is the frac^{l} part of the quot^{t}, or as we commonly express it, 2 is the rem^{r}; *i. e.* the last digit of a number in scale 10 is found by dividing the entire number by the radix 10. Also, if we continue successively to divide the quot^{t} $5 \times 10^2 + 3 \times 10 + 7$ by 10, we shall get as rem^{rs} 7, 3, and 5; *i.e.* the successive digits from right to left are obtained by dividing by the radix, until there be no dividend left. So also, if we divide the left-hand side of (II.) by the new radix 8, the quotient is $a \times 8^2 + b \times 8 + c + \frac{d}{8}$ or the remainder d is the last digit, as 2 was, just above. Now the two sides of equation (II.) have of course precisely the same value, and since when I divide the *left*-hand side of (II.) by 8, I obtain the last digit in scale 8, therefore I shall obtain the same digit by dividing the *right*-hand side 5372 by 8.

$8\,)\,\underline{a \times 8^3 + b \times 8^2 + c \times 8 + d}$			$8\,)\,\underline{5372}$		
$8\,)\,\underline{a \times 8^2 + b \times 8 + c}$	$+\frac{d}{8}$ or rem^{r} d		$8\,)\,\underline{671}$	$-\frac{4}{8}$ or rem^{r}	4
$8\,)\,\underline{a \times 8 + b}$	$+\frac{c}{8}$,,	c	$8\,)\,\underline{83}$	$-\frac{7}{8}$,,	7
$8\,)\,\underline{a}$	$+\frac{b}{8}$,,	b	$8\,)\,\underline{10}$	$-\frac{3}{8}$,,	3
	$\frac{a}{8}$,,	a	$8\,)\,\underline{1}$	$-\frac{2}{8}$,,	2
			$\underline{0}$	$-\frac{1}{8}$,,	1

Placing the two quantities in equation (II.) in parallel columns, and dividing both by 8, I obtain as quotients, $a\times 8^2+b\times 8+c+\frac{d}{8}$, and $671\frac{4}{8}$. Hence the quotients $a\times 8^2+b\times 8+c$, and 671 are equal, and the rem^{rs} d and 4 are equal; *i.e.* the first of the new digits to the right is 4. The second division by 8 gives equal quotients, viz. $a\times 8+b=83$, and a rem^r $c=7$. The next gives quotients a and 10 respectively, and a rem^r $b=3$. Hence the values of a, b, c, d, are 10, 3, 7, 4; but since in the scale of 8 there can be no single number $a=10$, I therefore divide the 10 still further, till I obtain *two* more digits, 2 and 1, in place of the 10, and the entire number of digits will be 1, 2, 3, 7, 4, &c.; and it now appears that there will be 5 figures in the number 5372 when expressed in the scale of 8, and the number will be represented by 12374.

And we can reconvert this number into the denary scale thus; it equals

$$\begin{aligned}&1\times 8^4+2\times 8^3+3\times 8^2+7\times 8+4 \qquad \text{(III.)}\\&=4096+1024+192+56+4\\&=5372,\text{ as before.}\end{aligned}$$

Hence we learn that if we have to transfer a number from the denary scale to any other scale of notation, we have only to divide the proposed number by the new radix, till there is no integral quotient. The successive remainders will be the new digits, the first one filling the units' place.

The process of reconverting from any of these new scales to the denary is shown in (III.), where the number 12374 in the octenary scale is restored to the denary, giving 5372.

The fact that 5372 in the scale of 10 is equal to 12374 in the scale of 8 is thus briefly exhibited;

$$(5372)_{10}=(12374)_8.$$

201. We have now seen how to convert from the denary scale into any other scale, and from any scale into the denary. But if we wish to change from any one of the new scales into any other one, as for example, from the scale of 5 to that of 7, we must first change from the quinary to the denary, and then from the denary to the septenary.

Ex. **Change 34201 from the quinary scale to the septenary.**

First, converting $(34201)_5$ into the denary scale, I have

$$\begin{aligned}(34201)_5&=3\times5^4+4\times5^3+2\times5^2+0\times5+1\\&=3\times625+4\times125+2\times25+1\\&=1875+500+50+1\\&=(2426)_{10}\qquad\qquad\text{(IV.)}\end{aligned}$$

$$\begin{array}{r}7\,\underline{)\,2426}\\7\,\underline{)\,346}\ \cdot\ 4\\7\,\underline{)\,49}\ \cdot\ 3\\7\,\underline{)\,7}\ \cdot\ 0\\7\,\underline{)1}\ \cdot\ 0\\\underline{0}\ \cdot\ 1\end{array}$$

Now, converting 2426 into the septenary scale, according to (200), as in the margin, we have as the new digits, 1, 0, 0, 3, 4, or $(2426)_{10}=(10034)_7$. And this equality may be proved by the reconversion of $(10034)_7$ into the denary scale; thus

$$\begin{aligned}(10034)_7&=1\times7^4+0\times7^3+0\times7^2+3\times7+4\\&=2401+21+4\\&=(2426)_{10}.\end{aligned}$$

Hence, we have $(34201)_5=(2426)_{10}=(10034)_7$.

Exs. 66.

1. **Change 345 from the common or denary scale to the binary.**
2. **„ 10101 from the binary to the denary.**
3. **„ 1375 „ denary „ ternary.**
4. **„ 89t4 „ undenary „ quinary.**
5. **„ 14328 „ nonary „ septenary.**
6. **„ 7854 „ denary „ duodenary.**
7. **„ 3t45 „ undenary „ binary.**
8. **„ 11111 „ binary „ denary.**
9. **„ 4444 „ duodenary „ quinary.**
10. **„ 21021 „ ternary „ senary.**
11. **Add together in the senary scale, 25341, 5423, 4021, 13450.**
12. **Find in the duodenary scale the value of 89t36e—4567t.**
13. **Express the result of 187t56 × t4789 in the duodenary scale.**
14. **Find the quotient of 18763÷456 in the nonary scale.**
15. **Find the area of a floor 15 ft. 11 in. by 7 ft. 9 in., by multiplying in the duodenary scale.**

202. It is a very good exercise for pupils to be able, without using any specific rules, to work examples which are generally wrought by rules; or, as it is sometimes expressed, to work them according to the principles of common sense, of course employing the four Simple and Compound Rules, *and* Reduction.

The most favourable examples for this exercise are those generally wrought by the Rules given in Simple and Compound Proportion. But here, as in (133), I will not assume proportionality in the quantities involved.

Ex. I. If 9 men earn £15 10*s*. in 3 months, or 13 weeks, what does each man earn per week?

If in 13 weeks 9 men earn 310*s*.

then, in 13 weeks 1 man earns $\frac{310}{9}s.$

or in 1 week $\frac{1}{13}$th of $\frac{310}{9}s.$

or $\frac{310}{117}s.$ or $2s.\ 7\frac{31}{39}d.$

Ex. II. If £240 be paid for bread sufficient to serve 49 persons for 18 months, when wheat is 48*s*. per quarter; how long will £235 find bread for 92 persons, when wheat is 56*s*. per quarter?

At 48*s*. per qr. £240 will serve 49 persons for 18 months,

∴	,,	£1	,,	1 person for $\frac{49\times18}{240}$ months,
and at 1*s*. per qr.		£1	,,	1 person 48 times $\frac{49\times18}{240}$ m.
Hence, at 56*s*. per qr.		£1	,,	1 person $\frac{48}{56}\times\frac{49\times18}{240}$ months;
∴	,,	£235	,,	92 persons $\frac{235}{92}\times\frac{48}{56}\times\frac{49\times18}{240}$ m.

or, $8\frac{17}{368}$ months.

This question worked as a Compound Proportion Example, according to (140), would furnish the following statement.

$$\left.\begin{array}{l}£240\\ 92\text{ men}\\ 56s.\end{array}\right. : \left.\begin{array}{l}£235\\ 49\text{ men}\\ 48s.\end{array}\right. :: 18\text{ months.}$$

and the fourth term would $=\frac{235\times49\times48\times18}{240\times92\times56}$ months,

the same as before.

Ex. III. Find the present worth of £169 18*s*. 4*d*., due 15 months hence, at 5 per cent.

Here £100 in 15 months at 5 per cent. produces £6 5*s*., or amounts to £106 5*s*.

i. e. the present worth of £106¼ is £100,

and ∴ of £1 is $£\dfrac{100}{106\frac{1}{4}}$,

and ∴ of £169 18s. 4d., or of £169$\frac{11}{12}$, is $\dfrac{169\frac{11}{12}\times 100}{106\frac{1}{4}}$ £,

the result which would be obtained by a statement, such as in (149).

Ex. IV. If 7 men can do a piece of work in 15$\frac{1}{2}$ days, in what time will 8 men and 7 boys do the same, reckoning a boy's labour worth $\frac{5}{8}$ that of a man?

$$7 \text{ boys} = \frac{5}{8} \text{ of } 7 \text{ men};$$

$$\therefore 8 \text{ men and } 7 \text{ boys} = \left(8+\frac{5}{8}\text{ of } 7\right)\text{ men}, = (8+4\tfrac{3}{8})\text{ men} = 12\tfrac{3}{8}\text{ men}.$$

Now, if 7 men do the work in 15$\frac{1}{2}$ days,

1 man will do it in 15$\frac{1}{2}$ × 7 days,

$$\therefore 12\tfrac{3}{8}\text{ men} \quad ,, \quad \text{in } \frac{15\frac{1}{2}\times 7}{12\frac{3}{8}} \text{ days},$$

$$\text{or, in } \frac{\frac{33}{2}\times 7}{\frac{99}{8}} \text{ days},$$

$$\text{or, in } \frac{\cancel{33}}{\cancel{2}}\times\frac{7}{1}\times\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{99}}} \text{ days, or } \frac{28}{3} \text{ days, or } 9\tfrac{1}{3} \text{ days}.$$

Exs. 68 A.

EASY MISCELLANEOUS EXAMPLES.

1. In £100 how many piastres at 3*s*. 7*d*. each?

2. What will be the expense of carpeting a room $23\frac{3}{4}$ feet long, and $16\frac{3}{4}$ feet wide, at 2*s*. 9*d*. per square yard?

3. Express the sum of the fractions $1\frac{19}{21}$, $\frac{3}{8}$ of $\frac{11}{14}$, and $\dfrac{4}{5\frac{1}{10}}$ by a fraction in its lowest terms.

4. What sum will discharge a debt of £56 5*s*., an abatement of 15 per cent. being made for present payment?

5. What will the carpeting of a room $16\frac{1}{2}$ feet square amount to, at 4*s*. $10\frac{1}{2}$*d*. a square yard?

6. If a brick be 9 inches long, 4 wide, and 3 thick, how many will be required to build a wall 1 foot 10 inches thick, 100 yards long, and $4\frac{1}{2}$ yards high?

7. An estate of 750 acres, which pays an average rent of £1 12*s*. 6*d*. an acre, is burdened with a mortgage of £2500, for which interest is paid at the rate of 4 per cent. per annum; what is the clear rental of the estate?

8. Reduce $\frac{3}{5}$ of $(6\frac{2}{3}+2\frac{1}{3})$, and $\dfrac{2\frac{1}{4}-\frac{3}{8}\text{ of }1\frac{5}{6}}{\frac{1}{6}\text{ of }3\frac{1}{3}+\frac{11}{36}}$ to their simplest forms.

9. *A*, who travels $3\frac{1}{4}$ miles an hour, starts $2\frac{1}{3}$ hours before *B*, who travels the same road at the rate of $4\frac{1}{3}$ miles an hour; when will *B* overtake *A*?

10. Extract the square root of 81144064 and of $\frac{3}{4}$ to three places of decimals.

11. Add together $\frac{5}{14}\times\frac{11}{15}$ of a guinea, $\frac{5}{8}$ of a shilling, and $\frac{1}{12}$ of half-a-crown.

12. Reduce $\frac{3}{5}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards to the fraction and decimal of a furlong.

13. Find the breadth of a room, the length of which is $17\frac{1}{4}$ feet, and the area $250\frac{1}{8}$ sq. feet.

14. How much money must be invested in the 3 per cents. at 84 to produce an annual income of £150?

15. What fraction of £3 10*s*. is £2 5*s*. 6*d*.? and reduce the result to a decimal.

16. How many cubic feet and inches are there in a solid, whose breadth is 9 feet 3 inches, length 11 feet 5 inches, and height 3 feet 2 inches?

17. Find the cost of 2627 sacks at 7*s*. 8¼*d*. per sack.

18. Find the price of a piece of wainscoting 7 ft. 9 in. long, and 9 ft. 3 inches broad, at 2*s*. per ft.

19. What ready money will discharge a debt of £85 due 5 months hence, if an abatement be made at the rate of 13*s*. 4*d*. per cent. per month?

20. What is land let for per acre, when the rent received for 56 acres, 3 roods, 17 perches, is £85 5*s*. 8¼*d*.?

21. What length of carpet ¾ yds. wide will cover a room whose length is 42 ft. 5 in. and breadth 31¾ ft.?

22. How much money must a person invest in the 3 per cents. at 90⅜ to produce a half-yearly income of £50?

23. A cubic foot of water weighs 1000 ounces avoirdupois; what is the weight of a cubic inch in ounces?

24. Goods are bought for £193 12*s*. and sold for £217 16*s*.; what was the gain per cent.?

25. If 6 horses in 2 days of 12 hours each plough 17 acres, how many roods will 2 horses plough in 8 hours?

26. Find the G. C. M. of 36·595 and 57·980.

27. Extract the square roots of 202500 and ·000576.

28. If the length of the year be taken at 365¼ days, instead of 365·242264 days, which is the true length, what will the error amount to in a century?

29. Bought a quantity of goods for £150 ready money, and sold them again for £200 payable ¾ of a year hence, what was the gain in ready money, allowing discount at 4½ per cent.?

30. If discount be treated as interest, find the gain in the last question.

31. If the expense of 3 persons on a tour of 5 months be £315, how many persons will spend £378 during a tour of 9 months?

32. If 2 tons, 3 cwt. 3 qrs. be bought for £112 10*s*., at what rate per cwt. must the article be sold, so as to clear 50 per cent.?

33. How much stock must be bought at 88⅓ per cent. in order that by selling out when the stocks are at 90¼, twenty guineas may be gained?

34. Find the value of $\frac{3}{7}$ of $1\frac{2}{5}$ of $12\frac{1}{2}$, and divide the result by $13\frac{1}{4}$.

35. Find the value of $\cdot 2\dot{7}$ of £1, and reduce 13 lbs. 15 oz. to the *decimal of a* cwt.

36. What sum of money laid out in the 3 per cents. at $89\frac{1}{2}$ will give an income of £50?

37. Add together 3·14159, 16·321, ·02, 4·59241; and divide the sum by 7500.

38. The weight of a cubic foot of water is 1000 ounces; find the weight of water necessary to fill a cistern whose length is 4 ft. 6 in., breadth 3 ft., and depth 4 ft.

39. Find the difference between the amounts at simple and compound interest, of £895 16*s.* for 2 years at $3\frac{1}{2}$ per cent.

40. What will be the expense of boarding the floor of a room, at 2*s.* $1\frac{1}{2}$*d.* per square yard, the length of the room being 21 ft. 10 in. and the breadth 18 ft. 4 in.?

41. What is the expense of a diagonal drain in a field 100 yds. long, and 80 yds. broad, at 1*s.* 3*d.* per yd.

42. At what rate per cent. will £625 amount to £765 12*s.* 6*d.* in 5 years, simple interest?

43. What fraction of 3 half-crowns is $3\frac{1}{3}$*s.*; and what decimal of £1 is $\frac{2}{7}$ of 2 guineas?

44. A person earns £175 a year, and pays an income-tax of 14*d.* in the £; what is his net income?

45. If 4 acres of land will maintain 6 horses for 8 weeks, for how long will 10 acres maintain 12 horses?

46. If 3 lbs. of tea be worth 8 lbs. of coffee, and 3 lbs. of coffee be worth 10 lbs. of sugar, how many lbs. of sugar can be got for 15 lbs. of tea?

47. If I lose $1\frac{1}{3}$*d.* in 3*s.* 4*d.* how much is that per cent.?

48. Find the true and ordinary discount of £85 for 16 months at 5 per cent.

49. If 14 horses eat 56 bushels of oats in 16 days, how many days' consumption would there be in 120 bushels for 20 horses?

50. A person, who was owner of $\frac{2}{5}$ of a copper mine, sold $\frac{3}{4}$ of his share for £1800; what was the value of the whole mine?

51. At what rate per cent. will £956 amount to £1314 10*s.* in $7\frac{1}{3}$ years, simple interest?

52. If $\frac{3}{5}$ oz. (Avoirdupois) of an article cost $\frac{7}{8}$ of a shilling, what will $\frac{5}{8}$ lb. of the same cost?

53. If 850 men are besieged in a fortress, and have provisions only for 3 months, how many men must leave the fortress, so that the rest may have provisions for 5 months?

54. A gentleman bequeaths £12,000 to his three sons *A*, *B*, and *C*: to *A* a sum unknown, to *B* twice as much as to *A*, and to *C* as much as to *A* and to *B*. What had each son?

55. If 8 cannon in 1 day expend 48 barrels of powder, in how many days will 24 cannon expend 3168 barrels?

56. If the carriage of 5 cwt. 3 qrs. for 150 miles cost £3 9*s.*, what must be paid for the carriage of 7 cwt. 2 qrs. 25 lbs. for 64 miles at the same rate?

57. In what time will £547 15*s.* amount to £629 18*s.* 3*d.* at 5 per cent. per annum?

58. What principal put out to simple interest for 5 years will amount to £570 16*s.* 6*d.* at 4 per cent. per annum?

59. Find the true discount and present worth of £187 5*s.* 6*d.* due at the end of 3 yrs. at $4\frac{1}{2}$ per cent.

60. Find the smallest number that can be divided by 8, 12, 18, 24, 27, without leaving any remainders.

61. What is the least number which taken from 387564 leaves a remainder divisible by 70; and what is the least number which added to 387564 gives a sum divisible by 65?

62. Simplify the following expressions:

$$37\frac{1}{2}-\frac{3}{4} \text{ of } 15\text{; and } \left(37\frac{1}{2}-\frac{3}{4}\right) \text{ of } 15.$$

63. In 40 guineas how many sevenths of £1, and how many fifteenths of 1 shilling?

64. What fraction added to the sum of $1\frac{2}{3}$, $\frac{3}{4}$ of $1\frac{5}{8}$, and $7\frac{1}{2}$, will amount to $17\frac{5}{8}$?

65. State what is necessary in comparing the magnitude of different fractions; and hence find the comparative value of $\frac{11}{12}$, $\frac{15}{16}$, $\frac{20}{21}$, $\frac{24}{25}$.

66. Arrange in order of magnitude, beginning with the lowest, the fractions $\frac{3}{4}$, $\frac{4}{5}$, $\frac{3+4}{4+5}$.

67. Shew that the simple fraction which expresses the value of $\frac{1}{2}$ of $\frac{\frac{2}{7}}{\frac{1}{14}}$ of $\frac{\frac{3}{5}}{1+\frac{2}{3}}+\frac{3}{7}$ of $\frac{2}{9}$ is equal to $\frac{1}{5}$ of $\frac{2}{7}$ of $\frac{1}{3}$ of $30\frac{3}{5}$.

68. What fraction having 12 for its denr is equal to $\frac{5\frac{1}{3}}{16}$?

69. Shew, without assuming any *Rule*, that the area of the rectangle whose adjacent sides are $5\frac{1}{6}$ ft. and $3\frac{3}{4}$ ft. $=19\frac{9}{24}$ sq. ft.

70. Shew by a diagram that $30\frac{1}{4}$ sq. yds. $=1$ sq. pole.

71. Shew also by diagrams that $(\frac{1}{2}\text{ ft.})^2=\frac{1}{4}$ sq. ft., and that $\frac{1}{6}$ ft. $\times\frac{1}{5}$ ft. $=\frac{1}{30}$ sq. ft.

72. Shew by a diagram that there are 640 acres in a square mile.

73. From a square containing 1 acre there are subtracted 16 rectangular plots, each 25·2 yds. long, and 10·5 yds. broad; how much is left?

74. Assuming that $(\sqrt{5}-\sqrt{3})\times(\sqrt{5}+\sqrt{3})=5-3$, and similarly of any other pairs of numbers, find the value to three places of decimals of the quantities $\frac{1}{\sqrt{5}-\sqrt{3}}$, $\frac{\sqrt{3}}{\sqrt{5}-1}$, and $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$.

75. I pay away $\frac{1}{6}$ of my money, and $\frac{3}{8}$ of the remainder; then $\frac{1}{4}$ of what remains, and lastly, $\frac{1}{10}$ of the original sum; what were the fractional parts remaining after the 3rd and the last payments?

76. Divide £1500 in the ratio of 7, 10, and 13.

77. If 35 bushels of oats are eaten in 17 days by 15 horses, how many days will $115\frac{1}{2}$ bushels last 102 horses?

78. How many cubes, whose edges are 15 inches, could be filled from a full cistern $4\frac{1}{2}$ ft. long, $3\frac{1}{3}$ ft. wide, and $2\frac{1}{2}$ ft. deep?

79. *A* and *B* do a piece of work in 7 days, which *A* alone could accomplish in $35\frac{1}{2}$ days; in what time would *B* do it by himself?

80. What must be the poundage so that a person whose rent is £130 shall pay a rate of £8 2*s.* 6*d.*?

81. What is the superficies of a cube whose edge is $3\frac{1}{4}$ inches?

82. Find the volume of the same cube.

83. What number multiplied by $\frac{3}{5}$ of $\frac{1}{4}$ of $7\frac{1}{2}$ will give as a result $3\frac{1}{12}$?

84. The product of two numbers is 40, and the square of the less is 1296; required the numbers.

85. What would 5*s.* 10*d.* per day come to in a year?

86. What is the price of 4108 articles at 2*s.* $11\frac{1}{4}$*d.*?

87. Find the cost of 1003 articles at 19*s.* $11\frac{3}{4}$*d.*?

88. At what rate per cent. will £165 gain £22 5*s.* 6*d.* in 3 years?

89. In what time will £55 12*s.* 6*d.* amount to £65 12*s.* 9*d.* at 3 per cent. per annum?

90. Required the rate per cent. at which £250 will amount to £267 3*s.* 9*d.* in $2\frac{1}{2}$ years, Simple Interest.

91. What is the simple interest of £265 10*s.* for 5 years 15 weeks, at 3 per cent., reckoning 52 weeks a year?

92. If linen be bought at 1*s.* 5*d.* a Flemish ell, and sold at 2*s.* 4*d.* a yard, what is the gain per cent.?

93. A house was bought for £550, and sold for £570; what was the gain per cent.?

94. A grazier bought 8 oxen for £97, in addition to which they cost him in keep $7\frac{1}{2}$ per cent. of this money. He sells half of them for

£15 10*s*. each; at what price each must he sell the rest, to gain 15 per cent. on his entire outlay?

95. If 1*s*. gain $2\frac{1}{2}d$. what is the gain per cent.?

96. Find the value of 8888 articles at £1 0*s*. 8*d*.

97. What is the worth of 3303 articles at £1 19*s*. $7\frac{1}{2}d$.?

98. Find the discount of £415, due 9 months hence, at 5 per cent. Simple Interest?

99. What is the present worth of £1281 5*s*., due 6 months hence, at 5 per cent. per annum?

100. What sum will amount to £306 5*s*. in 9 years, at $2\frac{1}{4}$ per cent. Simple Interest?

101. Find the value of 5555 articles at 1*s*. $6\frac{1}{4}d$.

102. A man in walking takes ·88 yds. each step, and is 5 hrs. 24 min. in going a journey of 12·15 miles; required the number of times he steps in a minute.

103. If an ounce of tea cost ·175 of 1*s*. 8*d*., how much can be bought for 8 guineas?

104. From ·725 crowns take 2·25 shillings, and reduce the remainder to the decimal of half-a-crown.

105. What fraction of an English ell is $\frac{4}{9}$ of an inch?

106. From $22\frac{2}{5}$ take $11\frac{1}{3}$; and multiply the remainder by 5 times $7\frac{1}{3}$.

107. If 2 cwt. of coal cost 1*s*. $11\frac{1}{2}d$., what will $12\frac{3}{5}$ tons cost?

108. A person in going a journey of 40·3 miles performs the first 14 miles in 1·7 hours; at what rate must he go the other part, so as to complete the whole journey in 4·2 hours?

109. Which is the greatest, and which the least of the following fractions, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{7}{9}$, $\frac{8}{11}$?

110. Find the value of $16\frac{3}{4}+2\frac{7}{24}+22\frac{7}{30}+17\frac{5}{6}+3\frac{1}{5}$.

111. Divide the difference between $1\frac{3}{5}$ of $5\frac{1}{2}$ and $1\frac{2}{3}$ of $6\frac{2}{3}$ by the product of $\frac{3}{4}$ and 9.

112. What fraction of a French ell is $\frac{5}{9}$ of an English ell?

113. Divide $15\frac{3}{8}$ equally amongst 9 persons.

114. Reduce $\frac{15}{16}$ to a complex fraction whose denominator is $39\frac{1}{4}$.

115. How many twelfths are there in $29\frac{1}{6}$?

116. The income-tax is 15*d*. in the pound; what is that per cent.?

117. By selling coats at 19*s*. 3*d*. each, 5 per cent. is lost; what should they be sold at, so as to gain 10 per cent.?

118. If ribbon be bought for $2\frac{1}{4}d.$ per yard, and sold at $5\frac{1}{4}d.$, what is the gain per cent.?

119. If a ship be bought for £7331 5*s.* and sold for £8797 10*s.*, what was the gain per cent.?

120. What is the value of £3750 stocks in the 3 per cents., at $87\frac{5}{8}$?

121. How much money must be invested in the $4\frac{1}{2}$ per cents. at $99\frac{3}{8}$, so as to yield an annual income of £360?

122. What amount of $3\frac{1}{4}$ per cent. Stock would yield £81 5*s.* per annum?

123. Find what decimal multiplied by 72 will express the sum of $\frac{1}{8}$, $\frac{7}{18}$, and $4\frac{1}{4}$.

124. Find the value of 4107 articles at £9 9*s.* 9*d.*

125. What is the exact discount on £5 12*s.* 9*d.*, due 12 months hence, at $2\frac{1}{2}$ per cent.?

126. What is the difference between the interest and discount on £75 18*s.* 9*d.* for 3 months at 5 per cent.?

127. What is the gain per cent. by buying eggs at 15 for a shilling, and selling them at a penny each?

128. What should an article which cost 8*s.* 4*d.* be sold for, so as to gain $15\frac{1}{2}$ per cent.?

129. Find the cost of 4107 articles at £1 0*s.* $11\frac{1}{2}d.$ each.

130. A cow was bought for £10 15*s.* and sold at a loss of $7\frac{1}{2}$ per cent.; what did she fetch?

131. If 1*s.* gain $1\frac{1}{3}d.$, what is the gain per cent.?

132. A court-yard is 30 ft. 10 in. long, and 12 ft. 11 in. broad; what will the paving come to at 16*d.* per sq. ft.?

133. Of what amount must I take $\frac{11}{12}$ to produce a guinea?

134. In 1851 the population of a town was 400,000; supposing at every decennial census it be found to have increased 20 per cent. on the preceding one, what will be the population in the year 1901?

135. A man builds 12 cottages at a cost of £925, and lets them at £5 10*s.* per annum. One on an average is always vacant; and of the rents that are due he loses $7\frac{1}{2}$ per cent. If the yearly repairs cost $\frac{3}{8}$ per cent. of the outlay, what rate of interest does he obtain for his money?

136. What is the commission on £4695 at 5*s.* 9*d.* per cent.?

137. What is the amount of ·0125 per cent. of a population of a million?

138. The duty on sales by auction was once 5 per cent. on the purchase-money, and an additional 5 per cent. on this duty; what was the total cost of an article knocked down at £51?

139. If 5*s.* 3*d.* gain 2*s.* $1\frac{1}{2}$*d.*, what is the gain per cent.?

140. Find the cost of repairing 7 m. 7 fur. 35 p. of a road, at £3 17*s.* $4\frac{1}{2}$*d.* per mile.

141. Of what fraction must I take $\frac{3}{5}$ to receive $\frac{11}{12}$?

142. A banker receives in deposits £16250 a year, for which he pays $3\frac{1}{2}$ per cent. yearly, and he lets it out at 5 per cent. payable half-yearly; what will be his annual profit?

143. The price of ·0625 lbs. of coffee being $\cdot 458\dot{3}$ shillings, what is the cost of ·075 of a ton?

144. How many bricks will be required for a wall 25 yds. long, 15 ft. high, and 1 ft. $10\frac{1}{2}$ in. thick, each brick being 9 in. long, $4\frac{1}{2}$ in. wide, and 3 in. thick?

145. If the papering of a room 28 yds. round and 4 yds. high cost £5 10*s.*, what will be the cost of similarly papering another room, 24 yds. round and $3\frac{1}{2}$ yds. high?

146. Find the equated time of payment of a debt, of which $\frac{1}{2}$ is due in 4 months, $\frac{1}{3}$ in 7 months, and the rest in 8 months.

147. Find the side of a square area, the paving of which at 4*s.* 2*d.* per sq. yd. cost £56 0*s.* 10*d.*

148. What must be paid for painting the outside of a box $5\frac{1}{2}$ ft. long, $3\frac{3}{4}$ ft. wide, and $2\frac{5}{8}$ ft. deep, at 1*s.* 8*d.* per sq. yard?

149. Extract the cube roots of 59319, and $4\frac{31}{4913}$.

150. A square field contains 15 a. 2 r. 20 p.; find its side in chains.

Exs. 67.

MISCELLANEOUS EXAMPLES.

1. WHAT will be the net value of a legacy of £333 6*s.* 8*d.*, after paying duty at the rate of 3 per cent.?

2. Two clocks point to 2 at the same instant; one loses 7 seconds, and the other gains 8 seconds in 24 hours; when will one be half an hour before the other, and what time will each clock then shew?

3. A bankrupt owes 3 creditors £10,000, 10,000 guineas, and 10,000 shillings respectively; but his property is worth only £7000: find how much in the pound he will pay, and how much each creditor will receive?

4. If 100 articles are bought at 3 a penny, and 100 more at 2 a penny; at what price must they be retailed so as to gain 25 per cent.?

5. A vessel containing 384 gallons is emptied by 3 taps; the first and second together empty it in 32 minutes; the first and third in 24 minutes; and the second and third in 16 minutes; how many gallons will each tap discharge in a minute?

6. How many square feet of board will be required to make a rectangular box, of which the length, breadth, and depth are $3\frac{1}{2}$ ft., $2\frac{1}{4}$ ft., and 1 ft. $2\frac{1}{4}$ in. respectively?

7. If a room be 32 ft. long, 26 ft. wide, and 14 ft. high, what will be the expense of papering it at 3*s.* per square yard, allowance being made for three windows, each 10 ft. by $6\frac{1}{4}$ ft., and two fire-places, each $9\frac{1}{2}$ ft. by 6 ft.?

8. What sum of money must be paid down, in order to receive £360 10*s.*, 2 yrs. hence, allowing $3\frac{1}{2}$ per cent. Compound Interest?

9. Find the simple fraction which expresses the value of $7+\sqrt{6\frac{1}{4}}$, when divided by $6\frac{1}{3}$ times $(3+\sqrt[3]{3\frac{3}{8}})$.

10. How many cubes, whose edges are $\frac{3}{4}$ in. long, can be contained in a box, of which the base is 18 sq. inches, and height $7\frac{1}{2}$ inches?

11. The volumes of spheres are in proportion to the cubes of their radii: the radii of two spheres are in the ratio of 4 to 5; and the weights of equal portions of the smaller and larger spheres are as 12 : 7; given that the weight of the smaller sphere is 256 lbs., find the weight of the larger one.

12. At what time between one and two o'clock will the hour and minute hands of a watch make an angle of 60 degrees with each other?

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13. If a pound Troy of English standard gold $\frac{11}{12}$ fine be worth £46 12*s*. 6*d*., what is the value of a coin weighing 7 dwts. 11 grs., in which 924 parts in 1000 are pure gold?

14. The wheels of a cart are $2\frac{1}{4}$ yards asunder, and the inner wheel describes the circumference of a circle of radius 20 yds.: find the difference of the paths of the wheels, having it given that the circumference of a circle $= 3{\cdot}1416$ times its diameter.

15. Two men are walking in the same direction, the distance between them at starting being 100 yds.; the first walks 45, and the second 49 yds. in 50 steps; how many steps will have been taken when they are together?

16. How much paper, $\frac{3}{4}$ yd. wide, will be sufficient to paper a room 22 feet 5 inches long, 12 feet 1 inch broad, and 11 feet 3 inches high? and how much will it cost at $4\frac{1}{2}$*d*. per yard?

17. A river 30 feet deep, and 200 yds. wide, is flowing at the rate of 4 miles an hour; find how many cubic feet of water run into the sea per minute; also the number of tons, supposing a cubic foot of water to weigh 1000 ounces.

18. A clock gains $3\frac{1}{2}$ minutes per day; how should its hands be placed at noon, that it may point out the true time at $7\frac{1}{2}$ in the evening?

19. A person performs $\frac{2}{7}$ths of a piece of work in 13 days; he then receives the assistance of another person, and the two together finish it in 6 days; in what time could each do the whole work by himself?

20. A tradesman buys goods for £189 15*s*. 6*d*., and sells them in 6 months for £253 0*s*. 8*d*. ready money: how much is that per cent. per annum profit?

21. If the value of £1 sterling varies from 25·15 francs to 26·75 francs, what is the variation in value of 100 guineas?

22. A person receives his rent, and after paying an income-tax of 7*d*. in the pound has £553 7*s*. 6*d*. left; what did he receive?

23. A field of $7\frac{1}{2}$ acres is planted with potatoes in rows; the distance between each row is 15 inches; how many yards of potatoes are there in the field?

24. Three masons, *A*, *B*, *C*, are to build a wall; *A* and *B* could together build it in 12 days, *B* and *C* in 20, and *A* and *C* in 15; in what time will they build it when they all work together?

25. If 3 miles, 4 furlongs, 93 yards be run in 6 minutes 4 seconds, how much is that short of the rate of a mile per minute?

26. Multiply $\dfrac{2\frac{3}{8}}{4\frac{3}{4}}$ by $\dfrac{2\frac{1}{52}}{13\frac{5}{13}}$, and divide $\sqrt{2\frac{1}{4}}$ by $\sqrt[3]{3\frac{3}{8}}$.

27. The comparative weights of coal and water are as 1·12 and 1; also a cubic foot of water weighs 1000 oz.; find the edge of a cubical block of coal which weighs 2000 tons.

28. The discount on £500 due 4 years hence is 250 marks; find the rate of interest.

29. Shew what factor is wanting in the number 32 to make it a perfect cube.

30. Required the least number which multiplied by 64 will make it a perfect 5th power.

31. *A* sets out from Cambridge to London ($51\frac{1}{2}$ miles) at the rate of 8 miles an hour, and *B* sets out at the same time from London to Cambridge at the rate of $9\frac{1}{2}$ miles an hour; at what distance from each place will they meet?

32. A tradesman marks his goods with two prices; one for ready money, and the other for credit of 6 months: what fixed proportion ought the two prices to bear to each other, allowing 5 per cent. Simple Interest?

33. What must be the least multiplier of the number 225, so that the product may be a perfect cube?

34. If the ratio of the diameter of a circle to its circumference be 113 : 355; and if the length of $\frac{1}{360}$th part of the earth's circumference be $69\frac{1}{12}$ miles, what is the earth's diameter?

35. How much per cent. is 14*s.* 6*d.* of £3 10*s.*?

36. *A* and *B* together can do a piece of work in 30 days; *B* by himself can perform the same in 70 days: in what time could *A* finish it by himself; and how much more of the work does *A* do than *B*?

37. The sun's longitude is increased by 360 degrees in 365 d. 5 h. 48 m.: what is his average daily motion?

38. Simplify the following expression, leaving one surd in the numerator of the resulting fraction:—

$$\sqrt{6^2-5^2}\times\frac{5}{\sqrt{6^2+5^2}}+\sqrt{6^2+5^2}\times\frac{5}{\sqrt{6^2-5^2}}.$$

39. *A* and *B* can do a piece of work together in 30 days; *A* does $\frac{1}{4}$ more than *B*; in what time can they do it separately?

40. Find the square root of the sum of the squares of ·2, ·4, ·6, ·86.

41. If an acre of land be bought for 6*d.* per foot, at what price per yard must it be sold to gain £136 2*s.* 6*d.*?

42. From a rectangular plank 1 foot broad and 2 inches thick, what length must be cut off to be worth 25*s.*, the value of the timber being 5*s.* per cubic foot?

43. Simplify the following expressions:—

$$1+\cfrac{2}{3+\cfrac{1}{4+\cfrac{1}{5\frac{2}{3}}}}. \qquad \frac{\sqrt{1+\frac{1}{3}} \div \sqrt{1-\frac{1}{5}}}{\sqrt{1+\frac{1}{3}} \times \sqrt{1-\frac{1}{5}}}.$$

44. Find the value of $3 \div \sqrt{10}$ to four places of decimals.

45. If the volume of a cylinder be obtained from this product, height × area of the base; find the area of the base of a cylinder whereof the volume is 1 cubic foot, and the height $7\frac{1}{2}$ inches.

46. Compare the volumes of two cylinders whose bases are in the ratio of 3 : 4, and altitudes as 5 : 6.

47. A person rents a piece of land for £120 a year. He lays out £625 in buying 50 bullocks. At the end of the year he sells them, having expended £12 10*s*. in labour. How much per head must he gain by them, in order to realize his rent and expenses, and 10 per cent. on his original outlay?

48. A cubical box contains 37 solid feet and 64 solid inches: find (1) the number of linear inches in the edge; (2) in the diagonal of each face; and (3) in the diagonal of the box.—(*See Appendix, Note to Art. Ratio.*)

49. A person finds that his net income, after deducting the income-tax of 7*d*. in the pound, is £233; find the amount of income.

50. A pile of cannon shot has a rectangular base, the sides of which contain 7 and 6 shot respectively; find the number of shot in the whole pile.

51. What is the unit of measurement when a mile is 160 units? Determine that common unit which will express 12960 minutes and 20160 minutes in the smallest possible integral numbers.

52. The areas of circles are proportional to the squares of their radii; find the ratio of the areas of two circles which have 2 feet and $\frac{1}{2}$ an inch as the values of their respective radii.

53. Find the ratio of the diameters of two circles, such that the area of one may $= 5\frac{4}{9}$ times that of the other.

54. Two cubical boxes have edges respectively $\frac{3}{4}$ inch, and 2 ft. 3 in.; find the ratio (1) of their surfaces, and (2) of their volumes.

55. A clock has its face marked so as to shew 24 hours in a day; and on a certain evening half an hour after sunset it was set at 24 o'clock. The morning following it was 8 min. past 4 by a common clock when it was 4 minutes past 8 by this clock. Find the time of sunset the previous evening.

56. Given that the ratio of the circumference of a circle to the diameter $= 3\cdot1416 : 1$; also that the length of an arc of a circle opposite to any angle at the centre is proportional to the degrees, &c. in that angle: find the length of the radius of that circle, in which an arc of 4000 miles is opposite to an angle containing 8·58 seconds.

57. The space through which a body falls in vacuo near the surface of the earth is proportional to the square of the number of seconds in the time of falling: if a body fall through 16·1 feet in the first second, find how far it will fall in 8 seconds, and in the ninth.

58. The time of oscillation of the pendulum of a clock $= 3\cdot1416 \times \sqrt{\frac{39\cdot2}{32\cdot2}}$ seconds, where 39·2 inches is the length of the seconds pendulum; find the alteration in the time of oscillation when the pendulum is lengthened $\frac{1}{20}$ of an inch. Find also how many seconds the clock will gain or lose in 24 hours.

59. Taking the same value as in the last question for the time of oscillation, where 32·2 is the measure of the force of the earth's attraction, find the alteration of that measure, if the length of the pendulum be increased by $\frac{1}{1000}$th part, and the time of oscillation be the same.

60. The difference between the year in the Julian calendar and the true year is ·007736 days; the Gregorian calendar corrects by omitting 3 days in 400 years: find how much error would have accumulated under that calendar, from A.D. 325 to A.D. 1848, and how soon the error will amount to a day.

61. In a block of wood, a hole is made 12 in. long and 1 sq. inch in section: the largest possible cylinder is placed in the hole; how much is unoccupied?—(*See question* 45.)

62. A cubical box, 1 inch high, is filled with water; 8 equal spheres of $\frac{1}{2}$ in. diam. are placed in it; what volume of water will remain,—having it given that the volume of a sphere $= \frac{4}{3} \times 3\cdot1416 \times$ the cube of the radius.

63. Will $\frac{21}{120}$ produce a circulating decimal?

64. A cube has an edge 2 ft. 6 in. long; find the ratio between the sum of the areas of the semicircles described on its edges, and the whole surface of the cube, having it given that the area of a circle $= 3\cdot1416 \times$ the square of the radius.

65. A wall is 15 ft. 8 in. long, and 11 ft. 6 in. broad, and has in it a door-way 6 ft. 3 in. by 2 ft. 4 in.; find the number of bricks of $165\frac{7}{12}$ solid inches contained in it, when the thickness is 11 inches.

66. If a sovereign weigh 5 dwts. $3\frac{1}{4}$ grs., 1 part out of 12 being copper, and the rest pure gold; find what fraction of a cubic inch the gold constitutes, having given that a cubic inch of water weighs 252·458 grains, and that gold is 19·362 times as heavy as water.

67. Reduce the following Arithmetical expression to its simplest form:

$$\left\{\left(\frac{9}{10}+2\frac{1}{3}\right)-\left(2\frac{1}{5}-1\frac{3}{4}\right)\right\}\times\left\{\left(5\frac{1}{4}+7\frac{2}{3}\right)\div 16\frac{1}{10}\right\}.$$

68. A cistern has 3 pipes, A, B, and C; A and B can fill it in 3 and 4 hours respectively; and C can empty it in 1 hour; if these pipes be opened in order at 1, 2, and 3 o'clock, find when the cistern will be empty.

69. A person sells out of the $3\frac{1}{2}$ per cents at $93\frac{1}{2}$ as much stock as produces £9350; at what price must the 4 per cents be, so that the above sum when invested in them shall produce an increase of £10 income?

70. If $\frac{1}{3}$ of a sheep be worth £$\frac{3}{4}$, and $\frac{2}{3}$ of a sheep be worth $\frac{1}{12}$ of an ox, what sum must be given for 50 oxen?

71. A garden walk, 4 ft. wide, is carried round a circular plot 3 yds. in diameter; find the price of gravelling the walk per foot, if the whole cost be 6*s*.

BOOK-KEEPING.

THE object of Book-keeping is to enable a merchant to ascertain

(1) How he stands with respect to all the persons with whom he has any money dealings.

(2) The money value of all that he possesses, and all that he owes, which are technically called his *Assets* and *Liabilities.* If the Assets exceed the Liabilities, he is said to be *solvent;* otherwise he is termed *insolvent.*

(3) To ascertain how, if he is insolvent, he has come to be so; and if not, in what manner his profits have been realized, so as to render him solvent, after all trade and domestic expenses have been paid.

This third account is termed the Profit and Loss account.

To ascertain (1), the merchant must have accounts open under the name of every *person* with whom he deals; and these are called *personal* accounts: and if he keeps his accounts only in this simple form, he can, by merely balancing these personal accounts, and taking an inventory of stock and cash, tell at any given time how he stands. And this system of Book-keeping is termed Single Entry.

But this will not tell him how he came into his present position; or shew upon what goods he has made or lost money. But if he produces (2), i.e. accounts of *things* bought or sold, paid or received, in addition to the accounts of *persons,* then he is enabled to tell how he stands with reference either to his original capital, both in goods and money, or to the state of that capital at any given time.

These accounts, being headed by the names of things and not of persons, are called *Nominal* Accounts. And it is the keeping of these, in addition to the Personal Accounts, which constitutes Double Entry.

The books required for book-keeping are (1) an Invoice-book; (2) a Day-book, or Waste-book; (3) a Cash-book; (4) a Ledger; (5) a Bill-book. This last is seldom used where the transactions are small. A memorandum of any transaction recorded in any one of these books is termed an Entry. A specimen of each of the above books will be given.

By Single Entry, every transaction comes direct from the books

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already mentioned into the Ledger; in each book the page of the Ledger is marked where the entry may be found, and in the Ledger the page or folio is also marked of the book from which it is brought. But in Double Entry there is generally used an additional book called the Journal, which is an intermediate book between those mentioned and the Ledger, so that nothing passes into the Ledger except through this book.

The transferring of the entries from these various books into the Ledger is termed *posting*.

Before proceeding to give specimens of these books, it will be advisable to call attention to a few preliminary principles, as to the mode of keepiug accounts.

All books are arranged with two sides, a *debtor* to the *left*, and a *creditor* to the *right*. These are briefly written Dr. and Cr.

Under any personal or nominal account, whatever is paid away, or parted with, by the person or name heading the account, is put to the Cr. side. And whatever has been received by the same, and which must therefore be accounted for, is Dr. In fact, we may say briefly, whatever is *in*, is Cr.; whatever is *out*, is Dr. When any item is thus put to the *credit* side of an account, the entry is preceded by the word "By." And when it is put to the debtor side of an account, it is preceded by the word "To."

Hence, if an article, as tea, is sold to a person on credit, there is an entry under his personal account, "Dr. To Tea;" and when he pays for it in cash, or bill (of which more will be said presently), there is an entry under the same account to his credit, "Cr. By Cash," or "By Bill;" and the two sides of the account balance.

But when the above goods, as tea, were sold to him, while he was the receiver, the article or articles delivered were payers of the goods; and under the nominal account of "Tea," there would be an entry "Cr. By Tea to A. B." And when this is paid, the nominal account headed "Cash" would have put under it the entry "Dr. To Cash from A. B." So that we have this arrangement; a Dr. entry to a personal account is balanced by a Cr. entry to a nominal account for goods: and a Cr. entry to a personal account for cash received is balanced by a Dr. entry to the nominal amount headed Cash.

Again, let the firm be buyers of tea from any merchant, then while under his personal account there is an entry "Cr. By Tea," there will be a corresponding Dr. entry under the nominal account headed "Tea." And when the tea is paid for by the firm, the merchant will be Cr. by *the amount*, and cash will be the Dr., since cash has received it.

Hence it is evident that if the posting be correct, all the entries on the Dr. side in the Ledger must exactly balance all the entries on the Cr. side. And this comparison of the totals of the two sets of Dr. and Cr. entries, for the purpose of testing the accuracy of the posting, is called a "Trial Balance," and is simply a trial, as to whether the entries have been correctly posted from the Journal, or the Day-book if journalized, into the Ledger.

All the above-mentioned Nominal accounts might have been classed under one head, viz. that of the firm, who manifestly are Drs. or Crs., according as each of their customers is Cr. or Dr.; and so a balance might be struck as before. But by the above classification, or division into several nominal accounts, each embracing but one class of goods, we are enabled to ascertain, not only *what* profits have been realized, but *how* they have been realized.

We will now describe the nature and use of

BILLS OF EXCHANGE.

A Bill is a promise to pay, usually at some bank in London, a certain sum mentioned on the bill, at the end of a time agreed upon, say 2, 3, or 6 months, and is drawn up on stamped paper in the following form.

LIVERPOOL, *Oct.* 27, 1860.

£56. 15*s.* 0*d.*

Three months after date pay to my order the sum of Fifty-six Pounds Fifteen Shillings for value received.

A. B.

To C. D. & Co.
Chesterfield.

Accepted payable at
E. T. & Co. Bankers.
London.
C.D.

Here C. D. owes the money to A. B. and by writing across the bill his name and that of the banker at whose bank it is made payable, he is said to *accept* the bill, and thus become responsible for its payment. A. B. is called the *drawer*, and C. D. the *acceptor*. A. B. who receives this bill writes his name on the back of it, if he parts with it, thereby making himself responsible to the person to whom he pays the bill, and he is then called the *indorser*; the person to whom he pays it, if his name is specially mentioned on the back of the bill, is called the first *indorsee*, and if he pays it to a third party, *he* must *indorse* it, i.e. write *his* name on the back also, and thus render himself responsible for the payment, if C. D. and A. B. fail to pay, or, as it is commonly termed, to *meet* it.

In any general statement of the business of C. D., commonly called a Balance Sheet, of which more will be said hereafter, all such bills which were accepted by C. D. and still unpaid or running, would be entered on the *debtor* side as "Bills payable." If in the course of his business he had received bills accepted by other parties, these would (subject to discount) be considered as cash, provided the acceptors were considered able to meet them: and they would be entered to his *credit* as "Bills receivable."

But such bills might also appear on the debtor side of his Balance Sheet, because he might have had some passing through his hands, accepted by parties that had since failed; and since he has indorsed them, he would be wholly, or in part, liable to be called upon for them. Hence in his Balance Sheet bills receivable in hand would be on his Cr. side, and bills receivable parted with, and not yet at maturity, would be on the Dr. side, so far as the acceptors are unable to meet them.

THE INVOICE-BOOK

This is used merely to record the invoices of all goods ordered into stock.

Specimen.

Chesterfield, *October* 27, 1860.

A. B.

Bought of C. D. and Co.

	£	s.	d.
4 Chests of Tea, each 56 lbs. at 3*s.* 4*d.* . . .	37	6	8
3 cwt. 3 qrs. of Sugar, at 56*s.* per cwt. . . .	10	10	0
	47	16	8

The above Invoice-book belongs to A. B., who is the buyer of the goods.

The more usual plan of keeping the Invoice-book is by pasting all Invoices received, in a book prepared for the purpose, according to their dates.

THE DAY-BOOK.

A Day-book, in its most limited sense, is one in which the transactions of each day are recorded. But sometimes it is made to contain a record of all the transactions of each day, whether of sales or purchases; bills accepted, or bills taken in payment; cash received or paid. And *these various items* from it have to be transferred to the Journal, the

Bill-book, the Ledger, and Cash-book respectively. If no Journal is kept, the Day-book may be made a record of all transactions both Dr. and Cr. that occur in the business, and it may be then said to be "journalized.'

Sometimes this book is called a Waste-book, especially when it embraces transactions as enumerated above. But if kept merely as an ordinary Day-book, it gives accounts only of sales and present payment. And in a very simple business, where but few articles are dealt in, if the amount of these sales be carried forward into a separate column at the end of each day, and then at the end of each month and year, the results for each year may be entered in a spare page of the Ledger, and thus give a view of the progress of the business.

F.L. stands for "Folio in Ledger," C.B. for "Folio in Cash-Book."

DAY-BOOK.

F. L.		C. B.	Bought.	Sold.	Miscellaneous.	Total.
	January 1, 1861.					
8	Bought of Phillips and Co.					
12	10 Chests of Tea (1050 lbs.) at 1/8		87 10 0			
13	8 pieces of Linen (128 yds.) at 33/6		13 8 0			100 18 0
1	Lodged in Joint Stock Bank				500 0 0	500 0 0
	2					
2	Sold J. Smith and Co.					
10	4 Bales of Cotton at £25 per			100 0 0		100 0 0
13	7 Pieces of Linen at 35/6 per			12 8 6		12 8 6
	4					
3	Bought of T. Martin					
	Shop fittings				45 10 0	45 10 0
	8					
2	Received from J. Smith & Co. in payment of their acct.; Cash				40 0 0	
	Bill at 2 mo. (4 Jan.—7 Mar.) No. 1.				72 8 6	112 8 6
	10					
4	Bought of B. Wilson					
	Gas fittings				17 15 0	17 15 0
	15					
5	Bought of Wm. Hinds					
10	10 Bales of Cotton at £20 per		200 0 0			
11	5 Hhds. of Port Wine at £16/5 p.		81 5 0			281 5 0
	Carried forward		382 3 0	112 8 6	675 13 6	1170 5 0

DAY-BOOK.

F. L.		C. B.	Bought.	Sold.	Miscella-neous.	Total.
	Amount brought forward		382 3 0	112 8 6	675 13 6	1170 5 0
	January 16					
3	Accepted T. Martin's Bill on me. No. 1, at 2mo. (16 Jan.—19 Mar.)				45 10 0	45 10 0
	18					
6	Sold Atkins and Co.					
11	4 Hhds. of Port Wine at £17/10 p.			70 0 0		70 0 0
6	Received from Atkins & Co. No. 2. Their Acceptance at 1 mo. (Jan. 11—14 Feb.)				70 0 0	70 0 0
	23					
1	Lodged in Joint Stock Bank				40 0 0	40 0 0
	February 6					
5	Accepted Wm. Hinds' Bill on me at 1 mo. (Jan. 24—27 Feb.) No. 2.				200 0 0	200 0 0
	14					
6	Received payment of Bill, No. 2. Accepted by Atkins and Co.				70 0 0	70 0 0
	26					
5	Paid Bill, No. 2. Accepted to Wm. Hinds				200 0 0	200 0 0
1	Drawn from Joint Stock Bank				200 0 0	200 0 0
	March 1					
7	Sold Robert Topham					
10	6 Bales of Cotton at £25			150 0 0		
12	8 Chests of Tea (840 lbs.) at 2/0			84 0 0	17 15 0	234 0 0
4	Paid B. Wilson for Gas fittings, &c.					17 15 0
	7					
2	Received payment of Bill, No. 1. Accepted by J. Smith and Co.				72 8 6	72 8 6
	18					
3	Paid Bill No. 1. Accepted to J. Martin				45 10 0	45 10 0
	29					
1	Drawn from Joint Stock Bank				175 0 0	175 0 0
	Carried forward		382 3 0	416 8 6	1811 17 0	2610 8 6

The Specimen of a Day-book represents what we have called a journalized arrangement, because under the head "Sold" is exhibited at one view the total amount of business done; and this can be carried forward, as described in the last paragraph. Of course the total amount of the first three columns is the same as that of the fourth, headed "Total."

THE CASH-BOOK.

The Cash-book contains simply the amounts received and paid every day. It must therefore according to p. 226 have two sides, a *debtor* side, on which are entered all the sums *received*, and for which therefore the cash must be considered responsible, i.e. a *debtor*: and a creditor side, on which are entered all the sums *paid*, and for which therefore the cash must be considered a *creditor*.

For a specimen of the Cash-book we refer to that portion of the Ledger headed Cash. Fo. 15, which will be found to contain all the entries from the Day-book involving cash or bills, both paid and received. All these payments or receipts have been entered in the Ledger under the names of the parties in connexion with whose transactions they have occurred; but they will be on the reverse sides of the Ledger, because for every sum paid or received by a customer there will have been a corresponding sum received or paid by the Cash-book, or, which is the same thing, by the Cash account in the Ledger.

There may appear also a third and different kind of entry; as, when a merchant has drawn a cheque on his own banker, and paid away either a part or the whole of the money. If he has paid the whole, there will be two equal entries on opposite sides; but if he has paid only part of it, there will be one entry on the Dr. side, of the whole cheque drawn; and one smaller entry on the Cr. side, leaving a corresponding balance in hand at the end of the day's transactions.

When a bill comes back dishonoured, and the drawer has to pay it by a cheque on his banker, he must enter the amount on both sides of his Cash-book as money received from his banker, and at the same time as paid away through the same banker to take up the bill. If the acceptor afterwards pays the money, the amount should be entered on the Dr. side of the drawer's Cash-book; and if paid to the banker again it will appear on the Cr. side, or it may remain as a balance in the Cash-box.

THE LEDGER.

The Ledger is a book containing the state of the account of every customer, with the firm: and its entries are obtained from the Day-book and the Cash-book.

Thus, all records of *sales* which have been entered in the Day-book will be transferred to the *debtor* side of the respective accounts in the Ledger because the parties have *received* goods; and all entries in the Day-book or Cash-book of cash received or bills received in lieu of cash, will be entered on the *creditor* side, because the parties have delivered the same.

It might happen occasionally that cash had been advanced to a debtor of the firm, to enable him to meet an acceptance or otherwise; then of course the item would appear on his debtor side in the Ledger.

LEDGER.

Dr.		Joint Stock Bank		Fo. 1.			Contra		Cr.
1861			F. B.	£ s. d.	1861			F. B.	£ s. d.
Jan.	1	To Cash	1	500 0 0	Feb.	26	By Cash	2	200 0 0
„	28	To do.	1	40 0 0	Mar.	29	„ do.	2	175 0 0
				540 0 0			„ Balance		165 0 0
Mar.	31	To Balance brought down							540 0 0
				165 0 0					

Dr.		J. Smith & Co.		Fo. 2.			Contra		Cr.
1861					1861				
Jan.	2	To Cotton	1	100 0 0	Jan.	2	By Cash	1	40 0 0
		To Linen	1	12 8 6			„ Bills Receivable	1	72 8 6
				112 8 6					112 8 6

Dr.		J. Martin		Fo. 3.			Contra		Cr.
1861					1861				
Jan.	16	To Bills payable (1)	1	45 10 0	Jan.	4	By Shop fittings	1	45 10 0

Dr.		B. Wilson		Fo. 4.			Contra		Cr.
1861					1861				
Feb.	28	To Cash	2	17 15 0	Jan.	10	By Gas fittings	1	17 15 0

LEDGER.

Dr.		Wm. Hinds		Fo. 5.	Contra				Cr.
1861			F. D.B	£ s. d.	1861			F. D.B	£ s. d.
		To Bills payable (2)	1	200 0 0	Jan.	15	By Cotton	1	200 0 0
Mar.	31	„ Balance		81 5 0			„ Port Wine	1	81 5 0
				281 5 0					281 5 0
					Mar.	31	By Balance brought down		81 5 0

Dr.		Atkins & Co.		Fo. 6.	Contra				Cr.
1861					1861				
Jan.	18	To Port Wine	1	70 0 0	Jan.	18	By Bill Receivable(2)		70 0 0

Dr.		R. Topham		Fo. 7.	Contra				Cr.
1861					1861				
Mar.	1	To Cotton	2	150 0 0					
		„ Tea	2	84 0 0					
				234 0 0	Mar.	31	By Balance		234 0 0
	31	To Balance		234 0 0					

Dr.		Phillips & Co.		Fo. 8.	Contra				Cr.
1861					1861				
					Jan.	1	By Tea (10 Chests)	1	87 10 0
							„ Linen (8 pieces)	1	13 8 0
Mar.	31	To Balance		100 18 0					100 18 0
					Mar.	31	By Balance brought down		100 18 0

Dr.		Cotton		Fo. 10.	Contra				Cr.
1861					1861				
Jan.	15	To W. Hinds (10 B.)	1	200 0 0	Jan.	2	By Smith & Co. (4 B.)	1	100 0 0
Mar.	31	„ Profit and Loss		50 0 0	Mar.	1	„ R. Topham (6 B.)	2	150 0 0
				250 0 0					250 0 0

Dr.		Port Wine		Fo. 11.	Contra				Cr.
1861					1861				
Jan.	15	To W. Hinds (5 Hhds.)	1	81 5 0	Jan.	18	By Atkins & Co. (4 Hhds.)	1	70 0 0
Mar.	31	Profit and Loss		5 0 0	Mar.	31	„ Balance (1 Hhd. in hand)		16 5 0
				86 5 0					86 5 0

LEDGER.

Dr.		Tea		Fo. 12.			Contra		Cr.
1861			F. D.B	£ s. d.	1851			F. D.B	£ s. d.
Jan.	1	To Phillips & Co. (10 Chests)	1	87 10 0	Feb.	28	By R. Topham	2	84 0 0
Mar.	31	,, Profit and Loss		5 5 0	Mar.	3	,, Balance (Two Chests in hand)		8 15 0
				92 15 0					92 15 0

Dr.		Linen		Fo. 13.			Contra		Cr.
1861					1861				
Jan.	1	To Phillips & Co. (8 pieces) 33/6	1	13 8 0	Jan.	2	By Smith & Co.	1	12 8 6
Mar.	31	,, Profit and Loss		14 0	Mar.	31	,, Balance (1 piece in hand)		1 13 6
				14 2 0					14 2 0

Dr		Cash		Fo. 15.			Contra		Cr.
1861					1861				
Jan.	2	To Cash, Smith & Co.	1	40 0 0	Jan.	1	By Cash, J. S. Bank	1	500 0 0
Feb.	14	,, Bills receivable (2)	2	70 0 0		28	,, do.	2	40 0 0
	26	,, J. S. Bank	2	200 0 0	Feb.	26	,, Bills payable (2)	2	200 0 0
Mar.	7	,, Bills receivable(1)	2	72 8 6	Mar.	1	,, B. Wilson	2	17 15 0
	29	,, J. S. Bank	2	175 0 0		18	,, Bills payable (1)	2	45 10 0
	31	To Balance		245 16 6					803 5 0
				803 5 0			By Balance brought down		245 16 6

Profit and Loss.

To Cotton	50 0 0		
,, Port Wine	5 0 0		
,, Tea	5 5 0		
,, Linen	14 0		
	60 19 0	Balance	60 19 0

In Fol. 1 there is an excess of £165 on the Dr. side over the Cr.; this added to the Cr. side of course makes the two sides agree; and it should then be brought to the Dr. side, as representing the state of the account, or as commencing a fresh account. In Fol. 5 is an example of the position of the balance being reversed.

In the Profit and Loss account, had there been any losses, they *would have* diminished the balance of £60 19*s*.

THE BILL-BOOK.

The bills payable and receivable account contains all the bill transactions.

(1) The Bills receivable. The Dr. side of this account contains all the bills received from other parties, or drawn upon them; the parties on whom they are drawn or from whom they are received being the corresponding Crs. The Cr. side of the same account contains all the same bills, the parties to whom they are paid being the corresponding Drs.

(2) The Bills payable. The Cr.* side of this account contains all the bills issued by the firm, the parties drawing them being the corresponding Drs.; the Dr. side contains the same bills, the parties paying them at maturity being the corresponding Crs., generally bankers.

But if bills payable or receivable are paid without the intervention of a banker, who is the corresponding Cr.? For example, if I take a bill for any given sum, and at maturity present it to the acceptor, and receive payment in cash, where is the proper credit entry? Not to the acceptor, as he has been credited once when he gave the bill; but the bills receivable account must be credited, because when the acceptor was credited, the bills receivable account was debited, and by the new entry to the Cr. side, the account is balanced.

Similarly, if we pay any acceptance in cash, we must not credit our own personal account, which has already been credited when we gave the bill; but we must credit the bills payable account, which was debited with it when we gave the bill.

* I here begin with Cr., whereas under "Bills receivable" I began with Dr. A bill receivable account begins with Dr., a bill payable with Cr.

BILLS RECEIVABLE.

Dr. Cr

No.	Received	On whose acct. Cr.	F. L.	Drawn by	Upon	Payable at	Time	Due	Amount	When	and how disposed Dr.	F. L.	
	1861												
1	Jan. 8	Smith & Co.	2	Self	Smith & Co.	L. & W. Bank	2 mo., Jan. 4	Mar. 7	72 8 6	Mar. 7	Cash Book	15	72 8 6
2	„ 18	Atkins & Co.	6	„	Atkins & Co.	Do.	1 mo., Jan. 11	Feb. 14	70 0 0	Feb 14	C. B.	15	70 0 0
									142 8 6				142 8 6

BILLS PAYABLE.

Dr. Cr.

When paid	By whom paid Cr.	F. L.		No.	Accepted	On whose acct. Dr.	F. L.	Payable to	Time	Where payable	Due	Amount
					1861							
Mar. 18	Cash Book	15	45 10 0	1	Jan. 16	J. Martin	8	Drawers	2 mo. Jan. 16	S. P. & S.	Mar. 19	45 10 0
Feb. 26	Do.	15	200 0 0	2	Jan. 24	Wm. Hinds	5	Do.	1 mo. Jan. 24	S. P. & Co.	Feb. 27	200 0 0

TRIAL BALANCE.

31 Mar. 1861.

£	s.	d.	F		F	£	s.	d.
				Joint Stock Bank . .		165		
				Smith & Co. . . .				
				Martin				
				Wilson				
81	5			Hinds				
				Atkins & Co. . . .				
				Topham		234		
100	18			Phillips & Co. . . .				
				Cotton				
				Port Wine		16	5	
				Tea		8	15	
				Linen		1	13	6
245	16	6		Cash				
				Bills Receivable . . .				
				„ Payable . . .				
60	19			Profit and Loss . . .				
				Trade Expenses . . .		63	5	
488	18	6				488	18	6

We have explained the meaning of a Trial Balance. We have yet to consider a General Balance. This is a complete statement of the position of a firm or company; and when presented in the form of a document is termed a Balance Account. An abstract of this balance account is called a "Balance Sheet." This consists of two parts:

(1) A Statement of Assets and Liabilities, showing whether the party is solvent, or insolvent.

(2) A Statement of Profit and Loss, explaining how the above result of solvency or insolvency has been brought about.

The following are specimens of Balance Sheets, taken from the "Money Market and City Intelligence" of "The Times."

The former is from the accounts of a bankrupt estate, and consists only of a statement of Assets and Liabilities; the second is that of a joint-stock bank, and comprises the statement of the Profit and Loss, as well as the Assets and Liabilities.

In the bank account, the pounds sterling only have been given, for the sake of brevity.

Specimen of a Balance Sheet.

Dr.

		£ s. d.
To Creditors on open account		1632 9 6
By Bills payable	58271 11 3	
Less, goods consigned	102 11 10	
		58168 19 5
By Liabilities on bills receivable	9959 11 6	
Expected to be duly met	7977 2 11	
	1982 8 7	
Cash in hand at Joint Stock Bank	1835 4 10	
		147 3 9
		59948 12 8

Cr.

By Cash at bankers held as security against bills receivable discounted		[659 3 9]*
Cash in travellers' hands		142 0 11
By debtors, good		3096 17 8
" " doubtful 843 11 1	estimated at	153 0 0
" " bad 567 5 7		
By Bills receivable, in hand,	2667 2 11	
	estimated to produce	1857 16 10
By Property		2348 18 2
By Stock		25569 15 11
		33168 9 6
Less, Creditors under £10	55 11 6	
Rent, paid in full	123 18 4	
		179 9 10
		32988 19 8
Deficiency		26959 13 0
		59948 12 8

thus shewing a probable dividend of 10*s.* or 11*s.* in the pound.

The following Balance Sheet shews (1) what money has been received as paid-up capital, and the reserve fund from previous profits not divided, but retained to meet contingencies, and the amount received from depositors, or customers; (2) what the bank has to shew for all the above money, i.e. either money invested, or spent upon preliminary expenses, or securities for money advanced.

* Not available as an asset.

Dr.		£			Cr. £
To Capital paid up		300,000	By investments in Government securities, &c.		282,374
Reserve fund	20,000		Freehold premises	40,000	
Half-year's interest at 4 per cent.	400		in occupation of the bank	35,000	
		20,400			75,000
Amount due by bank on current accounts		1,599,373	By bills discounted, loans, &c.		1,321,845
Amount carried to credit of Profit and Loss account,	43,934		By cash in hand, and at call		264,564
Less amount paid to customers for interest on balances	13,794		By preliminary expenses		4,116
		30,140*	By lease and buildings at Charing Cross		2,014
		1,949,913			1,949,913

Profit and Loss account.

Dr.	£		Cr. £
Half-year's current expenses, bad debts, &c.	10,904	By balance of Profit and Loss, brought forward from last half-year	1590
Rebate of interest on bills discounted, not yet due, carried to profit and loss new account	3,815	Ditto for current half-year	28550
Amount written off, preliminary expenses	700		
To dividend at 5 per cent. per annum upon £300,000	7500		
Amount carried to credit of Reserve fund	6600		
Balance carried to Profit and Loss new account	621		
	30,140		30,140

* This is the most important item, and is in reality the balance between the two sides of the capital account and indicates the gross profit. Had the bank lost more than it had made during the half-year, there would have been an item on the *other* side of the capital account, indicating a loss.

In the above, the Cr. side shows all that there is to dispose of from the gross profit indicated in the capital account, and the balance undisposed of in a previous half-year. The Dr. side explains how it is proposed to dispose of it; after all expenses are paid, and a proper addition is made to the reserved fund there remains £8121; and out of this a dividend of £7500 is paid to the shareholders, being at the rate of 5 per cent. per annum; and the remaining balance is carried forward to next half-year's Profit and Loss account.

UNITS OF VALUE, MEASURE, &c.

In dealing so frequently as we do with the various weights, measures, and moneys in use, we must necessarily have observed that there are certain fixed *standards* of each. An investigation into the circumstances which determined the choice of these standards is not uninteresting.

The particular coins, weights, or measures, which have been chosen as standards, whereby we measure all other quantities, are termed *units*. Thus, of the pound, shilling, or penny, any one may be considered as a unit, though, if they preserve their mutual relation unchanged, the determination of one unit will at once give us the knowledge of all the others.

Similarly, the determination of the length of the yard will give the value of all other units of length, as miles, feet, &c. if their relation to the yard be fixed. And so also the determination of the pound avoirdupois will be found to fix the value of every other standard of weight.

Now it will be found in (p. 245), under the head of Decimal Coinage, that in anything like a perfect system, as the French metrical system, there is no unit chosen but what is derived from some natural standard; and in that system we shall see that all the units of weight, value, &c. are derived from one simple unit of length, which again is derived from a measurement accurately taken upon the earth's surface.

But in England, few things can be imagined less influenced by uniformity of principles than the determination of our several units of value, weight, capacity, &c., and their multiples and submultiples.

Thus, the relations of farthings, pence, shillings, and pounds, which involve the multiplication by 4, 12, 20, proceed upon no scientific principle; and the same may be said of nearly all the weights and measures, especially in Dry Measure, and Avoirdupois Weight.

The penny is the real unit among our coins, all others being multiples

of it, except the farthing, which is a fraction of it. The recent bronze coinage has completely overthrown the relation which has hitherto been supposed to subsist between the intrinsic value of the metal and its current value; and may probably lead to very important results, in the further substitution of a currency which has rather the nature of a token than an embodiment of value. Copper Coinage is a legal tender only up to 12*d.* Silver Coinage only up to 40*s.*

The standard silver coin is $\frac{37}{40}$ pure. From 1 lb. Troy of this metal are coined 66*s.*, so that the mint price of standard silver is 5*s.* 6*d.* per oz., and that of pure silver $= \frac{40}{37}$ of 5*s.* 6*d.* $=$ 5*s.* $11\frac{13}{37}d.$

The standard gold coin is $\frac{11}{12}$ pure. From 1 lb. Troy of this metal are coined $46\frac{29}{40}$ sov. $=$ £46 14*s.* 6*d.*; so that the mint price of standard gold is £3 17*s.* $10\frac{1}{2}d.$; and that of pure gold is $\frac{12}{11}$ of 77*s.* $10\frac{1}{2}d.$ $=$ £4 4*s.* $11\frac{5}{11}d.$

A *Carat* is $\frac{1}{24}$ part of 1 lb. of gold: hence the fineness of gold is often expressed by saying that it is so many *carats fine*, meaning so many parts out of 24; thus our standard gold is said to be 22 carats fine, and jewellers' gold is 18 carats fine.

The following coins are worth notice, since they occur in ancient documents:

Groat $=$ 4*d.* Tester $=$ 6*d.* Noble $=$ 6*s.* 8*d.* Angel $=$ 10*s.*
Merk, or Mark $=$ 13*s.* 4*d.* Jacobus $=$ 25*s.*

The adoption of the unit of length is indeed traceable to scientific principles. Thus it is obtained from the length of the seconds' pendulum, oscillating in vacuo at the latitude of London. This length is determined as accurately as possible by experiment, and measured—and when this is divided into 39·13921 equal parts, 36 of them form our yard. And all other measures of length are either multiples of it, as the fathom, pole, furlong, mile, league; or sub-multiples, as the foot, inch, barleycorn. This last is obsolete in practice; and an inch is either divided decimally into tenths, hundredths, &c., or into 12 equal parts called *lines.*

It is worth enquiring whence come the two different measures of a degree, viz. 60 geographical miles, or $69\frac{1}{2}$ British miles. A geographical mile is $\frac{1}{60}$th part of a degree of longitude at the equator, but a British mile $=1760$ yds.; and on comparing these two miles we find that 60 of the former are equal to about $69\frac{1}{2}$, or, in round numbers, 70 of the latter. The advantage of having a geographical mile as a unit, is that it is a convenient length to take as a knot or nautical mile, which knot bears an ascertainable ratio to the length of a degree of longitude, though that ratio varies in different latitudes.

Nautical miles, or knots, can be readily converted into British miles, or the converse; for since 60 knots $=70$ B. miles; $\therefore$ 1 knot $=\frac{7}{6}$ mile $=1\frac{1}{6}$ miles; and 1 mile $=\frac{6}{7}$ knot $=(1-\frac{1}{7})$ knots; so that to convert knots into miles we add $\frac{1}{6}$th of the number; and to convert miles into knots we take off $\frac{1}{7}$th of the number.

For estimating long periods the year is taken as the unit of time; and for shorter periods, the day. The length of the day may be taken from two or three sources, which give results slightly different. These are as follows:

(1) The interval between any star's leaving the meridian of a place and returning to it. This interval is found never to vary, and is termed a *sidereal day*. It would not, however, be convenient as the ordinary measure of time; for its commencement, depending upon the presence of some particular star upon the meridian, would not be marked by a sufficiently striking phenomenon, and in the course of a year would happen when the Sun was at various distances above and below the horizon.

(2) The time of the Sun's leaving the meridian of any place and returning to it; i.e. the interval between two successive noons; and this interval is termed a *solar day*. But as the Sun's apparent motion in the heavens is not uniform, this solar day varies, and is therefore not a good standard.

(3) The third and most suitable standard is the interval which would be occupied by the Sun, if the equalities of its motion were corrected, that is, if it travelled at the same uniform rate as it now does on an average. This interval is termed the *mean solar day*. A clock which shows a change of 24 hours in such a day is said to keep true, or mean solar time; but the time indicated by it will almost always be different from that shown by a sundial, on account of the Sun's irregular motion; and hence we observe in the almanacs that the clock is sometimes said to be before, and sometimes after the Sun. In fact the clock and the sundial agree only four times in a year.

When therefore we say that the year consists of so many days, we speak of mean solar days. And having thus settled the length of a day, we divide it into 24 equal parts, and call each an hour; divide the hour into 60 equal parts, and call each a minute; divide a minute into 60 equal parts, and call each a second. We then take a pendulum of such a length that it will oscillate in a vacuum, in the latitude of London, $60\times60\times24$ times, or 86400 times, in one day: and we thus determine the length of the seconds' pendulum.

The length of the year itself is an important unit of time. This is

found by observing the interval between the Sun's leaving an equinox or solstice, and returning to the same equinox or solstice. This interval is termed the *Tropical Year*, and is found most accurately by comparing observations made at distant periods. It is found to be 365d. 5h. 48m. 47·58091 secs. mean solar time. As it would be very inconvenient for civil purposes that a year should consist of a broken number of days, (for we should in that case sometimes have a part of a working day in one year, and the remainder in another) it is agreed to take 365¼ days as a year; and to reckon 365 days as a year for 3 consecutive years, and then with the 4 quarters that have been accumulating in 4 years, make an extra day; so that every 4th year consists of 366 days. This day is called an *intercalary* day, and is added to the shortest month, viz. February, causing what is commonly termed Bissextile* or Leap Year.

By observing, however, the difference between the true value of the tropical year, 365·24222, and the assumed value, 365·25, we find there is an excess of 11 min. per annum, which amounts to about three days in 400 yrs. Hence, if we omit three leap years in 400 yrs., we shall very nearly rectify the error. Now every year, divisible by four, is ordinarily a leap year; hence every year that closes a century, since it is divisible by 4, should be a leap year; thus, 1600, 1700, 1800, 1900, &c. should all be leap years. But as we must omit three leap years in 400 yrs., it is agreed that three out of the above four years which terminate successive centuries shall not be leap years. To determine which are the three, we divide by 4 the above numbers without the final ciphers; and we find that 16 can be divided by 4 without remainder, but 17, 18, 19 cannot; hence 1600 is made a leap year, and 1700, 1800, 1900 are not.

The above error of 11 minutes per year went on accumulating from the institution of the Julian calendar till 1582 when the alteration was introduced by Pope Gregory XIII, from whom it was called the Gregorian calendar. Now, the vernal equinox, which in 325, at the Council of Nice, fell on the 21st of March, in 1582 happened on the 11th of March; therefore, Pope Gregory caused 10 days to be omitted, making the 15th of October immediately succeed the 4th; so that the next year the vernal equinox again fell on the 21st of March. This correction was not introduced into England till 1751, when it had become necessary to omit 11 days in the current year.

The time before the alteration is termed *Old Style*, and after it *New Style*. And as in Old Style the year began at the 25th of March, any

* From *bis*, twice, and *sextilis*, sixth; because in this year the *sixth* day before the calends of March was repeated twice.

event which occurred between January 1st and March 25th, would be referable to both years, and was thus noted, Jan. 17th, $16\frac{41}{42}$, or 1641-2. But with the change of style the year was made to commence at the 1st of January.

Russia still adheres to the Old Style, and the error now amounts to about 12 days.

The standard of weight is a lump of platinum, marked P.S. 1 lb. 1844, and called the parliamentary standard pound, and preserved in the Exchequer. This is the pound Avoirdupois, and $\frac{1}{7000}$th of this is termed a *grain*. 5760 of these grains make what is termed a pound Troy. Formerly, there was a lump of bronze, preserved as a standard of the pound Troy, and of course $\frac{1}{5760}$th of it was a grain. But this standard was destroyed in the burning of the Houses of Parliament in 1835. From the above avoirdupois lb. all the other multiples and submultiples are derived. Apothecaries retail by the troy pound and troy grain, but they subdivide the pound in a different way.

The standard of fluid measure is a gallon; and the true definition of it is that it contains 10 lbs. avoirdupois of distilled water weighed in air at a temperature of 62° F. when the barometer is at 30 inches.

The weight of a cubic inch of water, expressed in grains, can be best obtained from the following data:

$$
\begin{aligned}
&3{\cdot}937079 \text{ Eng. inches} &&= 1 \text{ decimètre},\\
&1 \text{ cubic decimètre} &&= 1{\cdot}000012 \text{ kilogrammes},\\
&1 \text{ kilogramme} &&= 15432{\cdot}349 \text{ grains}.
\end{aligned}
$$

Hence, the weight of a cubic inch of water is found to be 252·5977 grains.

$$\text{Also, since } 1 \text{ grain} = \frac{1}{252{\cdot}5977} \text{ cub. in. of water},$$

$$\text{and } 1 \text{ gallon, or } 10 \text{ lbs.} = 10 \times 7000 \text{ grains},$$

$$1 \text{ gallon} = \frac{70000}{252{\cdot}5977} \text{ cub. in.} = 277{\cdot}72 \text{ cub. in.}$$

DECIMAL COINAGE.

The question of Decimal Coinage, which seemed to interest the public mind some time since, appears now to have lost much of that interest; but it is not lost sight of by any who desire to see the facilities for education extended, or commercial intercourse simplified.

From what has been said above, (p. 240), about our present coinage, it is manifest that there has been no attempt made to proceed upon any system in the construction of our coinage, weights, and measures; but that the choice of every unit has been the result either of accident or arbitrary selection. And though there is primâ facie evidence of the superiority of a decimal over any other system of coinage, seeing that it substitutes simple arithmetic for compound; yet it is not till we come to the consideration of the weights and measures, that we see, to the full extent, how cumbrous our system is, if it deserves the name of system at all. But when we observe the almost infinite variety of weights and measures in use in various parts of the country, especially in the article of grain, so that a merchant who wished to buy or sell by the bushel would have to take a different standard of weight for a bushel in almost every considerable corn market in England, one would think that our present position would hardly find a defender.

The only system of decimal coinage which has been brought extensively and authoritatively under the eye of the British public, is termed the pound and mil system, which was recommended before a Committee of the House of Commons, in 1853, by many able witnesses, among whom were Professors Airy and De Morgan.

But the system which has by far the strongest recommendation for simplicity and completeness is the metrical system of France, which, as we shall presently see, embraces coinage, and weights and measures, linear, superficial, solid and liquid.

The advantages of *any* decimal system of money, weights and measures, in simplifying the operations of Arithmetic, are as follows.

I. There would be no such thing as Compound Arithmetic.

II. Reduction would no longer be treated as a separate and very

troublesome process, because all reduction to a lower name would be performed by merely moving the point to the right, and all reduction to a higher by moving the point to the left.

III. There would not be the same necessity for the acquirement of dexterity in the use of Vulgar Fractions, since they would for almost all business purposes be superseded by Decimals*.

IV. The rule called Practice would be entirely abolished; since Practice is only another method of working examples in Compound Arithmetic; and all questions hitherto wrought by it would be worked by Simple Multiplication.

V. Proportion and its numerous applications would of course partake of the great simplification arising from the rejection of all the present processes of Reduction and Compound Rules. The calculations of Interest, Stocks and Exchange, would offer the most noticeable instances of abbreviation.

Probably, under a decimal system, the fluctuations in the price of Stock would be reckoned by *tenths*, instead of as now by *eighths;* so also the brokerage would be £$\frac{1}{10}$, and not £$\frac{1}{8}$, per cent. And these two changes would very materially increase the simplification.

So that probably the whole subject of Arithmetic would be taught in half the time that it is now; and much greater accuracy would be generally obtained.

The pound and mil system of coinage is briefly as follows.

It adopts the present pound, as the unit, and its tenths, hundredths, and thousandths.

Thus, one *tenth* of £1 is the present *florin*,
one *hundredth* of £1 would be a new coin, called a *cent*,
and one *thousandth* of £1 would be a new coin, called a *mil;*
so that 10 mils = 1 cent, 10 cents = 1 florin, 10 florins = 1 pound.

Other coins would be used, but their value on paper must all be expressed in the above four coins, called *money of account.*

Also, in writing a sum of money consisting of pounds, and sums below £1, we should place a dot or point (·) after the pounds, and put the florins in the first place after the dot, the cents in the second, and the mils in the third.

* All the theory of decimals will of course remain unaltered; and it will still be advisable that pupils should learn the whole system of decimals, including the converting of Vulgar Fractions into Decimals and the converse; Circulating Decimals, &c. But in consequence of the general tendency to supersede Vulgar Fractions by decimals, these parts of the subject may be viewed not so much as a *practical* acquisition, but rather as an instrument of *educational* discipline, an important Arithmetic very much overlooked by many, both teachers and learners.

Thus, £23, 4 florins, 5 cents, 7 mils, would be written £23·457; and if each of these quantities be written separately, in four sums under one another, they would stand thus:

£
23.
·4
·05
·007

where in the third line a cipher is placed, to keep the 5 cents in the second column after the point, *i. e.* in the column of cents; and in the fourth line, two ciphers are placed, so as to keep the 7 mils in the column of mils. But in 23·457, the 4 keeps the 5 in its proper column, and the 45 keeps the 7; and therefore no ciphers are required.

But a sum of 10 mils, or more, being equal to a cent, will appear in the second place, and 100 mils or 10 cents, being really a florin, will fill the first place.

Thus, 17 mils = 1 c. 7 m., or in terms of £1 = £·017
75 cents = 7 fl. 5 c. „ „ = £·75
And, £·457 may be read 4 fl. 5 c. 7 m.
or 4 fl. 57 mils.
or 457 mils.

This system would have the following coins; those below the sixpence, or quarter florin, are new.

					£	£
Sovereign	=1000 mils, or written in pounds				1·000 or	1.
Half Sovereign	=	500	...	...	·500 „	·5
Crown	=	250	...	...	·250 „	·25
Florin	=	100	...	...	·100 „	·1
Half Florin or Shilling	=	50	...	...	·050 „	·05
Quarter Flo. or Sixpence	=	25	...	...	·025	
Two Cent Piece	=	20	...	...	·020 „	·02
Cent	=	10	...	...	·010 „	·01
Five Mil Piece	=	5	...	...	·005	
Two Mil	=	2	...	...	·002	
One Mil	=	1	...	...	·001	

In discussing the question of Decimal Coinage, our attention cannot but be forcibly directed to a due consideration of the Metrical System of France, which, on account of its simplicity, completeness and beautiful arrangement, stands preeminent among all other systems.

It is called the *Metrical* System, because from the single unit, the Mètre, are derived the weight and diameter of its coins, and all its weights, and its measures, whether linear, superficial, or cubic. Of course it is plain that in a system so simple and complete as this, all arithmetical operations are performed by means of the four Elementary Rules; and there is no such thing in all French computations as Compound Arithmetic.

The Mètre was obtained from the subdivision of a meridian line drawn upon the earth's surface. By the actual measurement of an arc (from Dunkirk to Barcelona) a computation was made of the length of a quadrant, or one-fourth part of a complete meridian. A ten-millionth part of this quantity was taken as the unit of linear measurement, and is equal to about 39·371 English inches.

All *multiples* of this, proceeding tenfold *higher*, are named from the mètre and *Greek* prefixes, as deca, hecto, kilo, representing 10, 100, 1000 times the mètre. And all *submultiples* of the mètre, proceeding tenfold lower, are named from the mètre and *Latin* prefixes, as deci, centi, milli, representing $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$ of the mètre. Hence we have the following table of the values of the French mètre, and its multiples and submultiples, expressed in terms of an English inch.

Myriametre	or	10000	metres	=	393710	English inches.
Kilometre	,,	1000	,,	=	39371·	,,
Hectometre	,,	100	,,	=	3937·1	,,
Decametre	,,	10	,,	=	393·71	,,
METRE	,,	1	,,	=	39·371	,,
Decimetre	,,	$\frac{1}{10}$	,,	=	3·9371	,,
Centimetre	,,	$\frac{1}{100}$	,,	=	·39371	,,
Millimetre	,,	$\frac{1}{1000}$	,,	=	·039371	,,

Here it can hardly be needful to remark, that every one of these denominations can be converted into the next lower, by simply moving the point to the *right*, *i. e.* by *multiplying* by 10 for each step in the reduction; and to the next higher, by moving the point to the *left*, *i. e.* by *dividing* by 10 for each step.

It will next be most convenient to discuss the weights. The unit is a Gramme, and as in the mètre, so the multiples of the Gramme proceeding tenfold higher each step are called decagramme, hectogramme,

kilogramme; and the submultiples, tenfold lower, are called decigramme, centigramme, milligramme.

The Gramme is the weight of the volume of distilled water, at the temperature of its greatest density, contained in a cube whose edge is a centimetre; and therefore, of course

the weight of	10	such cubes	forms a	decagramme,
,,	100	,,	,,	hectogramme,
,,	1000	,,	,,	kilogramme.

And in like manner

the weight of	$\frac{1}{10}$	of such a cube	forms a	decigramme.
,,	$\frac{1}{100}$	,,	,,	centigramme.
,,	$\frac{1}{1000}$	,,	,,	milligramme.

A gramme is equivalent to 15·432 grs. or nearly 16 grs. Troy,

$\therefore$ a kilogramme ,, 2·68 lbs. or about $2\frac{2}{3}$ lbs. Troy.

Again, the volume of a kilogramme of water is taken as the unit of capacity, and is called a Litre: and its multiples, proceeding tenfold higher, are, upon the same principle as before, termed a decalitre, hectolitre, kilolitre; and the submultiples, proceeding tenfold lower, are called a decilitre, centilitre, millilitre.

A litre is equivalent to 1·760773 pts. or about $1\frac{3}{4}$ pints,

$\therefore$ a kilolitre ,, 220·09668 gallons, or about 220 galls.

The principal unit of superficial measure is the *are* or square decamètre, *i. e.* a square having 10 mètres or linear units for its side.

$$\text{And since 1 sq. mètre} = \left(\frac{39\cdot371}{36}\right) \text{sq. yds.} = 1\cdot196046 \text{ sq. yds.}$$

$$\text{Hence 1 are} = 100 \text{ sq. mètres} = 119\cdot6046 \text{ sq. yds.}$$

$$\text{a decare} = 1196\cdot046.$$

$$\text{a hectare} = 11960\cdot46.$$

This last will be found to be from $2\frac{1}{3}$ to $2\frac{1}{2}$ acres English, and is commonly employed in France for measuring large pieces of land.

The coins are also derived from the mètre both as to their weight and measure. There are but two moneys of account, the centime, and the franc = 100 centimes; or, considering the centimes as decimal parts of a franc, the name of but one coin is required. Thus, 25 francs, 37 centimes may be written 25·37 francs.

The centime is a coin the diameter of which is 1 centimetre, and

weight is 1 gramme; so that 100 centimes touching one another and arranged in a straight line would extend one metre; and treated as weights, they would be equal to 100 grammes, or one-tenth of a kilogramme. Hence, every centime forms at the same time, a coin, a weight, and a measure.

The various coins that are needed and generally used to avoid carrying a multitude of smaller coins are, in silver, $\frac{1}{5}$ franc = 20 cents, $\frac{1}{4}$ franc = 25 cents, $\frac{1}{2}$ franc = 50 cents, 2 franc and 5 franc pieces: and in gold, 5 franc, 10 franc, 20 franc, and 40 franc pieces. Their weights and diameters are as follows:

SILVER.

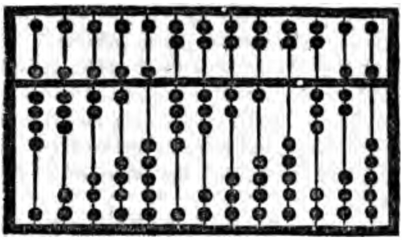
francs.		wt. in grammes.		diam. in millimetres.
$\frac{1}{5}$	......	1	......	15
$\frac{1}{4}$	......	1·25	......	15
$\frac{1}{2}$	......	2·5	......	18
1	......	5.	......	23
2	......	10.	......	27
5	......	25.	......	37

GOLD.

5	......	1·6129	......	17
10	......	3·2258	......	19
20	......	6·4516	......	21
40	......	12·9032	......	26
50	......	16·129	......	28
100	......	32·258	......	35

THE ABACUS.

THE *Abacus* or *Swan-pan*, as it is called in China, is an instrument for the quick performance of the operations of the four Simple Rules. As it is applicable only to *Simple* Addition, Subtraction, &c., it cannot be used for commercial purposes, excepting where a decimal coinage is current. The annexed cut represents the instrument. The



9 8 7 6 5 4 3 2 1 0·3 7 5

number of parallel wires is optional: I have inserted ten at the left of the decimal point, and three after it, so that we can count as high as 1000 millions of the unit, and as low as thousandths of the same unit.

Each ball or counter in the upper part of the frame stands for *five;* and each one below for *one*. When any one of the counters is used, it must be brought close to the cross bar which divides the frame; when the balls touch the outer bars, they are out of use for the time. The diagram will shew the number of balls required to represent any one of the digits from 0 to 9; the balls now in use correspond to the numbers below the frame.

In adding together any number of quantities, the operator performs the addition as each line is read over to him*; so that when the whole of the rows of figures have been read, the position of the counters after his last operation will give the total of all the sums that were to be added. And since it is said that in China the operator can perform the additions as rapidly as any one will ordinarily read the various amounts, and with perfect accuracy, there must be a considerable saving of time.

* The pupil should learn to add, beginning from the left, because all sums in money will generally be read over from left to right.

The following principles must be remembered, and the corresponding operations performed with facility.

I. To add *ten* to any row is to carry *one* to the next row.

II. If two *fives* are in use on any row, they must be rejected, and *one* carried to the next row.

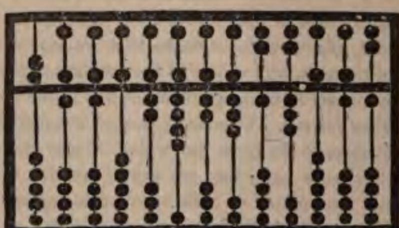
III. If *five* counters are in use on any bar in the lower frame, it is better to reject them, and substitute a single one in the upper frame.

In performing additions, when there are counters, not in use, sufficient to perform the addition, it is very easy; thus, I can add 2, 3, or 4 to 1, or to 5: or, I can add 6, 7, or 8, to 0 and 1.

But the number of counters will often be too small, especially when we are adding to numbers which already require nearly all the counters to be in use. Thus, I cannot *at one process* add 3 to 8, or to 9, or to 4. In this case, instead of adding 3 directly, I add its equivalent, 5—2, or 10—7, *i. e.* I add 5 and subtract 2, or add 10 and subtract 7.

When the number to which I am adding is *more than five*, I add the larger equivalent, as 10—7, instead of 5; or 10—8, instead of 2: but if it be *less than five*, I add the smaller equivalent, as 5—2, instead of 3; or 5—3, instead of 2.

We have seen that the first diagram represents the number 9876543210·375. The second gives the result of the annexed sum in



9876543210·375
786862774·527
2384276·421
1000·000
1·007
6532·485
18·746
10665797813·561

Simple Addition. Each row is added singly, according to the directions given above; and if the work is performed correctly, the operator will find the arrangement of the balls as in the adjoining diagram. I have used numbers involving three places of decimals, in order to show the application of the instrument to a decimal currency. The operation of Subtraction will of course be just opposite to that of Addition: thus, where there are balls enough to enable me to subtract directly, as 2 from 4, or 8, I do so: otherwise, I must subtract indirectly, as was done in

Addition. To subtract 4, *i. e.* 5—1, from 5, 9, 7, or 8, I subtract 5, and add 1: to subtract 8, *i. e.* 10—2, from any number less than 8, I subtract 10, (that is, 1 from the previous row,) and add 2. And so on for other numbers.

Multiplication by the numbers 2 to 12, or by composite numbers, whose factors are not greater than 12, is easily performed; of course each line, as it is produced mentally, is recorded on the abacus, and takes the place of the previous one. The same may be done with any multiplier, as 257, if the top line or multiplicand be written on a slip of paper, and kept before the eye: for of course, in such an example, if I commence by marking the top line upon the instrument, the arrangement is changed as soon as I commence the multiplication; and for working the second and third line I must have the multiplicand written before me.

Short Division may be very readily performed, but not Long Division; and therefore only the numbers from 2 to 12, and those which are composed of factors not greater than 12, can be used as divisors.

Those who have learnt the Multiplication Table as far as 20 times 20, can of course extend their use of the abacus accordingly.

Obs. In the construction of the abacus, it would be well to have the balls of a different colour after every three rows, reckoning from right to left, in order to assist the eye in catching quickly the place of one thousand, one million, &c.

APPENDIX.

FRACTIONAL QUOTIENT.

It was mentioned in (22) that the word quotient is not inapplicable, where one number is to be divided by another, and yet the division cannot be performed; thus in the division of 2 by 5, we said that the quotient was $\frac{2}{5}$.

This may be seen more clearly, if we remember that the word *quotient* means *how often* the divisor is contained in the dividend,—and if we observe the following illustration. Suppose that I have a rod five inches long, and with it I am to measure the depth of water in several full vessels, of which the depth is known to be 1, 2, 3, 4,...... inches. If now I ask *how many* times the length of this rod is contained in the depth of the water; the reply is, that in the first, one inch or one-fifth of the rod was immersed, *i. e.* the rod *went into* the depth one-fifth times; and therefore the genuine quotient of 1 when divided by 5 is one-fifth or $\frac{1}{5}$. Similarly, the quotient in the case of the second, third, &c. vessels, would be $\frac{2}{5}$, $\frac{3}{5}$, &c. And when I come to measure the depth of the five-inch vessel, the quotient ought to be $\frac{5}{5}$, or 1, *i. e.* the rod is just once immersed. And when I measure the seven-inch depth, I find the quotient to be $\frac{7}{5}$ or $1\frac{2}{5}$; *i. e.* it takes one immersion of the rod, and $\frac{2}{5}$ of another, to measure the seven inches.

ABSTRACT AND CONCRETE.

An *abstract* number is a number of repetition, or the amount of repetition a certain number goes through; but a *concrete* number is the number which thus goes through the process of repetition.

In the process of multiplication, the number that follows the sign is of course by the above definition, abstract, unless we can show that the order of multiplication is immaterial. A concrete number multiplied by a concrete number is inconceivable; when it appears to occur, as in the case 3 ft. $\times$ 2 ft. $=$ 6 sq. ft., the expression would be more accurately written 3 sq. ft. $\times 2 =$ 6 sq. ft. See *Art.* 178.

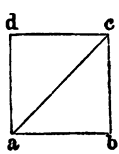
Division is the converse of Multiplication, and may be defined as the art of finding how often one quantity is contained in another. It is of

two kinds: 1st, where two concrete quantities of the same kind are given, and it is required to find how many times one must be repeated to obtain the other: here, $\frac{\text{concrete dividend}}{\text{concrete divisor}}$ gives as quotient an abstract number, *i. e.* so many times. 2ndly, where an abstract number is given as the divisor, and a concrete number, as before, for the dividend; here the question takes the shape "What number requires to be repeated as many times as the divisor indicates, in order to become the dividend?" And the corresponding definition is, Division is the process of finding what number it is that requires to be multiplied by the divisor to produce the dividend. The resulting number may be the same in both cases, the results differing only in kind: thus, we should have $\frac{35 \text{ lbs.}}{5 \text{ lbs.}} = 7$ times; and $\frac{35 \text{ lbs.}}{5} = 7$ lbs.

RATIO.

It was observed in (174) that quantities which cannot be accurately represented by numbers, are said to be incommensurable. And since the Exs. in Ratio and Proportion, which we have considered, have always involved only commensurable quantities, it might be thought that the application of the principles of Ratio and Proportion was limited to such quantities.

But by observing (66), from whence our definition of Ratio was taken, we learn that a Ratio can exist between any two quantities whose magnitude can be represented, as A and B are, *i. e.* geometrically: and since it can be shown* that magnitudes which cannot be represented accurately in numbers, can yet be correctly represented geometrically; therefore a ratio can exist between two or more quantities, even if one or all of them be incommensurable, though the value of that ratio cannot be accurately represented in numbers. Thus, in Fig. 1, page 172, the ratio of the diagonal of a square inch to its side is represented by the fraction $\frac{a\text{———}c}{a\text{———}b}$; or arithmetically, $\frac{1\cdot4142\ldots\ldots}{1}$ or $1\cdot4142\ldots\ldots$



* Some knowledge of Geometry is necessary to enable us to find the length of the line *ac*.

Since *abc* is one angle of a *rectangular* figure, or a *right angle*, we learn from Euclid, I. 47, that the square of which *ac* is the side = the sum of the squares of which *ab* and *bc* are the sides. Now, in Fig. 1, p. 172, we took $ab = 1$, and $ac = 1$; therefore the sum of the squares of *ab* and $ac = 1^2 + 1^2 = 2$; therefore the square of $ac = 2$, or *ac* itself $= \sqrt{2} = 1\cdot4142\ldots$

CIRCULATING DECIMALS.

Sometimes a decimal of very long period may be easily carried out to many places, without performing the division throughout, as in the following Ex.

To reduce $\frac{1}{19}$ to a decimal.

$$\begin{array}{l} 19)\ 1\cdot00\ (\cdot05263 \\ \underline{\ \ \ \ \ 95} \\ \ \ \ \ \ \ 50 \\ \underline{\ \ \ \ \ \ 38} \\ \ \ \ \ \ \ 120 \\ \underline{\ \ \ \ \ \ 114} \\ \ \ \ \ \ \ \ \ \ 60 \\ \underline{\ \ \ \ \ \ \ \ \ 57} \\ \ \ \ \ \ \ \ \ \ \ \ 3 \end{array}$$

By division we have

$$\frac{1}{19} = \cdot05263\frac{3}{19} \qquad \text{(H)}$$

therefore, multiplying both sides by 3,

$$\frac{3}{19} = \cdot15789\frac{9}{19}$$

and substituting for $\frac{3}{19}$ in (H) we have

$$\frac{1}{19} = \cdot0526315789\frac{9}{19};$$

therefore $\frac{9}{19} = \cdot4736842101\frac{81}{19} = \cdot4736842105\frac{5}{19}$;

and hence $\frac{1}{19} = \cdot052631578947368421 05\frac{5}{19}$; and by continuing this process, it is plain that we double at every step the number of figures previously obtained. This decimal, it will be seen, circulates after the eighteenth figure; so that $\frac{1}{19} = \cdot\dot{0}526315789473642\dot{1}$.

PROPORTION.

The pupil who is acquainted with Book VI. of Euclid's Ele-

FIG. 9.

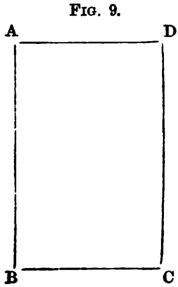
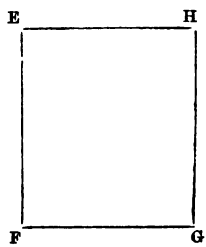


FIG. 10.



ments of Geometry, will know that if two rectangular figures, AC, EG, be equal, the four sides which contain a pair of the equal angles, ABC, EFG, are proportional in the following order,—AB, EF, FG, BC; so that

$$AB : EF :: FG : BC \qquad \text{(J)}.$$

We can show that this proportion is such as would be obtained from the statement of a question in Rule of Three, in which two surfaces AC, EG, are required to be equal; and until we know of this proportion, we have no right to employ the ordinary process of Simple Proportion.

Ex. How wide a piece of cloth, 15 feet long, will cover a floor 13 feet 6 inches long and 10 feet wide?

Let AB be the length = 15 feet; BC the breadth, which is yet to be found; EF = 13 ft. 6 in.; FG = 10 ft.; then, by the usual statement, and writing BC as the fourth term, we have

$$15 \text{ ft.} : 13 \text{ ft. } 6 \text{ in.} :: 10 \text{ ft.} : BC;$$

and this will be found to be the same as (J).

In the language of Geometry, we say that the sides about the equal angles ABC, EFG, are *reciprocally* proportional.

The solution of such a question as the following is worth notice.

The shadow of a steeple is 105 yards long, and that of a stick $4\frac{1}{2}$ feet long is 15 yards: find the height of the steeple.

FIG. 11.

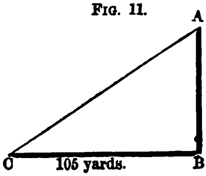
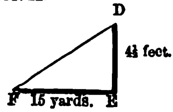


FIG. 12.



Let AB, BC, represent the steeple and its shadow; and let DE, EF represent the stick and its shadow: I have to find the length of AB.

Now, most pupils, seeing this question under the head of Rule of Three, would immediately take it for granted that the three terms given would form a statement; *i. e.* that the ratio between the lengths of the two shadows is equal to that of the steeple and stick by which those shadows are cast. This is quite true; and my object in explaining this sum is merely to show what authority we have for believing that these two ratios are equal. Join AC, DF,—these lines AC, DF, will represent the direction of rays of light from the sun, and being from the same distant body are considered parallel: hence, since AB, DE, are parallel,

as are also FE, CB; the triangles, ABC, DEF, are said to be similar: and from a geometrical property of such triangles, we have the following:

$$EF : BC :: DE : AB;$$

or, substituting the value of EF, BC, DE, which are given by the question, we have

$$15 \text{ yards} : 105 \text{ yards} :: 4\tfrac{1}{2} \text{ feet} : AB;$$

and therefore $AB = \dfrac{105 \text{ yards}}{15 \text{ yards}} \times 4\tfrac{1}{2} \text{ ft.} = 31\tfrac{1}{2}$ feet.

The following Ex. will show that it is not always safe to assume that a question which is apparently a Rule of Three Ex. will at once furnish a statement; *i. e.* that two quantities which appear to be connected, as in ordinary Exs., are really proportional to one another, as in (75) and (76).

Ex. If a ball falling from rest drop through a space of $64\frac{2}{5}$ feet in two seconds, through what distance will it have fallen in three seconds?

Now, if it were true that the distance fallen in any time were *directly proportional* to the time, the statement would be

$$2 \text{ seconds} : 3 \text{ seconds} :: 64\tfrac{2}{5} \text{ feet},$$

and the fourth term would be $96\frac{3}{5}$ feet. But a knowledge of the laws of mechanics corrects this supposition, and teaches us that the distance fallen in any time is not directly proportional to the number of seconds, but to the *square* of that number; and that in the above statement I ought to have put 2^2 and 3^2, or 4 and 9, instead of 2 and 3: hence the statement is

$$4 : 9 :: 64\tfrac{2}{5} \text{ feet};$$

and the distance fallen in three seconds $= 144\frac{9}{10}$ feet.

EXCHANGE.

To show that £1 = 25·17 francs, nearly.

£1 of English gold 22 carats fine is $\frac{11}{12}$ pure, or contains $91\frac{2}{3}$ gold in 100 parts. £1 of French gold contains 90 parts in 100. 1 kilogramme is coined into 155 Napoleons, or 3100 francs; therefore 1 kilogramme of English gold is worth 3157·4 francs; or deducting the charge for coining, namely 6·7 francs per kilog., £1 Eng. gold = 3150·7 francs. Also 1 oz. Troy, or 480 grains = 31·1 grammes, at the rate of 15·432 grains to 1 gramme. The Mint price of standard gold in England = 77*s.* $10\frac{1}{2}$*d.* per oz., because 1 lb. Troy is equivalent to $44\frac{1}{2}$ guineas, or $934\frac{1}{2}$ sh.; therefore 1 oz. is worth $934\frac{1}{2}$*s.* ÷ 12, or 77·875*s.*

Collecting these facts, we have

$$£1 = 20s.$$

$$77{\cdot}875s. = 1 \text{ oz. Troy}$$

$$1 \text{ oz. Troy} = 31{\cdot}1 \text{ grammes},$$

$$1000 \text{ grammes of Eng. gold} = 3150{\cdot}7 \text{ francs};$$

$$\therefore £1 = \frac{20 \times 31{\cdot}1 \times 3150{\cdot}7}{77{\cdot}875 \times 1000} \text{ francs} = 25{\cdot}17 \text{ francs, nearly.}$$

MISCELLANEOUS QUESTIONS.

ON FRACTIONS AND THE PRINCIPLES OF PROPORTION.

1. EXPLAIN the terms "Common Measure," and "Common Multiple."

2. Show how to find the Least Common Multiple of any set of whole numbers. Take any numbers as an Example.

3. When are numbers said to be prime to one another? Give an example of three numbers which are prime to one another, but none of them primes.

4. What do you mean by prime *factors?* Resolve 1512 into its prime factors.

5. Show what is the G. C. M. of 27, 144, 96, by breaking them into their prime factors.

6. Explain the terms *proper* and *improper* fractions. Give an Ex. of each.

7. State the two modes of multiplying fractions by whole numbers. Take an Ex., and show that the modes are true.

8. State also the two modes of *dividing* fractions by whole numbers, and prove their correctness in any example.

9. What do you mean by reducing a fraction to lowest terms? Show the correctness of the process employed.

10. Taking any Ex. of the above reduction, draw a figure which shall enable us to see by inspection that the fraction when so reduced remains unaltered *in value.*

11. For what purpose do we require the L. C. M. of any numbers?

12. Show how to reduce improper fractions to mixed numbers, and the contrary; giving an example of each kind of fractions.

13. What must be done with fractions before they can be added or subtracted? Take as Exs. $\frac{3}{4}+\frac{5}{6}$; $\frac{7}{8}-\frac{3}{5}$.

14. How do you compare two proper fractions, at sight, so as to ascertain which is the larger? Ex. Compare $\frac{7}{11}$ and $\frac{9}{13}$.

15. Prove that $\frac{5}{7}\times\frac{2}{3}=\frac{10}{21}$; and that $\frac{3}{5}$ of $\frac{2}{7}=\frac{6}{35}$.

16. Hence show that *of* and the sign ($\times$) placed between two fractions have the same meaning.

17. Show that the common rule for division of fractions, viz. "In-

vert the divisor, and proceed as in multiplication," is correct. Ex. Prove that $\frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \times \frac{7}{2}$.

18. What do you understand by the brackets in the following Ex. $(\frac{3}{5}+\frac{2}{7}) \times (\frac{2}{3}+\frac{3}{4})$?

19. Explain the process of exhibiting any fractional quantity in positive terms. Ex. Express $\frac{7}{8}$ of 27*s.* in shillings, pence, and fractional parts of a penny.

20. What is the meaning of the term RATIO? How do you express the ratio of 4 to 5, and of 5 to 4?

21. Give distinctive names to the ratios 2 : 5 ; 3 : 3 ; 5 : 2.

22. Exhibit the ratio of 3*s.* to $7\frac{1}{2}$*d.* as an abstract number.

23. What is Proportion? The test of proportionality among 4 quantities is whether or no the product of the extremes equals the product of the means. Prove this.

24. Explain the reason of the following operations in Rule of Three; 1st, The reduction of the 1st and 2nd terms to the same name: 2nd, The multiplying of the 2nd and 3rd terms together and dividing by the 1st.

25. Explain how the term *proportional* is used to embrace two kinds of proportion. Give an Ex. of each.

26. Give a description of the mode of working the following questions:

(1) Reduce 3*s.* 6*d.* to the fraction of £1.

(2) What fraction of a half guinea is equivalent to a moidore?

27. Work the following Ex., and be particular in forming a correct fractional quotient in the pence. Ex. (£8 17*s.* $9\frac{1}{4}$*d.*)$\div 6\frac{7}{9}$.

28. Show how to compare fractional quantities. Ex. Compare $\frac{3}{5}$, $1\frac{2}{7}$, $\frac{5}{9}$.

29. Compare also $\frac{5}{7}$£, $\frac{7}{9}$ of a guinea, and $\frac{5}{8}$ of 16*s.* 8*d.*

30. If $\frac{7}{15}$ths of a piece of work were done in 1 hour, how soon would all the work be done? What relation is there between the work done in 1 hour, and the time of doing all the work?

ON DECIMALS.

1. Explain the term "Decimal Fractions;" and show the principle upon which we write tenths, hundredths, thousandths, &c. decimally.

2. Show how to convert a *vulgar* fraction into a *decimal* fraction, written as a vulgar fraction.

3. Explain what is meant by a power of any number; and write 16, divided by the 4th power of 10, (1) as a vulgar fracn; (2) as a decimal.

4. State the method of converting into a decimal, a fraction which contains a power of 10 as a denominator.

5. How do you multiply a decimal by a power of 10? Ex. $3{\cdot}75 \times 10^4$.

6. How do you divide a decimal by a power of 10? Ex. $18{\cdot}76 \div 10^5$.

7. State what fractions produce *terminating* decimals, and what produce *non-terminating*. Explain the reason.

8. If the den^r of a fr^n, in its lowest terms, which produces a circulating decimal, be known, what may be known concerning the length of the period of the decimal? Illustrate by the Ex. $\frac{6}{7}$.

9. Show how to convert terminating decimals into vulgar fractions. Ex. $\cdot0605$.

10. Explain fully the mode of converting non-terminating decimals into vulgar fractions. Take as Exs. $\cdot\dot{3}4\dot{3}$, $\cdot36\dot{7}2\dot{5}$.

11. State the mode of adding together decimals—(1) terminating, (2) non-terminating.

12. State the mode for subtraction.

13. Show the truth of the common rule for the multiplication of decimals. Ex. Find the value of $1{\cdot}75 \times {\cdot}037$.

14. Explain the mode of performing Long Division in Decimals; and show what varieties may occur in fixing the decimal point in the quotient. Give Exs. in illustration.

15. Show how to perform the operations of multiplication and division of circulating decimals.

16. Find the value of $3{\cdot}756 \times 21{\cdot}9875$, correct to 2 places of decimals: and explain the principle and mode of working by the contracted form of multiplication in decimals.

17. Find the product of $13{\cdot}0586$ by $12{\cdot}758$, without either assuming the rule for multiplication of decimals, or converting them to vulgar fractions.

18. Give Exs. of reduction of decimals,
    (1) From a decimal of £1 into positive terms, as shillings, pence, &c.
    (2) From a quantity involving pounds, shillings, and pence, to the decimal of a moidore. Work both your Exs.

ON PRACTICE.

1. What do you mean by the term "aliquot parts"? Give Exs.

2. What is the highest aliquot part of £1? What of 1*s.*?

3. Write out tables of aliquot parts, (1) of 1*s.* (2) of £1.

4. Find by Practice the cost of 7108 lbs. at $9\frac{3}{4}d$. Explain all the work.

5. Write down one of every variety of Examples in Practice.

6. Explain the mode of finding the cost of $5605\frac{6}{7}$ articles at any price.

7. Show also how to find the value of 1105 yds. at $7s.\ 6\frac{9}{10}d$.

8. Give an Ex. in Practice in which the usual mode of taking aliquot parts of the highest denomination in money cannot be employed. Show how the Ex. is to be worked.

ON PROPORTION.

1. How many terms are generally found in proportion?

2. What is the general object of a question in Rule of Three?

3. In stating a Rule of Three sum, which term do you write down first, and how do you select it?

4. When the statement is completed, describe the remaining steps of the work.

5. Write down the fractional form of the fourth term, in terms of the other three; and hence derive the rules for completing a sum after it is stated.

6. From the fractional form mentioned in the last question, shew that the fourth term will be of the same nature as the third term.

7. Give an example in Rule of Three involving four terms, of which one will not appear in the statement.

8. Show how three terms are to be obtained from the following Ex. What is the length of a floor which is 16 feet broad, and equal in area to a floor 24 feet square?

9. What is the difference between Simple and Compound Proportion?

10. Prove that one statement may be made to produce the same result as two or more separate statements by Single Rule of Three. Construct an Example in illustration.

ON THE APPLICATIONS OF PROPORTION.

1. Explain the terms *Principal, Rate, Interest, Amount.*

2. Distinguish between Simple and Compound Interest.

3. What other questions come properly under the head of Simple Interest?

1 sq. foot × 1 linear inch.
1 sup. pr. × 1 linear inch.
1 sq. inch × 1 linear inch.

8. Show by actual computation of the volume of a block, 4 ft. 6 in. long, 2 ft. 4 in. wide, and 1 ft. 9 in. thick, that the process given by Cross Multiplication is correct.

9. Find the value of the above volume by multiplying together the above dimensions, as fractional parts of a foot.

10. Explain the Gunter's Chain; and show how to convert an area expressed in links into acres, &c.

11. What is meant by an incommensurable quantity? Give an example of such a quantity.

EXTRACTION OF ROOTS.

1. Explain the terms INVOLUTION and EVOLUTION.

2. What is meant by the following expression $\sqrt[5]{18}$?

3. Explain the terms perfect square, cube, &c.

4. What will be the form of a square root of a number which is not a perfect square?

5. Give a name to such quantities as $\sqrt{2}$, $\sqrt[3]{5}$.

6. Explain the mode of *pointing*, previous to the extraction of the square root, both of whole numbers and decimals.

7. Show how to find a multiplier which shall make any proposed number a perfect square, or a perfect cube.

Ex. Find the multiplier which shall make 45, (1) a perfect square, (2) a perfect cube.

8. Write down the squares of all the digits.

9. Give a rule for the extraction of the square root.

10. Show why the incomplete divisor does not always give a correct figure in the root.

11. How do you find any power of a fraction? How do you extract the root of a fractional quantity?

12. Give two methods of extracting the square root of a fractional quantity, of which the denominator is not a perfect square.

13. Give a rule for the extraction of the Cube Root.

14. Give a rule for the extraction of any root whatever.

15. Extract the square root of 576, and the cube root of 1728, in an algebraical form.

CAMBRIDGE: PRINTED AT THE UNIVERSITY PRESS.

By the same Author,

ARITHMETIC, Part I. comprising the Elementary Rules, Reduction in two Parts, and Decimals applied to the Simple Rules. 1*s*. 6*d*.

Parts I. and II. 4*s*. 6*d*. or with Answers, 5*s*. 6*d*.

The Questions in Part II. 1*s*.

Answers to both Parts, 1*s*.

KEY to Lund's Mensuration, 3*s*.

ANSWERS

1.

40; 70; 60; 30; 90; 20; 50; 10; 80; 100.

Twenty; Sixty; Ninety; One Hundred; Fifty; Thirty; Eighty; Seventy; Forty.

2.

300; 800; 700; 100; 1000; 400; 200; 500.

Six Hundred; Eight Hundred; Three Hundred; One Hundred; One Thousand; Nine Hundred; Four Hundred; Seven Hundred; Two Hundred; Five Hundred.

3.

5,000; 9,000; 30,000; 300,000; 80,000; 70,000; 700,000; 1,000,000.

Four Thousand; Eighty Thousand; Six Hundred Thousand; Eight Hundred Thousand; One Thousand; Seventy Thousand One Thousand Thousand, or, One Million.

4.

4,000,000; 9,000,000; 70,000,000; 100,000,000; 300,000,000; 800,000,000.

Forty Millions; Eight Millions; Sixty Millions; Two Millions; Fifty Millions; Eighty Millions; Six Hundred Millions; One Thousand Millions.

5.

14; 22; 61; 77; 85; 93; 48; 51; 34; 91; 19.

Twenty-eight; Thirty-seven; Forty-five; Eighty-nine; Seventy-three, Sixty-four; Fifty-eight; Seventeen; Twenty-one; Eighty-eight; Thirty-three; Ninety-nine.

6.

507; 1819; 4017; 920; 811; 9416; 2004.

Three Hundred and Two; Four Hundred and Twenty-seven; One Hundred and Nine; Seven Hundred and Four; Eighteen Hundred and Forty-five; Nineteen Hundred; Seven Thousand Three Hundred and Twenty-four; Eight Thousand Nine Hundred and Seven; Fourteen Hundred and One; Three Thousand Eight Hundred and Seventy; One Thousand and Forty-four; Nine Thousand and Nine; Ten Thousand.

7.

50,006; 170,018; 405,039,001; 3005,000,609; 1,000,043.

Seven Hundred Thousand Four Hundred and One; One Million Four Hundred Thousand Nine Hundred and Six; Forty Thousand and Ten; Eighty Millions Forty Thousand and Seventeen; Four Hundred Millions Four Hundred and One Thousand and Ninety; Thirty-six Thousand and Eleven; Three Thousand Two Hundred and Forty-five Millions Sixty-eight Thousand and Eighteen.

8.

26; 36; 18; 27; 33; 53; 82; 90; 109; 111; 164; 354; 433.

9.

1.	3294	10.	30805	19.	1675085	28.	11816604
2.	5776	11.	142076	20.	2048802	29.	21004076
3.	3843	12.	122620	21.	2546960	30.	7013186
4.	35226	13.	540321	22.	25224832	31.	23458319
5.	50155	14.	529409	23.	8339252	32.	38746717
6.	35942	15.	37661	24.	13002041	33.	10027804
7.	36048	16.	493616	25.	25620569	34.	113908648
8.	33397	17.	1668015	26.	17412631	35.	5529093
9.	49863	18.	373821	27.	14231579	36.	7629344

10.

1.	24812	9.	3257368	17.	67870123	25.	864197532
2.	7713	10.	6675391	18.	72110823	26.	57074598
3.	10101	11.	5264254	19.	378128289	27.	987654322
4.	333333	12.	7461266	20.	101541012	28.	8877706050
5.	91122	13.	864999	21.	301098907	29.	80627217
6.	410778	14.	3329989	22.	777608981	30.	8377784877
7.	5108775	15.	1660687	23.	85438111	31.	2818644445
8.	7040206	16.	16575639	24.	244549286	32.	7966214609

11.

1. 2634
2. 21899
3. 93183
4. 15391
5. 60
6. 2707
7. 5856
8. 908006
9. $307+1001 = 1400 - 92$
10. $70+18+5004 - 709 = 4383$
11. One thousand diminished by four hundred and fifty seven, increased by one hundred and ninety-three, diminished by seventy-five, amounts to six hundred and sixty-one.
    or, more briefly, 1000, minus 457, plus 193, minus 75, equals 661.
12. The sum of three thousand and forty-five, and of six thousand two hundred and eight, is equal to the difference between ten thousand and one, and seven hundred and forty-eight.
    or, 3045, plus 6208, equals 10001, minus 748.
13. 4198.
14. $1850 - 401 = 1449$.

12.

I.	II.	III.	IV.
6913578	19232862	7502788	43282743
10370367	25643816	9378485	49465992
13827156	32054770	11254182	55649241
17283945	38465724	13129879	
20740734	44876678	15005576	
24197523	51287632	16881273	

13.

I.	II.	III.
34567890	15712640	41114709000
345678900	23568960	365464080
34567890000	471379200	2740980600
	6285056000	319781070000

14.

1. $7 \times 4 + 3 - 8 = 23$
2. $19 + 325 = 43 \times 8$
3. $45 + 119 = 41 \times 4$
4. $1000 - 355 = 43 \times 15$

15.

I.	II.	III.	IV.
68015739	103852824	29900356089	1340721082368
74198988	2906272822	385545173346	3062399370432
80382237	5402149770	446902471888	2630622007968
86565486	6775719634	47987095224	3333181579776
92748735			
98931984			

1. 19043772736	7. 665991118517072	13. 46534780060041
2. 2209088063364	8. 6355275989629	14. 1327855904511542
3. 1472624992321	9. 51242170689540	15. 2758422761998619
4. 765899100120	10. 33607289157312	16. 214303349934732
5. 3634106380016	11. 71297820058620	17. 68843201227955
6. 14558651711651	12. 24987612372412	18. 330979824248580

16.

1. 170
2. 126
3. Circumf. × ht. = 32 × 6 = 192
4. 225
5. 26250

17.

I.	1924814		III.	29752718	2 rem^r
	1283209	1 rem^r		21251941	5 „
	962407			16529288	
	769925	3 „		18595449	
II.	14960720		IV.	573461501	7 rem^r
	12823474	2 rem^r		509743557	2 „
	11220540			417062910	5 „
	.9973813	3 „		382307667	11 „

18.

I.	2688806	7 rem^r	II.	34960547	20 rem^r	III.	2393690	18 rem^r
	2352705	11 „		24278158	7 „		3517259	7 „
	2091293	17 „		15607387	23 „		1424344	74 „
	1792537	14 „		10790292	43 „			

19.

1.	10732008	19 rem^r	7.	43104115	29 rem^r
2.	138453712	5 „	8.	548361197	17 „
3.	852636688	6 „	9.	41152205	11 „
4.	93728911	72 „	10.	48754142	10 „
5.	47086124	74 „	11.	593354917	64 „
6.	54156011	97 „	12.	781459741	80 „

20.

1.	1551727	21 rem^r	4.	1065702283	38 rem^r
2.	50652426	3 „	5.	94521982	5 „
	99722706	3 „	6	177902356	17 „

7.	55477849	10 remr	16.	51472699	838 remr
8.	2771561329	1 „	17.	11034128	10 „
9.	4506092	91 „	18.	810259	616 „
10.	19406738	15 „	19.	89966740	1018 „
11.	5011466	273 „	20.	13246104	2288 „
12.	19846833	359 „	21.	78113255	655 „
13.	17540722	331 „	22.	759210	12589 „
14.	21310159	147 „	23.	252842	14157 „
15	4868620	466 „	24.	981476	69434 „

21.

I.	3874910	2 remr	II.	12937457	17 remr	III. 87269	863 remr
	387491	2 „		8624971	27 „	9817	6963 „
	38749	102 „		6468728	37 „	157	42963 „
				517498	157 „		
				43124	5157 „		

22.

1. 10
2. 145
3. $2812 \div 76 = 37$
4. Twice the breadth = 36 in.
   Area of top and bottom $= 36 \times 20$
   $= 720$
5. Length $= 225 \div 15 = 15$
6. $175 \div 25 = 7$
   $175 \div 35 = 5$
7. Length of 1st oblong = 24 ft.
   Breadth of „ = 6 ft.
   Circumf. of 1st oblong = 60 ft.
   Circumf. of 2nd „ = 76 ft.

23.

1. 35822259.
2. Quott = 2.
3. 603.
4. 384116499.
5. 8226.
6. 1364142.
7. £5.
8. 10,200,000.
9. 35517.
10. Less = 14606.
    Product = 456685802.
11. 324.
12. 94,080,000.
13. 30492.
14. 109.
15. 150 yds. 11250 sq. yds.
16. 6120.
17. 6075.
18. Each = 2711430.
19. $3846 \times 705 = 51517170 + 19$
20. $368979 + 335342 = 85 \times 709$
    $+ 11501 \times 56 = 70432.$

21. 11825 lines
532125 letters.
22. 90756.
23. 668025.
24. 856.
25. $(75992 - 242) \div 202 = 375$.
26. $(37200 \div 496) - (45696 \div 238) = 117$.
27. Multiply the divisor by the quotient, and add in the remainder;
Dividend $= 345 \times 178 + 27 = 61437$.
28. 70043.
29. 12317.
30. 613331.
31. 39960.
32. 2700.
33. 98.
34. 5066250.
35. 118.
36. 14960 lines
790570 letters.
37. $375 \times 4685 = 1756875$
Rem^r $= 1756976 - 1756875 = 101$.
38. 32 periods.
39. 875 men $\div$ 125 men $= 7$.
There are 7 $\times 5$, or 35, officers; and $840 \div 35$, or 24 privates to one officer.
40. See Art. 26.

24.

1. 18000 pence.
2. 129600 far.
3. 131040 pence.
4. 348300 pence.
5. 9600 farthings.
6. 26880 ounces.
7. 7454720 drams.
8. 1440 drams.
9. 4080 dwts. 97920 grs.
10. 12000 nails.
11. 3024000 seconds.
12. 362400 sheets.

25.

1. 18186 pence.
2. 20046 farthings.
3. 51028 pence.
4. 195330d. 781323 f.
5. 1739237 drams.
6. 98007 grains.
7. 1214 hf. pts.
8. 50579 scruples.
9. 11327 quarts.
10. 7559 nails.
11. 525948 minutes.
12. 261665 poles.

26.

1. 1383 crowns.
2. 40015 half crowns.
3. 73749 sixpences.
4. 22316 groats.
5. 28493 twopences.
6. 7908 parcels of 4 oz.
7. 4002 spaces.
8. 14060 parcels.
9. 873 quarts.
10. 25955 parcels.
11. 22592 spaces.
12. 13557 half quarters.

27.

1. £144 16s. 6d.
2. 74 guin. 16s. $7\frac{1}{4}$d.
3. 424 moid. 6s. 8d.
4. 131 lbs. 4 oz. 15 dwts. 3 g.

5. 3wks. 6d. 11h. 36m. 38s.
6. 71t. 12c. 21lbs. 13oz. 9dr.
7. 14473qrs. 7bush. 3pks. 1g. 1qt. 1pt.
8. 1038dys. 22hrs. 48min. 41sec.
9. 15058 hhds. 25 gals.
10. 26yrs. 154dys. 14hrs. 4min.
11. 6lbs. 8oz. 3drs. 0sc. 16grs.
12. 83 bar. 5 gals. 0 qts. 1 pt. 1 hf. pt.

28.

1. 423 moid. 4s.
2. 607 crs. 4s. 5¼d.
3. 2457 sixp. 2d. or, 491 hf. crs. 1s. 2d.
4. £4323 12s. 6d.
5. 915 moid. 10d.
6. 16609 pieces, 2s. 4d.
7. 21 portions, 5lbs. 4oz.
8. 4587 spaces.
9. 3824 reams, 16 quires.
10. 32 mea. 1 hhd. 11 gals.
11. 2921 c. 0 qrs. 8lbs. 1 par.

29.

1. 6110172 sq. in.
2. 27987 yds.
3. 22629200 yds.
4. 2401245 pence.
5. 4059072 halfpence.
6. 277 fath. 4 ft. 8 in.
7. 522 fath. 0 ft. 5 in.
8. 7 m. 1539 yds. or, 7 m. 6 fur. 39 po. 4½ yds.
9. 30 lea. 0 m. 4 fur. 29 po. 2½ ft.
10. 2186 ac. 2 ro. 36 po.
11. 4 ro. 15 po. 28¼ yds. 1 ft. 87 in.
12. 44 sq. m. 567 ac. 0 ro. 8 per.

30.

1. 225 coins 3s.
2. 1400.
3. 653 coins 6s. 4d.
4. 123 coins 14s. 3d.
5. 9234 pieces 1¼ nls.
6. 1868 portions 1 oz.
7. 3072.
8. 31015 interv. 7½ sec.
9. 317 dists. 1¼ fur.
10. 3750.
11. 1400.

31.

	£	s.	d.
1.	3462	18	11¾
2.	8852	13	7½
3.	14561	9	1
4.	58176	13	10½
5.	98110	13	8
6.	986124	1	3½
7.	101567	7	8¾
8.	297966	3	1¼
9.	356106	7	7¼
10.	675224	4	8¾

LONG MEASURE.

	yds.	ft.	in.	b c.
11.	91	0	4	0
12.	583	2	11	1

	mls.	yds.	ft.	in.
13.	1346	39	0	6
14.	12259	125	0	9

	fu.	po.	yds.	ft.
15.	2306	8	1½	2
16.	3344	28	4½	2

TROY WEIGHT.

	lbs.	oz.	dwts.	grs.		lbs.	oz.	dwts.	grs.
17.	1418	6	4	10	19.	5438	5	0	23
18.	7636	6	9	14	20.	3904	7	12	3

AVOIRDUPOIS WEIGHT.

	lbs.	oz.	drs.			cwts.	qrs.	lbs.	oz.
21.	457	12	8		23.	4676	3	9	15
22.	2655	9	7		24.	11224	2	11	4
	tons	cwts.	lbs.	oz.		tons	cwts.	lbs.	oz.
25.	9750	0	92	4	26.	5444	3	54	5

APOTHECARIES' WEIGHT.

	oz.	drs.	sc.	grs.		lbs.	oz.	drs.	sc.
27.	266	2	2	9	29.	2914	9	7	0
28.	754	3	2	2	30.	4388	6	4	2

CLOTH MEASURE.

	yds.	qrs.	nls.	in.		yds.	qrs.	nls.	in.
31.	819	0	1	1$\frac{3}{4}$	32.	1857	3	3	0$\frac{1}{4}$

WINE MEASURE.

	hhds.	gals.	qts.	pts.		hhds.	gals.	qts.	pts.
33.	307	10	3	1	35.	1383	54	0	1
34.	838	1	1	1	36.	829	12	2	0

ALE AND BEER MEASURE.

	hhds.	gals.	qts.	pts.		hhds.	bar.	kild.	gals.
37.	738	21	1	1	38.	687	1	0$\frac{3}{4}$	15

SQUARE MEASURE.

	ac.	ro.	po.	yds.		sq. m.	ac.	yds.
39.	8292	0	8	18$\frac{1}{2}$	41.	16766	182	3961
40.	38838	3	6	2$\frac{3}{4}$				

CUBIC MEASURE.

	sol. yds.	ft.	in.		sol. yds.	ft.	in.
42.	14909	7	277	43.	13274	7	1092

PAPER.

	reams	qui.	shts.		reams	qui.	shts.
44.	4027	8	5	45.	15436	18	3

32.

	£	s.	d.		£	s.	d.
1.	9	12	10½	7.	29605	18	6½
2.	57	18	8¾	8.	6697	0	9½
3.	723	6	8¾	9.	36882	15	7¾
4.	3692	19	1½	10.	39786	18	4¾
5.	4583	7	9¾	11.	828057	19	11½
6.	8891	13	1¾	12.	1476155	16	10¾

TROY WEIGHT.

	lbs.	oz.	dwts.	grs.		lbs.	oz.	dwts.	grs.
13.	85	3	2	17	15.	1818	8	3	18
14.	7656	1	18	12	16.	5157	9	14	11

AVOIRDUPOIS WEIGHT.

	tons	cwts.	qrs.	lbs.		cwts.	lbs.	oz.	drs.
17.	1465	12	2	19	19.	2676	1	12	9
	cwts.	qrs.	lbs.	oz.					
18.	6839	2	23	12	20.	2355	93	8	3

CLOTH MEASURE.

	yds.	qrs.	nls.	in.		Fr. Ells	qrs.	nls.	in.
21.	2907	2	1	1½	23.	256	1	1	0¼
	E. Ells	qrs.	nls.	in.		Fl. Ells	qrs.	nls.	in.
22.	106	3	2	1¼	24.	3808	2	1	1¾

WINE MEASURE.

	bar.	gals.	qts.	pts.		hhds.	gals.	qts.	pts.
25.	485	32	2	1	27.	7988	53	0	1
26.	1638	29	1	1	28.	1909	15	3	1

SQUARE MEASURE.

	sq. yds.	ft.	in.		ac.	ro.	sq. po.	sq. yds.
29.	6257	3	85	31.	357	2	23	25¼
30.	5646	8	88	32.	438	3	23	28¾

TIME.

	dys.	hrs.	min.	sec.		wks.	dys.	hrs.	min.
33.	177	17	33	36	35.	238	5	19	4
34.	79	10	22	49	36.	68	3	7	58

DRY MEASURE.

	wys.	qrs.	bush.	pks.		lasts	wys.	qrs.	bush.
37.	28	2	6	1	39.	235	1	4	4
38.	40	0	5	3	40.	1017	0	4	4

33.

	£	s.	d.		£	s.	d.		£	s.	d.
I.	39	14	4½	II.	94	7	6¼	III.	4112	7	5¼
	47	13	3		107	17	2		4569	6	0½
	55	12	1½		121	6	9¾		5026	4	7¾
	63	11	0		134	16	5½		5483	3	3

34.

	£	s.	d.		£	s.	d.		£	s.	d
1.	4863	16	8	4.	100004	3	2	7.	361313	6	6
2.	17408	1	0	5.	41907	1	6	8.	943786	0	0
3.	73165	16	8	6.	267327	2	0				

35.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	195	1	4½	9.	6204	10	7¼	17.	14511	7	10¾
2.	467	7	0	10.	31074	19	0	18.	4279	17	4¾
3.	689	19	4¾	11.	39000	0	0	19.	3682	19	4½
4.	1118	9	3¾	12.	37109	6	8	20.	14344	9	7½
5.	2189	12	11	13.	447	11	8¼	21.	139325	10	5
6.	3182	14	8¼	14.	3827	5	6	22.	11395	2	6
7.	8423	16	2	15.	15816	0	3¾	23.	76058	8	4
8.	22400	3	10	16.	1369	16	6¾	24.	16511	6	3

TROY WEIGHT.

	lbs.	oz.	dwts.		lbs.	oz.	dwts.	grs.
25.	553	8	15	28.	2054	8	17	5
26.	4316	1	1	29.	35961	8	13	8
27.	10501	7	11	30.	1523554	2	14	11

AVOIRDUPOIS WEIGHT.

	cwts.	lbs.	oz.	drs.		tons	cwts.	lbs.	oz.
31.	1086	16	7	12	34.	8031	14	20	2
32.	28424	102	2	5	35.	31857	0	105	0
33.	139828	88	3	13	36.	151969	6	26	11

APOTHECARIES' WEIGHT.

	oz.	drs.	sc.	grs.		lbs.	oz.	drs.	sc.
37.	2577	6	0	5	40.	6551	2	0	2
38.	2972	1	0	9	41.	29109	11	3	2
39.	1025	0	0	5	42.	15897	2	3	0

LONG MEASURE.

	mls.	fur.	po.	yds.
43.	15651	5	19	4
44.	25352	2	6	$4\frac{1}{2}$
45.	168612	5	25	$1\frac{1}{2}$

	fur.	po	yds..	
46.	9377	30	4	1
47.	353862	31	2	1
48.	658558	1	2	0

CLOTH MEASURE.

	yds.	qrs.	nls.
49.	34816	1	3
50.	173545	0	2
51.	373113	0	0

	E. ells.	qrs.	nls.	in.
52.	38337	3	2	1
53.	430187	2	0	$1\frac{1}{2}$
54.	1134654	0	0	$1\frac{1}{2}$

ALE AND BEER MEASURE.

	hhds.	gals.	qts.
55.	11265	49	2
56.	100831	32	1
57.	3607991	9	2

	bar.	gals.	qts.	pts.
58.	27988	0	0	1
59.	478612	31	2	0
60.	375158	2	1	1

WINE MEASURE.

	hhds.	gals.	qts.
61.	5716	40	3
62.	50500	46	2
63.	82071	55	3

	hhds.	gals.	qts.	pts.
64.	18428	54	1	1
65.	60112	7	0	0
66.	285157	21	0	0

SQUARE MEASURE.

	yds.	ft.	in.
67.	5519	7	3
68.	10018	1	108
69.	23084	5	29

	ac.	ro.	po.	yds.
70.	8881	1	30	$22\frac{3}{4}$
71.	31292	1	17	$14\frac{1}{2}$
72.	126748	0	2	$29\frac{1}{4}$

CUBIC MEASURE.

	sol. yds.	ft.	in.
73.	2925	5	440
74.	15526	19	576

	sol. yds.	ft.	in.
75.	35161	16	1496
76.	257007	20	1143

WOOL WEIGHT.

	sacks.	tods.	lbs.
77.	12626	7	15
78.	83861	5	12

	packs.	lbs.
79.	428313	15
80.	563410	125

TIME.

	hrs.	min.	sec.
81.	15717	11	15
82.	21484	48	56

	dys.	hrs.	min.	sec.
83.	217810	21	6	31
84.	468404	2	6	55

36.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
I.	29	7	6	;	17	12	6	;	12	11	9¼	5 rem^r			
II.	89	7	8½	;	53	12	7½	;	44	13	10¼	;	26	16	3¾
III.	129	7	9	;	86	5	2	;	64	13	10½	;	43	2	7

TROY WEIGHT.

	lbs.	oz.	dwts.	grs.				lbs.	oz.	dwts.	grs.		
1.	3	9	3	3	1	rem^r	4.	11	9	3	11	6	rem^r
2.	25	1	12	13	2	„	5.	32	8	14	11	5	„
3.	34	3	15	3	2	„	6.	152	3	16	17		

AVOIRDUPOIS WEIGHT.

	cwts.	lbs.	oz.	drs.				tons.	cwts	qrs.	lbs.		
7.	12	58	9	7	5	rem^r	9.	5	13	3	27		
8.	45	13	8	13	1	„	10.	13	1	0	8	1	rem^r

37.

	£	s.	d.				£	s.	d.		
1.	17	12	6			7.	19	17	3¼	35	rem^r
2.	8	18	9	89	rem^r	8.	18	16	9¼	66	„
3.	21	7	1½			9.	46	7	2¾	48	„
4.	17	12	7¼	26	„	10.	87	12	11¼	54	„
5.	9	8	10¼	45	„	11.	47	16	0¼	30	„
6.	28	8	6½	1	„	12.	47	17	11¾	98	„

LONG MEASURE.

	mls.	fur.	po.	yds.				fath.	yds.	ft.	in.		
13.		4	26	1	20	rem^r	16.		1	2	5	9	rem^r
14.	1	3	6	3	34½	„	17.	1	0	1	9	76	„
15.	1	1	22	4	70½	„	18.	7	1	2	9	41	„

TIME.

	yrs.	wks.	dys.	hrs.				yrs.	dys.	hrs.	min.		
19.	2	5	3	19	3	rem^r	22.	2	91	11	31	41	rem
20.	1	50	6	13	28	„	23.	12	201	2	51	66	„
21.	1	49	3	12	23	„	24.	8	97	3	43	107	„

38.

	£	s.	d.				£	s.	d.		
1.		7	10	108	rem^r	3.	1	2	8	273	rem^r
2.	1	0	1¾	157	„	4.	2	14	2¼	187	„

	£	s.	d.			£	s.	d.	
5.	2	17	3¼	574 remʳ	13.	2	8	10¾	674 remʳ
6.	9	19	10¼	227 „	14.	3	16	8	204 „
7.	11	0	2¾	671 „	15.		10	11¼	270 „
8.	9	2	1	204 „	16.	2	18	4¾	1879 „
9.	6	3	7	133 „	17.		15	0¾	5345 „
10.	6	2	8¼	407 „	18.		8	10¾	2964 „
11.	1	19	10¾	49 „	19.		16	0¼	2673 „
12.	3	2	5	780 „	20.	6	1	1¼	1058 „

SQUARE MEASURE.

	ac.	ro.	po.	yds.			sq. m.	ac.	ro.	po.	
21.	1	1	16	7	428¾ remʳ	24.	14	404	0	28	557 remʳ
22.			35	25	285¾ „	25.	9	585	3	14	858 „
23.			23	21	1177 „	26.	7	195	1	15	894 „

CUBIC MEASURE.

	sol. yds.	ft.	in.			sol. yds.	ft.	in.	
27.	2	26	58	136 remʳ	29.	6	2	1571	1449 remʳ
28.	2	9	1381	3963 „	30.	6	26	1179	424 „

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31. 17s. 4½d. 32. £28 4s. 4d. 33. £4 2s. 6d. 34. 15s. 2d.

39.

1. 844.
2. 16814. 4 ft. 2 in. over.
3. 161384. 2 ft. remʳ.
4. 18625.
5. 3 ft. 5 in.
6. 10 feet.
7. £526 11s. 6d.
8. 5067 gals. 0 qts. 1½ pts.
9. 59 parcels, 10¼ lbs. over.
10. 61 of each, 20 poles over.
11. 371 of each, 8s. 10d. remʳ. £470 7s. 6d.
12. 921 in all; 2a. 2r. 5p. remʳ
13. £2405.
14. 3438¾ yds.
15. 284 vessels, 15 gals. over.
16. 380 times; 12 oz. 4 drs. over.
17. 3506 intervals; ½ min. over.
18. 33 times.
19. 238 a. 0 r. 20 p.
20. 4 seconds for each change; or one-tenth of 4 sec. between each stroke.

40.

1. 13 doz. 2 spoons. 159 grs.
2. 384 of each.
3. 7848 hours.
4. 5850 dys. 20 hrs. 5⅓ min.

5. 4805 hours.
6. £3 0s. 11d.
7. 485760.
8. £11326 6s. 8d.
9. 266 coats $1\frac{3}{4}$ yds. over.
10. 4631.
11. Required number $= 385 \times 385 + (7 \times 5) = 4235$.
12. 94395; 1 min. 15 sec. over.
13. 5000 pieces.
14. 61 yds. 18 ft. 460 in.
15. 80 times.
16. 80 yards.
17. £2575.
18. £62 10s. 5d.
19. 94 sol. ft. 3 in.
20. 41666 persons, 104 rem^r.
21. 21 ac. 4785 sq. yds.
22. 19 seconds and one-fifth.
23. 5824.
24. $9\frac{1}{2}$ dozen.
25. £1501 14s. 10d.
26. £11 7s. 3d.
27. 3500.
28. £2648 8s. 9d.
29. £2 17s. 4d. 192 rem^r.
30. 523776.
31. £1654 13s. 5d.
32. £12125 10s. 6d.
33. 1560.
34. 176579 miles per sec. 498 rem^r.
35. Weight of rails $= 70 \times 80 \times 1760$ lbs.
    Weight of chairs $= 80 \times 1760 \times 2 \times 14$ lbs. $+ 56$ lbs.
    Whole weight $=$ 6160 tons 56 lbs.
36. 816 lbs. 8 oz.
37. $192\frac{1}{2}$ of each sort.
38. 84.
39. One minute would have elapsed since the last bell in the 44th round.
40. 7313464 intervals 2 min. 22 sec.
41. 23496.
42. £13 15s. 1d. to each child.
    £27 10s. 2d. to each woman.
    £55 0s. 4d. to each man.
43. 5s. $6\frac{1}{2}$d.
44. Area of court yard $=$ 50 yds. $\times$ 50 yds. $=$ 2500 sq. yds.
    Area of grassplot $=$ 34 yds. $\times$ 34 yds. $=$ 1156 „
    Area of walk $=$ 1344 „
45. He gains 5 yds. in 110 sec.
    or 1 yd. in 22 sec.
    therefore 880 yds. in 880×22 sec., or 19360 sec.
    or in 5 hrs. 22 min. 40 sec.

ON THE PRINCIPLES OF THE SIMPLE RULES.

1. Art. 1.
2. Arts. 2, 3, &c.
3. Art. 1.
4. $365+4000+18-1728-496 = 2159$.
5. Arts. 11, 12.
6. Art. 16.
7. By seeing if the sum of the lower line and the answer will give the upper line.
8. $17 + 8 = 36 - 11$.
9. $18 \times 12 = 2376 + 11$.
10. See end of Art. 19.
11. (Dividend – rem^r) ÷ quotient = divisor.
12. Art. 17.
13. Art. 24.
14. Art. 22.
15. See end of Art. 28.
16. Art. 25.
17. Multiply the divisor by the quotient, and add in the rem^r; the result should be the dividend.
18. Art. 23.
19. Art. 26.
20. Art. 28.

ON REDUCTION AND THE COMPOUND RULES.

1. Art. 30.
2. Art. 31.
3. Art. 29.
4. Art. 29.
5. Art. 29.
6. (1) Art. 34.
   (2) Art. 36.
7. Arts. 42, 43.
8. Art. 52.
9. Arts. 42—44.
10. Art. 39.
11. Art. 40.
12. What is the area of a surface, which can be divided into 55 oblongs, each 8 inches by 5? *Ans.* 55×40 sq. in. = 2200 sq. in.

ANSWERS.

1.

1.	$2 \times 3 \times 5 \times 7 \times 5$.	7.	$2 \times 2 \times 2 \times 2 \times 2 \times 2$.
2.	$5 \times 5 \times 5 \times 3 \times 7$.	8.	$3 \times 7 \times 11 \times 19$.
3.	$3 \times 3 \times 3 \times 11 \times 5$.	9.	$3 \times 11 \times 5 \times 2 \times 7$.
4.	$3 \times 5 \times 7 \times 11$.	10.	$2 \times 3 \times 7 \times 151$.
5.	$2 \times 2 \times 5 \times 7 \times 13$.	11.	$2 \times 2 \times 3 \times 3 \times 3 \times 53$.
6.	$2 \times 7 \times 7 \times 7 \times 7$.	12.	$2 \times 2 \times 2 \times 2 \times 73$.

2.

1.	24.	4.	1260.	7.	6552.	10.	17017.	13.	720.
2.	720.	5.	120.	8.	2520.	11.	3960.	14.	15120.
3.	9240.	6.	1260.	9.	5940.	12.	420.		

3.

1.	6.	3.	7.	5.	9.	7.	11.
2.	85.	4.	1.	6.	1.	8.	27.

4.

I. $\frac{5}{3}, \frac{5}{2}, \frac{20}{6}, \frac{25}{6}, 5, \frac{35}{6}, \frac{40}{6}, \frac{45}{6}$

II. $\frac{19}{8}, \frac{95}{24}, \frac{19}{4}, \frac{19}{3}, \frac{171}{24}, \frac{19}{2}$

III. $\frac{4}{9}, \frac{8}{27}, \frac{2}{9}, \frac{8}{45}, \frac{8}{54}, \frac{8}{63}, \frac{1}{9}, \frac{8}{81}$

IV. $\frac{18}{49}, \frac{12}{49}, \frac{9}{49}, \frac{36}{343}, \frac{36}{392}, \frac{4}{49}, \frac{2}{49}$

5.

1.	$\frac{3}{5}$	3.	$\frac{11}{17}$	5.	$\frac{171}{353}$	7.	$\frac{5}{7}$	9.	$\frac{153}{520}$	11.	$\frac{7}{11}$
2.	$\frac{5}{9}$	4.	$\frac{19}{43}$	6.	$\frac{83}{100}$	8.	$\frac{815}{937}$	10.	$\frac{11}{90}$	12.	$\frac{81}{49}$

6.

1. $3\frac{3}{7}$	4. $21\frac{15}{28}$	7. $101\frac{2}{99}$	10. $9\frac{835}{844}$
2. $7\frac{5}{11}$	5. $108\frac{7}{113}$	8. $99\frac{99}{100}$	11. $31\frac{4}{5}$
3. $19\frac{1}{5}$	6. 328	9. $9\frac{175}{176}$	12. $45\frac{86}{87}$

7.

1. $\frac{13}{7}$	3. $\frac{171}{13}$	5. $\frac{14863}{114}$	7. $\frac{30626}{175}$	9. $\frac{16100}{111}$
2. $\frac{33}{8}$	4. $\frac{8009}{29}$	6. $\frac{20061}{199}$	8. $\frac{15556}{343}$	10. $\frac{6937}{90}$

8.

1. $\frac{520,\ 576,\ 560,\ 198,\ 675}{720}$

2. $\frac{264,\ 504,\ 440,\ 297,\ 396}{1848}$

3. $\frac{8976,\ 9945,\ 10080,\ 11560}{12240}$

4. $\frac{1512,\ 2100,\ 2160,\ 2205,\ 2240}{2520}$

5. $\frac{80,\ 96,\ 105,\ 110,\ 83}{120}$

6. $\frac{945,\ 819,\ 770,\ 486,\ 574}{1260}$

7. $\frac{3528,\ 5832,\ 525,\ 6120,\ 4060}{7560}$

8. $\frac{2772,\ 306,\ 2057,\ 1734}{3366}$

9. $\frac{110,\ 118,\ 85,\ 1}{120}$

9.

1. $2\frac{157}{240}$	3. $3\frac{193}{720}$	5. $17\frac{73}{240}$	7. $28\frac{688}{693}$
2. $2\frac{763}{5400}$	4. $1\frac{1301}{1350}$	6. $32\frac{23}{44}$	8. $34\frac{543}{1820}$

10.

1. $\frac{1}{21}$	4. $\frac{17}{24}$	7. $1\frac{5}{56}$	10. $3\frac{5}{6}$	13. $10\frac{5}{16}$	16. $9\frac{13}{80}$
2. $\frac{19}{90}$	5. $\frac{283}{576}$	8. $6\frac{29}{60}$	11. $4\frac{2}{9}$	14. $\frac{13}{20}$	17. $13\frac{631}{945}$
3. $\frac{9}{56}$	6. $\frac{1}{342}$	9. $3\frac{9}{10}$	12. $16\frac{1}{2}$	15. $10\frac{8}{9}$	18. $16\frac{1187}{1260}$

11.

1. $\frac{16}{35}$	3. $11\frac{2}{3}$	5. 112	7. $14\frac{73}{180}$
2. $\frac{320}{1323}$	4. $2\frac{531}{2432}$	6. $\frac{1}{28}$	8. $\frac{49}{55}$

12.

1. $1\frac{3}{8}$ 3. $11\frac{1}{4}$ 5. $1\frac{3}{4}$ 7. $\frac{33}{290}$ 9. $\frac{17}{22}$

2. $1\frac{1}{2}$ 4. $21\frac{7}{12}$ 6. $\frac{121}{195}$ 8. $\frac{12}{31}$ 10. 1.

13.

I. $1\frac{50}{77}$; $2\frac{13}{77}$; $5\frac{212}{231}$ III. $171\frac{1}{9}$; $\frac{232}{2541}$

II. $\frac{9}{245}$; $5\frac{33}{34}$; $6\frac{23}{306}$ IV. $\frac{2399}{3080}$; $\frac{4601}{4704}$

14.

I. 6s. 8d.; £1 3s. $7\frac{1}{2}$d.; 2s. $2\frac{26}{35}$d.

II. 16 cwt. 1 qr. 12 lbs. 11 oz. $10\frac{8}{11}$ drs.; 540 sol. in.; 3 b. 2 p. $1\frac{1}{18}$ gls.

III. $39\frac{1}{3}$ gals.; 3 dys. 4 hrs. 21 min. $49\frac{1}{11}$ sec.; 159 dys. 19 hrs. $7\frac{1}{2}$ min.

IV. 1 qr. 1 nl.; 6 acres 1680 yds.

V. 4 sq. ft. $28\frac{4}{5}$ sq. in.; 1 lb. 4 oz. 5 dr. 1 sc.

15.

I. 35· $4\frac{2}{5}$; $\frac{8}{63}$ II. $\frac{31}{70}$; $\frac{1}{6}$; $7\frac{13}{20}$ III. $16\frac{2}{19}$; $20\frac{2}{17}$

16.

IV. 18; 27; $17\frac{1}{15}$; $11\frac{1}{4}$ V. $\frac{3}{16}$; $\frac{27}{175}$; $3\frac{45}{49}$

17.

1. $\frac{1}{16}$, $\frac{7}{8}$ 4. $\frac{3}{70}$, $\frac{1}{210}$ 7. $\frac{353}{1152}$ 10. 1

2. $\frac{47}{56}$, $\frac{1237}{1008}$ 5. $\frac{2880}{7}$, $\frac{1008}{5}$ 8. $\frac{9}{40}$ 11. $\frac{5}{166554}$

3. $\frac{177}{217}$, $\frac{158}{837}$ 6. $\frac{10}{9}$, $\frac{1}{54}$ 9. $\frac{16456}{3}$ 12. $\frac{1}{5632}$

18.

1. 14. 9d.
2. 1540 lbs. 14 oz.
3. 958 yds. 2 ft. 2 in.
4. 1 ac. 1 r. 36 p. 5 y. 1 ft. $65\frac{29}{35}$ in.
5. £224 7s. $10\frac{24}{75}$d.
6. 32 yds. 0 ft. $5\frac{63}{130}$ in.
7. 4 t. 18 cwt. 1 qr. 3 lb. 11 oz. $11\frac{1}{45}$ dr.
8. £301 6s. $10\frac{12}{35}$d.
9. £978 15s. $8\frac{2}{3}$d.
10. £367 4s. $11\frac{71}{105}$d.
11. 6491 yrs. 8 m. 0 w. $0\frac{9}{11}$ d.
12. £10 9s. $1\frac{187}{630}$d.
13. 35 lb. 6 oz. 11 dwt. $4\frac{88}{711}$ grs.
14. £237 9s. $3\frac{44}{65}$d.
15. 8s. $1\frac{319}{558}$d.

19. A.

2. $\text{Div}^r = \dfrac{368469 - 6}{263} = 1401$

3. 990,001.

4. $4 \times 7 - 14 \div 2 + 11 = 18.$

5. Division. £1 9s. 2d.

6. L.C.M. = 2520.

7. 95,040,000 miles.

8. 59 min. $8\frac{1}{4}\frac{3}{8}\frac{4}{7}$ sec.

9. $\dfrac{£79\ 9s.\ 6d.}{£4\ 13s.\ 6d.} = 17.$

10. £2 5s.

11. $\begin{cases} \quad\ 8s.\ 2\frac{4}{7}\frac{6}{8}d.\ \text{saved.} \\ £1\ 0s.\ 6\frac{4}{7}\frac{2}{8}d.\ \text{spent.} \end{cases}$

12. $\dfrac{2\frac{1}{2} \times 1\frac{7}{8}}{2\frac{1}{2} \div 1\frac{7}{8}} = 1\frac{7}{8} \times 1\frac{7}{8} = \dfrac{15}{8} \times \dfrac{15}{8} = \dfrac{225}{64} = 225 : 64.$

13. See Arts. 28 to 30.

14. $\dfrac{1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}}{\frac{4}{5} \text{ of } 1\frac{2}{3}} = \dfrac{47}{80}$

15. $\dfrac{3}{16}$ of debts = £7848, $\therefore$ debts $= \dfrac{£7848 \times 16}{3} = £41856$

and dividend $= \dfrac{3}{16}£ = 3s.\ 9d.$

B.

1. 120.

2. 2700.

3. $28\frac{7}{11}$ miles; or 28 m. 1120 yds.

4. £687 10s. Required No. of times $= \dfrac{£13\ \ 8s.\ 1\frac{1}{2}d.}{13s.\ 9d.} = 19\frac{1}{2}.$

5. $2\frac{8}{35}$ inches.

6. The sum = 18 times £1 3s. 6d. + 9 times $\frac{2}{3}$ of £1 3s. 6d. = £28 4s.

7. L. C. M. of all these measures = 45 gallons.

8. 3 oz. 16 dwts.

9. Required number $= \dfrac{£18\frac{9}{10}}{2\frac{5}{8}s.} = 144$

10. It is greater: Diminished by $\frac{1}{}$. (See **Exs. 15.**)

11. Cost price = 1s. 8d. per yard.

When $\frac{1}{3}+\frac{1}{4}$ are sold, rem^r = $\frac{5}{12}$ths = 25 yds.

$\therefore$ the whole = $\frac{12}{5}\times 25$ yds. = 60 yds.

Whole cost price = $60\times 1\frac{2}{3}$sh =	£5	0 0
Whole receipts = ... =	£5	18 4
Gain .... =		18 4

12. Product = $\frac{10}{21}$; Quotient = $2\frac{26}{35}$.

13. $\frac{219}{220}$.

14. £73 1s. 3d. : £250 10s. :: $\frac{1}{8}$: $\frac{1}{8}\times\frac{£250\frac{1}{2}}{£73\frac{1}{16}}$ or *Ans.* $=\frac{3}{7}$.

15. 36. *Art.* 5. *Note.*

C.

1. Four millions, seventy thousand, three hundred and eleven: Three millions, nine hundred and thirty thousand, three hundred and seven.
2. 4117 lbs.
3. Profit = $40000\times\frac{1}{8}$d. = £20 16s. 8d.
4. $46\frac{3}{7}$.
5. 500 yds.
6. 11844.
7. If 25 yds. cost £7 17s. 6d.

   1 yd. " $\frac{£7\ 17s.\ 6d.}{25}$

   $\therefore$ 36 yds. " $\frac{(£7\ 17s.\ 6d.)\times 36}{25}$ = £11 6s. $9\frac{3}{5}$d.
8. 27825.
9. $\frac{5}{16}$ seconds.
10. 4444 dys. 10 hrs. 40 min.
11. 216.
12. 45 : 26.
13. When $\frac{2}{5}$ are used, the rem^r = $\frac{3}{5}$.

    $\therefore$ by question $\frac{2}{3}$ of $\frac{3}{5}$, or $\frac{2}{5}$, brings £1 2s. $5\frac{1}{2}$d.;

    $\therefore$ the whole brings $\frac{5}{2}$ of £1 2s. $5\frac{1}{2}$d., or £2 16s. $1\frac{3}{4}$d.
14. $\frac{\frac{2}{5}\text{ of 7s.}+\frac{3}{4}\text{ of }10\frac{1}{2}\text{sh.}-\frac{2}{5}\text{ of 21s.}}{27\text{sh.}}=\frac{91}{1080}$
15. $5-\left(\frac{1}{2}+\frac{3}{5}+\frac{2}{7}\text{ of }\frac{5}{8}\right)=3\frac{101}{140}$.

D.

1. 69,0 8,911$\frac{18}{[illegible]}$, or 18 rem^r
2. 6 a. 3 r. 35$\frac{21}{121}$ p.
3. 367 oz.
4. 50688.
5. £9174 7s. 6d.
6. In 50 yds. *B* gains 5 feet

   ∴ in 1 yd. „ $\frac{5}{50}$ ft. or $\frac{1}{10}$ ft.

   ∴ in 1760 yds., or 1 mile, he gains 176 ft., or $58\frac{2}{3}$ yds.

   ∴ *B* is now in advance of *A* $8\frac{2}{3}$ yds.
7. 45,714,285$\frac{5}{7}$ lbs.
8. Required number $= \dfrac{1760 \times 1760 \text{ sq. yds.}}{50 \times 5\frac{1}{2} \times 5\frac{1}{2} \text{ sq. yds.}} = 2048.$
9. 5400 times.
10. See Arts. 23 to 27.
11. $\dfrac{396,\ 175,\ 168}{540}$, or 396 : 175 : 168.
12. $\dfrac{\frac{3}{16} \text{ of his share}}{\text{whole mine}} = \dfrac{£750}{£10000}$ ∴ his share $= \frac{16}{3} \times \frac{75}{1000}$ of the mine.

    $= \frac{2}{5}.$
13. 4387 : 977.
14. 1s. $2\frac{8}{103}$d.
15. See Ex. 5, Art. 90.

    A, *B*, and *C* do together $\frac{1}{2} + \frac{2}{5} + \frac{2}{7}$, or $\frac{83}{70}$, in 1 hour

    ∴ in 20 min., or $\frac{1}{3}$ of 1 hour, they do $\frac{1}{3}$ of $\frac{83}{70}$, or $\frac{83}{210}$.

20.

I. 80·34, 80340, 803400, 8034000.

II. 1·7504, ·0017504, ·00017504, 17·504.

III. 5, 500; ·0005, ·00005, ·0000005.

21.

I. ·3, ·011, ·19, ·00015, 100·1.

II. ·6, 1·4, ·5, 2·25, 1·375, 1·0625, ·72. ·006.

III. ·0546875, 5·046875, 11·0288, 94·25, 35·50078125

22.

I. $\cdot 1\dot{6}$, $\cdot \dot{1}4285\dot{7}$, $\cdot \dot{4}2857\dot{1}$, $\cdot \dot{1}\dot{8}$, $\cdot \dot{6}1538\dot{4}$, $\cdot \dot{7}$, $\cdot \dot{4}58\dot{3}$.

II. $\cdot \dot{3}$, $\cdot \dot{5}$, $1\cdot \dot{5}7142\dot{8}$, $\cdot 46\dot{4}2857\dot{1}$, $\cdot 2\dot{6}1538\dot{4}$, $\cdot \dot{1}4\dot{8}$, $\cdot 053\dot{5}7142\dot{8}$, $\cdot \dot{7}0\dot{3}$.

III. $8\cdot 41\dot{6}$, $17\cdot \dot{1}\dot{2}$, $375\cdot 1\dot{2}\dot{7}$, $4\cdot 9\dot{1}0256\dot{4}$.

23.

I. $\frac{1}{20}$, $10\frac{73}{100}$, $115\frac{1}{125}$, $\frac{1}{10000}$, $1\frac{1}{200}$, $\frac{343}{1000}$.

II. $12\frac{1}{100}$, $10\frac{1}{125}$, $\frac{29}{4000}$, $135\frac{11}{20}$, $\frac{2101}{20000}$, $9\frac{99}{100}$.

24.

I. $\frac{1}{9}$, $\frac{91}{333}$, $\frac{1}{11}$, $\frac{1}{99}$, $15\frac{25}{333}$, $\frac{1}{7}$.

II. $\frac{1}{6}$, $\frac{3}{110}$, $130\frac{2}{7}$, $\frac{19}{99000}$, $\frac{32143}{225000}$, $35\frac{1}{111}$

III. $\frac{5071}{60000}$, $60\frac{251}{24750}$, $100\frac{2}{55}$, $\frac{139868}{833325}$, $35\frac{1}{100}$ or $35\cdot 01$, 3501.

25.

1. $1449\cdot 53275$.
2. $1000\cdot 781091$.
3. $3976\cdot 836319$.
4. $8078\cdot 113601$.
5. $3609\cdot 366\dot{2}5750\dot{8}$.
6. $154\cdot 78\dot{8}9559\dot{6}$.
7. $6\cdot \dot{1}8404\dot{9}$.
8. $137\cdot 76869\dot{1}5834\dot{0}$.

26.

I. $3\cdot 425$; $174\cdot 7976$; $\cdot 01049$.

II. $\cdot 8514$; $\cdot 135$; $\cdot 61\dot{3}\dot{6}$; $\cdot \dot{2}$.

III. $\cdot 26\dot{3}$; $1\cdot 8025\dot{2}6\dot{7}$; $6\cdot 55\dot{5}1596\dot{9}$.

27.

1. $2\cdot 25$.
2. $8\cdot 265$.
3. $\cdot 473568$.
4. $462\cdot 07708$.
5. $\cdot 076775$.
6. $10\cdot 25325$.
7. $\cdot 0616$.
8. $\cdot 0459375$.
9. $\cdot 4453125$.
10. $279\cdot 40773798$.

28.*

I. $\cdot 05$; $3651\cdot 96$.

II. $146\cdot 1463$; $17\cdot 1821$.

III. 24; 3 but nearly 4

* Where, as in some of these Exs., there would be many places of decimals in the product formed by the ordinary method, and only two or three are required, it will be necessary generally to work for one more place than this required number, to insure correctness

29.

1. $\cdot 59\dot{3}$.
2. $\cdot 0582\dot{9}\dot{0}$.
3. $58\cdot 53\dot{5}5\dot{8}$.
4. $124\cdot\dot{5}1\dot{8}$.
5. $\cdot\dot{5}8089142247\dot{5}$.
6. $3\cdot 16\dot{4}\dot{1}$.

30.

1. 90.
2. 654·296875.
3. 135000.
4. 200.
5. 21·0705.
6. ·0024491 &c.
7. 4·835232, &c.
8. ·00250498046875.
9. ·00941204, &c.

31.*

1. 5·672 nearly.
2. 38·6348, &c.
3. 3·55, &c.
4. 13.

32.

1. $5\cdot\dot{0}\dot{5}$.
2. $413\cdot\dot{3}$.
3. 22·1987096, &c.
4. $\cdot 33\dot{8}5\dot{8}$.
5. ·0972.
6. $1\cdot\dot{9}6536\dot{7}$.

33.

1. £1 8s. $10\frac{1}{2}$d.
2. 9·072d.
3. £1878 19s. $4\frac{4}{5}$d.
4. 5s. 6·9d.
5. 1s. 1·34925d.
6. £772 17s. $10\frac{2}{25}$d.
7. 1s. $4\frac{1}{8}$d.
8. £16 5s. 4d.
9. 3 fur. 1 po. 2 yds.
10. 1 cwt. 0 qr. 19 lbs. 10 oz. 11·52 drs.
11. £1 10s. $5\cdot\dot{1}\dot{7}$d.
12. ·2s. or $2\frac{2}{5}$d.

34.

1. $\cdot 1\dot{6}$.
2. $\cdot 0358\dot{3}$.
3. $\cdot\dot{3}9682\dot{5}$.
4. ·1775.
5. $\cdot 0670\dot{4}\dot{5}$.
6. $1\cdot\dot{1}\dot{8}$.
7. $\cdot 00291\dot{6}$.
8. $\cdot 00\dot{1}8\dot{5}$.
9. 14·64.
10. $1\cdot 9\dot{5}$.

35.

1. ·0125.
2. $44\cdot 1\dot{6}$.
3. 15.
4. $\begin{cases} 5\cdot 25 \\ 2\cdot 87 \\ 1\cdot 855 \\ \cdot 2[illegible]6 \ldots\ldots \end{cases}$
5. $2\frac{337}{600}$
6. $13\cdot\dot{1}\dot{3}$.
7. £1 1s. $4\cdot\dot{3}$d.
8. $\frac{5}{16} = \frac{5 \times 625}{16 \times 625} = \frac{3125}{10000} = \cdot 3125$
9. ·054375.
10. $\frac{1}{\cdot 2} = \frac{\cdot 375}{\text{4th prop.}}$ $\therefore$ 4th prop. $= \frac{\cdot 375 \times \cdot 2}{1}$

$= \cdot 075$;

·005438 &c.; $2\cdot 3\dot{5}71428\dot{8}$.

---

* A caution, similar to that in 28., applies here.

36.

E.

1. 3656.
2. 3900.
3. $11\frac{1}{4}$.
4. £61035 3s. $1\frac{1}{3}$d.
5. 61160.
6. 12510.
7. 30 dys. $10\frac{2}{3}$ hrs.
8. $\frac{1984}{139}$
9. $\frac{97\frac{1}{3} \times 81 \times 144}{9 \times 4\frac{1}{2}} = 28032.$
10. $32 \div 2 + 4 \times 13 - 23 = 237.$
11. $\frac{120,\ 135,\ 144,\ 100}{180}$. See Art. 39.
12. $\frac{187}{200}$
13. $(\cdot 75 - \cdot 5) \times \cdot 008 = \frac{1}{500}$
14. $3\cdot 08\dot{3}$.
15. $\frac{\frac{2}{3} \text{ of 13s. 4d.}}{£1} = \frac{\frac{2}{3} \text{ of } \frac{2}{3}£}{£1} = \frac{4}{5 \times 3} = \frac{\cdot 8}{3} = \cdot 2\dot{6}.$

F.

1. 24000.
2. 80 miles 902 yds.
3. £423 10s.
4. £1 18s. $10\frac{3}{8}$d.
5. $\frac{1}{160}$ of $\frac{1}{2}$ml. + $\frac{1}{330}$ of 2 mls. 50 yds. − $\frac{1}{110}$ of 1 ml. 360 yds.

   + $\frac{1}{220}$ of 1500 yds. = $3\frac{19}{22}$ yds.; or *A* is $3\frac{19}{22}$ yds. above *B*.
6. 1st rem^r = $\frac{15}{28}$; 2nd rem^r = $\frac{3}{5}$ of $\frac{15}{28}$ or $\frac{9}{28}$

   ∴ if $\frac{9}{28}$ths = 1350, the whole = $\frac{28}{9} \times 1350 = 4200$.
7. $\cdot 1\dot{6}$ of a moid. + $\cdot\dot{5}7142\dot{8}$ of 2 guis. = $\frac{1}{6}$ of 27s. + $\frac{4}{7}$ of 42s. = £1 8s. 6d.
8. 72.
9. There are in (22 × 22) sq. yds. (100 × 100) sq. links,

   or in 484 sq. yds. 10000,

   or in 4840 yds., *i. e.* in an acre, there are 100,000.
10. 8s. $7\frac{1}{2}$d.
11. 7 ft. $7\frac{48}{49}$ in.

12. $777\frac{3}{8}$.

13. The number $= 17\frac{1}{9} \times 13\frac{7}{8} = 237\frac{5}{12}$.

14. $\dfrac{\frac{9}{10} \text{ of } 144 + \frac{3}{5} \text{ of } 25 - \frac{7}{8} \text{ of } 20}{17\frac{1}{9}} = 7\frac{659}{1540}$.

15. $\frac{2}{9}$. See Art. 57.

G.

1. £35,478,000.
2. 12051.
3. $£\dfrac{£230 \ 8s.}{\left\{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{20}\right\}} = 128$.
4. £83 8s. $10\frac{3}{8}$d.
5. $\cdot 07 + \cdot 005 + \cdot 0375 = \frac{9}{80}$
6. 12 yds.
7. 5 oz. 8 dwts. $16\frac{1}{5}\frac{3}{5}$ grs
8. £1343 15s.
9. Nitre $= \frac{15}{20}$ths of 16 cwt. $=$ 12 cwt.
   Charc[l] $= \frac{3}{20}$ „ $=$ 2 cwt. $44\frac{4}{5}$ lbs.
   Sulphur $= \frac{2}{20}$ or $\frac{1}{10}$th $=$ 1 cwt. $67\frac{1}{5}$ lbs.
10. Rem[r] $= 1 - \frac{1}{3} - \frac{1}{5}$ or $\frac{7}{15}$ths $=$ £15,
    $\therefore$ whole $= \frac{15}{7} \times$ £15 $=$ £32 2s. $10\frac{2}{7}$d.
11. See Arts. 28, 29.
12. Span of each arch next to middle $= \frac{9}{10}$ of 75 ft.
    „ next but one $= \frac{9}{10}$ of $\frac{9}{10}$ of 75; and so on;

$\therefore$ whole length $= 75 + 2\left\{\frac{9}{10} \text{ of } 75 + \frac{9}{10} \text{ of } \frac{9}{10} \text{ of } 75 + \frac{9}{10} \text{ of } \frac{9}{10} \text{ of } \frac{9}{10} \text{ of } 75\right\} + 1$
$= 560\cdot 85$.

13. $\frac{50}{1111}$s.
14. $8\frac{1}{5}$d.
15. £3 8s. $11\frac{7}{10}$d.

H.

1. 7596.
2. 387200.
3. 13230 pks. 198450 lbs.
4. 19200.

5. 61160.

6 $\dfrac{\text{Geog. m.}}{\text{Brit. m.}} = \dfrac{\frac{1}{60}\text{ deg.}}{\frac{1}{69\frac{1}{2}}\text{ deg.}} = \dfrac{69\frac{1}{2}}{60} = \dfrac{139}{120}$

Least number of miles = L. C. M. of $69\frac{1}{2} \times 60 = 4170$.

7. $\dfrac{\cdot 325\text{ m.}}{2\cdot 1\dot{6}\text{ feet}} = 792.$

8. $\dfrac{7}{256} = \dfrac{\cdot 875}{32} = \dfrac{\cdot 109375}{4} = \cdot 02734375$ See end of Art. 98.

9. 40. See Art. 122.

10. 75 hundredths; 2, and 324 thousandths; 17 and 1 millionth.

11. £2219 8s. $2\frac{23}{80}$d.

12. $\dfrac{10\frac{2}{5}\text{ m.}}{13\frac{8}{11}\text{ ft.}} \sim \dfrac{10\frac{2}{5}\text{ m.}}{9\frac{2}{7}\text{ ft.}} = 10\frac{2}{5}\text{ m.}\left\{\dfrac{1}{9\frac{2}{7}} - \dfrac{1}{13\frac{8}{11}}\right\}\text{ft.}$

$= 10\frac{2}{5} \times 1760 \times 3 \times \left\{\dfrac{7}{65} - \dfrac{11}{146}\right\} = 1776\frac{1872}{4745}$ times.

13. $\dfrac{£4 - £3\ 16\text{s. } 9\text{d.}}{1\frac{1}{2}\text{ guins.}} = \dfrac{3\frac{1}{4}\text{ sh.}}{31\frac{1}{2}\text{ sh.}} = \dfrac{13}{126}.$

14. $\frac{2}{225}$.

15. $\dfrac{147 \times \cdot\dot{2}\dot{7}}{147 \div \cdot\dot{2}\dot{7}} = (\cdot\dot{2}\dot{7}) \times (\cdot\dot{2}\dot{7}) = \dfrac{9}{121} = 9 : 121$

37.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	1	18	$8\frac{1}{4}$	5.	133	4	$9\frac{1}{4}$	9.	484	7	$1\frac{1}{2}$
2.	22	6	$11\frac{1}{4}$	6.	285	6	$5\frac{3}{4}$	10.	345	2	$4\frac{1}{4}$
3.	29	0	$8\frac{1}{2}$	7.	263	14	2	11.	189	11	2
4.	133	13	$1\frac{1}{2}$	8.	311	4	$10\frac{3}{4}$	12.	430	9	$9\frac{3}{4}$

38.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	649	7	0	7.	1749	9	$3\frac{1}{2}$	12.	4240	15	$9\frac{3}{4}$
2.	1194	12	$2\frac{1}{4}$	8.	1438	4	0	13.	622	7	11
3.	668	4	3	9.	2487	9	$4\frac{1}{2}$	14.	3043	2	3
4.	2423	15	$6\frac{1}{2}$	10.	3299	7	0	15.	5648	15	2
5.	399	13	$0\frac{3}{4}$	11.	5731	12	$4\frac{1}{2}$	16.	5908	10	$5\frac{1}{2}$
6.	216	5	$5\frac{1}{4}$								

39.

	£	s.	d.
1.	9564	19	$7\frac{1}{2}$
2.	2406	18	9
3.	3545	11	9
4.	6140	13	$10\frac{1}{4}$
5.	4327	16	$6\frac{3}{4}$
6.	9292	0	0
7.	1832	6	$2\frac{1}{4}$
8.	3199	11	6
9.	1850	12	0
10.	6523	0	9
11.	11030	1	$11\frac{1}{4}$
12.	12405	17	$6\frac{1}{4}$
13.	13370	11	$1\frac{1}{4}$
14.	3051	12	$1\frac{1}{4}$
15.	11679	19	$10\frac{1}{4}$
16.	8869	3	11

40.

	£	s.	d.
1.	1623	5	0
2.	8874	4	$8\frac{1}{4}$
3.	23212	6	0
4.	9346	5	9
5.	10764	7	6
6.	13776	7	5
7.	13646	17	6
8.	2590	10	6
9.	11943	0	$4\frac{1}{4}$
10.	32322	5	5
11.	3055	9	$2\frac{3}{4}$
12.	17444	3	$4\frac{1}{2}$

41.

	£	s.	d.
1.	21407	18	$7\frac{1}{4}$
2.	4983	11	$0\frac{3}{4}$
3.	33482	6	10
4.	12850	11	$7\frac{1}{4}$
5.	73524	12	10
6.	95875	6	6
7.	18036	0	0
8.	25648	8	9
9.	80697	7	$8\frac{1}{2}$
10.	96661	6	2
11.	47609	16	$6\frac{3}{4}$
12.	78502	0	6
13.	83375	5	11
14.	70504	8	6
15.	55038	10	$7\frac{1}{2}$
16.	81490	14	$4\frac{1}{2}$
17.	396453	18	6
18.	571688	0	0

42.

	£	s	d.
1.	152798	12	$6\frac{3}{8}$
2.	53749	18	$5\frac{46}{84}$
3.	69997	8	$10\frac{1}{14}$
4.	1780	13	$9\frac{8}{11}$
5.	3744	4	4
6.	4462	19	$7\frac{1}{2}$
7.	19421	11	$3\frac{11}{16}$
8.	9274	5	$8\frac{3}{8}$

43.

	£	s.	d.
1.	7829	16	$7\frac{2}{7}$
2.	32110	13	0
3.	15912	3	$7\frac{5}{8}$
4.	2311	13	$4\frac{5}{16}$
5.	4432	12	$6\frac{1}{4}$
6.	48275	14	$2\frac{1}{4}$

44.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	1	16	$6\frac{3}{4}$	5.	257	3	$4\frac{5}{8}$	8.	44	13	[illegible]
2.	421	19	$7\frac{5}{32}$	6.	391	15	$5\frac{1}{2}$	9.	20	13	$7\frac{37}{50}$
3.	11982	19	$5\frac{13}{25}$	7.	22	10	$6\frac{17}{350}$	10.	50	7	$9\frac{13}{28}$
4.	12869	14	$0\frac{43}{84}$								

45.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	147	15	$3\frac{3}{4}$	5.	2389	16	$10\frac{1}{2}$	8.	49	12	$7\frac{1}{4}$
2.	1082	16	3	6.	33	5	$3\frac{9}{16}$	9.	401	18	$3\frac{3}{10}$
3.	375	11	$9\frac{37}{40}$	7.	931	17	6	10.	107	13	$8\frac{1}{121}$
4.	2237	13	$4\frac{1}{8}$								

*$25\frac{1}{4}$ yds. $= \dfrac{25\frac{1}{4}}{30\frac{1}{4}}$ p., or $\dfrac{101}{121}$ of 1 pole, or $\dfrac{101}{242}$ of 2 poles.

and ∴ cost $\frac{101}{242}$ of (£1 11s. 3d.) = 13s. $0\frac{123}{242}$d.

46.

1. £63.
2. 45.
3. £3 17s. 6d.
4. 80 yds.
5. 528 yds.
6. 425 yds. 3 qrs. 1 nl.
7. £17 1s. $10\frac{7}{20}$d.
8. $\frac{56}{72} \times 15\frac{1}{2}$ cwt. $= 12\frac{1}{18}$ cwt.
9. $\dfrac{72}{13\frac{3}{4}} \times 4 \times 112$ lbs. $= 2345\frac{5}{11}$ lbs.
10. $\frac{1}{2}$ peck.
11. $\dfrac{£51}{12\frac{3}{4}\text{s.}} \times £1 = \dfrac{51 \times 20 \times 4}{51} \times £1 = £80.$
12. $3\frac{3}{4}$ yds.
13. $\dfrac{12\text{s.}}{7\text{s.}} \times 35\text{s.} = 60\text{s.}$
14. $\dfrac{£352}{2\frac{3}{4}\text{ s.}} \times £1 = \dfrac{352 \times 20\text{s.} \times 4}{11\text{s.}} \times £1 = £2560.$
15. $13\frac{1}{2}$d.

16. Profit on 1 yd. $= \frac{1 \text{ yd.}}{148 \text{ yds.}} \times$ £12 12s. 10d. = 1s. $8\frac{1}{2}$d.

The retail price = 2s. $7\frac{1}{2}$d. + 1s. $8\frac{1}{2}$d. = 4s. 4d.

17. $\frac{5\text{s.}}{6\frac{1}{4}\text{s.}} \times 10$ oz. = 8 oz.

18. £3 5s. 9[illegible]d.

19. Taxes = £570 8s. − £534 15s. = £35 13s.

£570 8s. : £1 :: £35 13s. : 1s. 3d. in the £.

20. $\frac{1550 \text{ links}}{100 \text{ links}} \times 4$ perches = 1023 feet.

21. In reality the 12 oz. produce $44\frac{1}{4}$ guins.,

$\therefore$ value of 5 oz. $= \frac{5 \text{ oz.}}{12 \text{ oz.}} \times 44\frac{1}{4}$ guins. = £19 7s. $2\frac{1}{4}$d.

22. I spend $\frac{365\text{d.}}{35\text{d.}} \times 12$ guins. = £131 8s.

whole income = £231 8s.

23. £8 14s. 11[illegible]d.

24. 21 ft. $10\frac{1}{2}$ in.

25. $19\frac{1}{2}$: 1 :: $17\frac{1}{2}$ ft. : $\frac{1}{19\frac{1}{2}} \times 17\frac{1}{2} \times 1728$ in. $= 1550\frac{10}{13}$ in.

26. No. of crowns in £250 = 250 × 4

$\therefore$ No. of dollars $= \frac{120}{111} \times 1000 = 1081\frac{3}{37}$.

27. From Mond. mid. to Thursday even. = 66 hrs.

Time gained $= \frac{66 \text{ h.}}{24 \text{ h.}} \times 7\frac{1}{2}$ min. $= 20\frac{5}{8}$ m

Time beyond 6 h. = 14 m. + $20\frac{5}{8}$ m. = $34\frac{5}{8}$ min.

28. 12s. 6d.; £172 17s. $2\frac{1}{4}$d.

29. $\frac{3\frac{1}{2} \text{ ft.}}{2\frac{3}{4} \text{ ft.}} \times 154$ ft. = 196 ft.

30. Tax $= \frac{£3827\frac{5}{8}}{£1} \times 7$d. = £111 12s. $9\frac{3}{8}$d.

Net income = £3715 19s. $8\frac{5}{8}$d.

31. Selling price $= \frac{236 \text{ gal.}}{3 \text{ qts.}} \times 8\frac{1}{2}$s. = £133 14s. 8d.

Profit = £133 14s. 8d. − £111 6s. 8d. = £22 8s.

32. No. of gallons at 11s. 6d. to make £45 10s.

$= \frac{£45 \text{ 10s.}}{11\text{s. 6d.}} \times 1$ gal. $= 79\frac{3}{23}$ gallons.

$\therefore$ water required $= (79\frac{3}{23} - 63)$ gallons.

$= 16\frac{3}{23}$ gallons.

33. Amount spent annually $= \frac{365\text{d.}}{146\text{d.}} \times £55 = £137\ \ 10\text{s.}$

Amount saved in 1 yr. $= £312\ \ 10\text{s.} - £137\ \ 10\text{s.} = £175$

Time in saving 1000 guins. = 6 years.

34. 40·5 Fr. deg.

35. $13\frac{2}{13}$ yds.

36. $\frac{50\text{s.}}{40\frac{1}{2}\text{s.}} \times 3\frac{1}{4}$ lbs. $= 4\frac{1}{81}$ lbs.

37. $\frac{1000}{80{\cdot}5} \times 2\frac{1}{2}$ sec. $= 31\frac{9}{161}$ sec.

38. $\frac{2\frac{1}{4}\text{ in.}}{3\text{ ft.}} \times 20$ lbs. $= 1\frac{1}{4}$ lbs.

39. Net sum for dividend $= £1520 - £205 = £1315.$

Debts partly paid $= \frac{8}{3} \times £1315 = £3506\frac{2}{3}$

Whole debts $= £3506\frac{2}{3} + £205.$

$= £3711\ \ 13\text{s. } 4\text{d.}$

	£	s.	d.
40. Amount spent $= \frac{52}{3} \times £16\frac{2}{3} =$	288	17	$9\frac{1}{3}$
$\frac{1}{12}$th of £500 given away =	41	13	4
Income Tax $= \frac{7}{240}$ of £500 =	14	11	8
Whole expenditure =	345	2	$9\frac{1}{3}$
Amount saved =	154	17	$2\frac{2}{3}$
	£500		

47.

1. 4th term $= \frac{20\text{ m.} \times 4\text{ d.} \times 30\text{ yds.}}{10\text{ m.} \times 8\text{ d.}} = 30$ yds.

2. $5\frac{17}{26}$ yrs.

3. $\frac{3\text{ boats} \times 20{,}000 \times 700\text{ her.} \times 8\text{ dys.}}{450\text{ boats} \times 6000\text{ her.}} = 124\frac{4}{9}$ dys.

4. £208 18s. $6\frac{6}{7}$d.

5. $\frac{5\text{ m.} \times 1800\text{ ft.} \times 960\text{ ft.} \times 14\text{ h.} \times 3\frac{1}{2}\text{ dys.}}{7\text{ m.} \times 800\text{ ft.} \times 700\text{ ft.} \times 12\text{ h.}} = 9$ dys.

6. £1 0s. 3d.

7. $\frac{23\frac{3}{4}\text{ lbs.} \times 5\frac{5}{12}\text{sh.} \times 1\frac{1}{4}\text{ sh.}}{6\frac{7}{8}\text{ lbs.} \times 4\frac{3}{4}\text{ sh.}} = 4\text{s. } 11\frac{1}{11}\text{d.}$

8. $\frac{7\frac{1}{2}\text{ h.} \times 4 \times 37260\text{ sheets}}{12\frac{1}{2}\text{ h.} \times 3} = 29808$ sheets.

9. $\left.\begin{array}{l}19\frac{1}{4}\text{ ft.}\\ 275\text{ m.}\end{array}\right\} : \left.\begin{array}{l}16\frac{1}{2}\text{ ft.}\\ 385\text{ m.}\end{array}\right\} :: 45\text{ rev}^{ns} : 54\text{ rev}^{ns}.$

10. £1 5s.

11. $\left.\begin{array}{l}3\frac{1}{4}\text{ ft.}\\ 7\frac{1}{2}\text{ in.}\\ 1280\text{ lbs.}\end{array}\right\} : \left.\begin{array}{l}2\frac{1}{4}\text{ ft.}\\ 8\text{ in.}\\ 2028\text{ lbs.}\end{array}\right\} :: 16\text{ ft.} : 18\frac{13}{33}\text{ ft.}$

12. Since a rate of 360 rev^{ns} is *slower* than a rate of 470 rev^{ns}, therefore from this pair of terms the 4th term must be *less* than 50: but a certain amount of rev^{ns} in 7 min. is at a *quicker* rate than in 8 min., therefore from this pair the 4th term is *more*.

$$\left.\begin{array}{l}470\text{ t.}\\ 7\text{ min.}\end{array}\right\} : \left.\begin{array}{l}360\text{ t.}\\ 8\text{ min}\end{array}\right\} :: 50\text{ rev}^{ns} : 43\tfrac{253}{329}\text{ rev}^{ns}.$$

13. 10s. $3\frac{5}{7}$d.

14. 12 hours.

15. If the *inner* wheel makes 800 rev^{ns}, the outer will make more;* (1)
If the *large* wheel make 800 rev^{ns}, the small wheel will make more (2)
If in describing $\frac{1}{7}$th of the path, 800 rev^{ns} are made, less will be made in describing $\frac{1}{8}$th. (3)

Hence we have as follows:—

$$\left.\begin{array}{ll}(1) & 7\\ (2) & 5\\ (3) & \frac{1}{7}\end{array}\right\} : \left.\begin{array}{l}8\\ 6\\ \frac{1}{8}\end{array}\right\} :: 800\text{ rev.} : 960\text{ rev.}$$

16. The new dimensions are 20, 10, and $12\frac{1}{2}$; hence we have

$$\text{4th term} = \frac{9760 \times 20 \times 10 \times 12\frac{1}{2} \times 5\frac{1}{3}\text{ sh.}}{100 \times 16 \times 8 \times 10} = £50\ \ 16\text{s. } 8\text{d.}$$

17. $\left.\begin{array}{l}9\text{ eng.}\\ 8\text{ hor.}\\ 5\text{ dys.}\\ 9\text{ hrs.}\\ 25 \times 3\text{ bush.}\\ 60\text{ lbs.}\\ 1\text{ alt.}\end{array}\right\} : \left.\begin{array}{l}5\text{ eng.}\\ 9\text{ hor.}\\ 3\text{ dys.}\\ 10\text{ hrs.}\\ 75 \times 2\text{ bush.}\\ 63\text{ lbs.}\\ 15\text{ alt.}\end{array}\right\} :: 1\text{ week} : 13\frac{1}{8}\text{ weeks.}$

18. If there were 3 engines of 4 pipes, there must be *more* of 3 pipes.
If there were 3 engines of 3 in. in section, there must be *less* of 5 in.
If there were 3 engines of a 20 stroke rate, there must be *more* of 17 stroke.
If there were 3 engines of any number of strokes in 3 minutes, there must be *less* of those making the same strokes in $2\frac{1}{2}$ min.

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* Of course I am here considering the effect of but one pair of terms; and from that pair alone, the answer would be more. The reasonings in lines (1), (2), and (3) produce the corresponding three statements.

If there were 3 engines discharging 4680 gals., there must be *more* for 20,000 gals.

If there were 3 engines discharging any quantity in 16 min., there must be *less* of those discharging the same in 30 min.

Hence the statement will be as follows:—

3 pipes		4 pipes				
5 in.		3 in.				
17 str.	:	20 str.	::	3 engines	:	5[illegible] engines.
3 min.		$2\frac{1}{2}$ min.				
4680 gals.		20,000 gals.				
30 min.		16 min.				

48.

	£	s.	d.		£	s.	d.
1.	16	4	0	8.	222	15	$8\frac{1}{10}$
2.	19	0	$5\frac{4}{25}$	9.	298	16	$1\frac{1}{8}$
3.	28	9	$1\frac{211}{400}$	10.	13344	11	5
4.	1109	11	3	11.	26	15	$3\frac{81}{180}$
5.	1842	6	$11\frac{67}{100}$	12.	157	17	$10\frac{1}{2}$
6.	2095	5	$5\frac{97}{180}$	13.	2183	6	$2\frac{13}{20}$
7.	30	12	$11\frac{9}{10}$	14.	2810	7	8

	£	s.	d.
15.	86	8	10
16.	324	13	1

17. Int^t for 1 year £38 3s. 10d. nearly
Am^t = £1359 5s. 7d.

18.	1431	0	$0\frac{11}{25}$

49.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	323	5	2	3.	152	4	11	5.	2392	7	1
2.	41	6	11	4.	1019	10	2	6.	2090	13	0 nearly

50.

1. Int^t of £62 = £71 6s. − £62.
 = £9 6s.

 Also, Int^t for 1 year of £62 = £3 2s.

 $$\therefore \text{time required} = \frac{\text{given interest}}{\text{1 year's interest}} = \frac{£9\ 6s.}{£3\ 2s.} = 3 \text{ yrs.}$$

2. $$\text{Time required} = \frac{\text{given interest}}{\text{int. of 1 year}}$$
 $$= \frac{£75\ 19s.\ 8\tfrac{1}{4}d.}{£30\ 7s.\ 10\tfrac{1}{2}d.} = 2\tfrac{1}{2} \text{ yrs.}$$

3. 5 years.

4. £100 produces £103$\frac{1}{2}$.

 £103$\frac{1}{2}$: £108 13s. 6d. :: £100 : required sum £105.

5. £100 will amount to £106 15s. in 3 yrs.

$\therefore$ £106$\frac{3}{4}$: £136 2s. 1$\frac{1}{2}$d. :: £100 : required sum £127 10s

6. £1050.

7. £95 15s. in 5 yrs. gains £16 15s. 1$\frac{1}{2}$d.

or in 1 yr. „ £3 7s. 0$\frac{3}{10}$d.

$\therefore$ If £95 15s. gains £3 7s. 0$\frac{3}{10}$d., £100 will gain £3 10s.

Ans. £3$\frac{1}{2}$ per cent.

8. 5$\frac{1}{4}$.

9. 4$\frac{1}{2}$.

10. 12$\frac{1}{2}$.

11 £100 at Comp. Int. for 3 yrs. amounts to £115 15s. 3d.

$\therefore$ if £100 gains £115 15s. 3d., £358 17s. 3$\frac{3}{10}$d. will be gained, by £310. *Ans.* £310.

12. 5s. 2$\frac{1}{2}$d.

13. £640 12s. 6d. See end of Art. 144.

14. £147 3s. 9d.

15 8s. 4$\frac{7}{260}$d.

16. Time required $= \frac{\text{given interest}}{\text{int}^{\text{t}}\text{ of 1 yr.}} = \frac{\text{£245 13s. 6d.}}{\text{£30 14s. }2\frac{1}{4}\text{d.}} = 8$ yrs.

17. 7 per cent. (See Ex. 7.)

18. If for 10 days the int$^{\text{t}}$ is 3s. 7d., we have for 1 yr., £6 10s. 9$\frac{1}{2}$d. or 5 per cent. (See the Statement of Ex. 7.)

51.

1. £408 2s. 2d.
2. £407 1s. 11d.
3. £620. See **50.** Ex. 11.
4. If £100 gives £3$\frac{1}{2}$, what will give £5200. *Ans.* £148571 8s. 6$\frac{6}{7}$d.
5. £206 5s.
6. If £10500 gives £367$\frac{1}{2}$, what does £100 give? *Ans.* 3$\frac{1}{2}$.
7. Number of years' purchase $= \frac{\text{£}100}{\text{£}3\frac{3}{4}} = 26\frac{2}{3}$.
8. *Ans.* $= \frac{\text{£}100}{33} = 3\frac{1}{33}$ per cent.
9. *Ans.* $= 100 \times \frac{\text{£}120}{4\frac{1}{4}} =$ £2823 10s. 7$\frac{1}{17}$d.
10. £1225. See preceding Ex.

52.

1. 6 months interest of £100 = £2 10s.
Add £100
£102 10s.

£102 10s. : £100 :: £100 . £97 11s. $2\frac{26}{41}$d. pres. worth of £100.

∴ discount required = £100 − £97 11s. $2\frac{26}{41}$d. = £2 8s. $9\frac{15}{41}$d.

	£	s.	d.		£	s.	d.		£	s.	d.
2.	1	5	$6\frac{86}{101}$	5.	952	7	$7\frac{3}{7}$	8.	396	1	$4\frac{22}{33}$
	6	15	$7\frac{187}{200}$	6.	850	0	0	9	2	14	$11\frac{18}{45}$
4.	22	11	$8\frac{1009}{2449}$	7.	117	11	$8\frac{233}{489}$				

). The bill is legally due on July 18; and from April 6th to July 18th is 103 days;

	£	s.	d.	
And interest of £325 8s. 4d. for 103 days, at 4 p.c. =	3	13	5	
And true discount on „ „ =	3	12	5	nearly
Banker's gain =		1	0	nearly

1. From Jan. 15th to June 4th, when the bill is due, is 140 days.

	£	s.	d
And interest of £90 7s. 6d. for 140 days at 5 per cent. =	1	14	7
True discount „ „ =	1	14	0
Banker's gain =			7

	£	s.	d.	
2. The present worth of the former half =	245	10	0	very nearly
„ „ latter half =	243	6	2	
Ans. =	488	16	2	

53.

1. £137 12s. 6d. : £151 7s. 9d. :: £100 : £110. *Ans.* 10 per cent
2. 3s. 6d. : 4s. 9d. :: £112 : £152 *Ans.* 52 per cent.
3. 4s. $10\frac{1}{2}$d.
4. 8s. $9\frac{3}{4}$d.
5. To lose $17\frac{1}{2}$ p. c. is to receive $82\frac{1}{2}$ for £100 worth of goods,

$$\therefore \frac{£82\frac{1}{2}}{£100} = \frac{2s.\ 9d.}{3s.\ 4d.} \qquad \textit{Ans. } 3s.\ 4d.$$

6. £3.
7. $\frac{£22\frac{1}{2}}{£27} = \frac{£85}{£102}$. *Ans.* £102, or 2 per cent. profit.

8. $\frac{£90}{£110} = \frac{15s.}{18s.\ 4d}$ *Ans.* 18s. 4d.

9. 2s. $1\frac{1}{3}$d.

10. Neither loss nor gain.

11. Cost price = £362 10s., $\therefore \frac{£362\ 10s.}{£100} = \frac{£36\ 5s.}{£10}$. *Ans.* £10 p.c. profit

12. $\frac{\text{Cost price}}{£50} = \frac{£100}{£95}$, $\therefore$ cost price =

	£	s.	d.	
$\therefore$ cost price	52	12	$7\frac{11}{19}$	
$\therefore$ prime cost of 4 pipes ..	= 210	10	$6\frac{6}{19}$	
Add 5 per cent. or $\frac{1}{20}$th ..	= 10	10	$6\frac{6}{19}$	
Money to be received for 4 pipes	= 221	1	$0\frac{12}{19}$	
Subtract selling price of first	50			
	3)171	1	$0\frac{12}{19}$	= sell[g] pr. of 3 pipes
	57	0	$4\frac{4}{19}$	= sell[g] price of each of the 3 pipes.

13. 20 per cent. on £20 = £4.
$\therefore$ he must realize £24.
But 10 per cent. on £10 = £1; $\therefore$ he makes £11 by the first half.
And $\therefore$ £24 − £11 or £13 by the second half. *Ans.* £13.

14.

		£	s.	d.
56 gals. at £$1\frac{1}{8}$	=	63	0	0
10 p.c.	=	6	6	0
5 p.c.	=	3	3	0
49 gals. must produce		72	9	0
$\therefore$ 1 gal.		1	9	$6\frac{6}{7}$

54.

1. £81 16s. $4\frac{4}{11}$d; and £68 3s. $7\frac{7}{11}$d.
2. £393 8s. $10\frac{14}{61}$d. £206 11s. $1\frac{47}{61}$d.
3. *A*, £492 6s. $1\frac{11}{13}$d. *B*, £276 18s. $5\frac{7}{13}$d. *C*, £230 15s. $4\frac{8}{13}$d.
4. £6, £3, £1.
5. £181 5s. to *A*; £468 15s. to *B*.
6. £5, £6 13s. 4d., £8 6s. 8d.
7. £300, £225, £180.
8. 4s. $5\frac{1}{3}$d.
9.

		£	s.	d.
Whole cost of 100 gallons	=	74	10	0
Profit of 10 per cent.	=	7	9	0

Retail price = $\frac{£81\ 19s.}{100}$ = 16 $4\frac{1}{4}$

	£	s.	d.
0.	694	8	$10\frac{3}{8}$
	347	4	$5\frac{1}{8}$
	208	6	8

. 55.

I.

1. $30\frac{1}{4}$ sq. yds.
2. Whole profit = £1000 − £75 = £925; gain of each = £92 10s.
3. 334800.
4. 15 ft. 267 in. − 3 × (4 ft. 375 in.) = 2 ft. 870 in.
5. Required No. = $\dfrac{15\frac{3}{4}\text{ ft.} \times 15\frac{3}{4}\text{ ft.} \times 144\text{ sq. in.}}{2\frac{1}{4}\text{ in.} \times 2\frac{1}{4}\text{ in.}} = 7056.$
6. L.C.M. = £378.
7. See Art. 58.
8. 9.
9. £827 1s. $1\frac{3}{4}$d.
10. £98 0s. $4\frac{29}{320}$d.
11. $\dfrac{\text{Prime cost}}{\text{Profit}} = \dfrac{\text{4s. 3d.}}{\text{1s. 6d.}} = \dfrac{£100}{£35\frac{5}{17}}$ *Ans.* £35 5s. $10\frac{10}{17}$d.
12. $\dfrac{\text{1st quality}}{\text{2nd quality}} = \dfrac{\text{1st price}}{\text{2nd price}} = \dfrac{\text{1s. 6d.}}{\text{42 in.}} \div \dfrac{\text{2s. 4d.}}{\text{56 in.}} = \dfrac{6}{7}.$
13. In 1s. there is $\frac{1}{66}$ of $\frac{37}{40}$ lbs.
    ∴ in 20s. there are $\frac{20}{66}$ of $\frac{37}{40}$ lbs. = 3 oz. 7 dwt. $6\frac{6}{11}$ grs.
14. $\dfrac{\text{8 cwt. } 3\frac{1}{2}\text{ qrs. } 1\frac{1}{3}\text{ st.}}{\text{15 tons.}} = \dfrac{\text{9 cwt. 2·8 lbs.}}{15 \times 2240\text{ lbs.}} = ·03008\dot{3}.$
15. $\dfrac{\text{1 perch}}{\text{1 sq. furlong}} = ·000625.$
16. $\frac{2}{13}$ths. See Art. 152.
17. $\frac{1}{14}$, $\frac{2}{29}$, $\frac{3}{14}$.

K.

1. 4836.
2. £3511 11s.
3. $7\frac{1}{3}\frac{1}{2}$ ft., by Comp^d Prop^n
4. £1610 14s. $10\frac{8}{32}\frac{1}{4}$d.
5. 1 Eng. Ell : 68 yds. :: £1 18s. : £$103\frac{9}{25}$ selling price.
   Profit = £$103\frac{9}{25}$ − £75 = £28 7s. $2\frac{2}{5}$d
6. £2 0s. $11\frac{6}{7}\frac{1}{3}$d.
7. See Arts. 74, 134.

8. Required tax = twice $\frac{7}{3}$rds of the former, $= 2 \times \frac{7}{3} \times$ £1080 = £5040.
9. £306.
10. See Art. 127.
11. $\frac{2\frac{1}{4}\text{ per.}}{1\text{ sq. mile}} = \frac{1}{40960}$.
12. Men's share $= \frac{2}{5}$ths or 60 yds.; the women's share $= \frac{3}{5}$ths or 90 yds.

$\frac{1}{10}$th of $\frac{2}{5}$ths to each man gives 6 yds.

$\frac{1}{10}$th of $\frac{3}{5}$ths to each woman gives 9 yds.

13. $\left.\begin{array}{lcl} 8\text{ oars} & . & 6\text{ oars} \\ 10\text{ str.} & : & 8\text{ str.} \\ 5 & & 6 \end{array}\right. :: 9\text{ m.} : 6\frac{1}{8}\text{ m.}$
14. 2s. 8·546d.
15. $\cdot 54903\dot{3}8095\dot{2}$.
16. If $\frac{17}{100}$ is thrown off, $\frac{17}{83}$ of the net price must be put on. See Art. 152.

L.

1. $3276 \times$ (£2 7s. 6d.) + 6d. = £7780 10s. 6d.
2 In $20\frac{5}{8}$ days, or on April 21st.
3. $\frac{\text{Girth at 10 ft.}}{6\text{ ft.}} = \frac{\text{whole length} - 10\text{ ft.}}{\text{whole length}} = \frac{40\text{ ft.}}{50\text{ ft.}}$

$\therefore$ girth at 10 ft. $= \frac{4}{5} \times 6$ ft. $= 4\frac{4}{5}$ ft. And girth at 20 ft. $= 3\frac{3}{5}$ ft.

4. £45 7s. $0\frac{3}{8}$d.
5. 3 ft. $2\frac{2}{17}$ in.
6. Expenses, beside printing, = 54 p. c.

. 46 p. c. or $\frac{46}{100}$ths of pub^g price = the printing price = 2s. 6d.

$\therefore$ pub^g price $= \frac{100}{46}$ths of 2s. 6d. = 5s. $5\frac{5}{23}$d.

7. £6 10s. $5\frac{5}{23}$d.
8. $1493\frac{1}{3}$.
9. 11 : 8 :: 77 : 56. *Ans.* 56.
10. Whole poundage = 3s. 4d. + $\frac{2}{3}$ of 3s. 4d. + $\frac{3}{5}$ of $\frac{2}{3}$ of 3s. 4d.

= 3s. 4d. + 2s. $2\frac{2}{3}$d. + 1s. 4d.

= 6s. $10\frac{2}{3}$d.

$\therefore$ whole rate paid $= 27 \times$ (6s. $10\frac{2}{3}$d.) = £9 6s.

whole cost = £36 6s.

11. $4\frac{53}{160}$.
12. $\frac{£3\cdot\dot{7}}{4\cdot 15\text{ guins.}} = \frac{£3\frac{7}{9}}{£4\frac{7}{45} \times \frac{21}{20}} = \frac{200}{231}$, or 200 : 231.

13. £52 6s. $3\frac{5}{8}$d.

14. In 1 hour there is put in $\frac{1}{2}+\frac{1}{8\frac{2}{7}}+\frac{1}{7\frac{1}{2}}$ or $\frac{1}{2}+\frac{7}{58}+\frac{2}{15}$ or $\frac{656}{870}$ths.

$\therefore$ the time required is $\frac{870}{656}$ hours, or $1\frac{107}{328}$ hours.

15. Disc^t = $\frac{2}{19}$ths; $\therefore$ $\frac{2}{17}$ths must be added to net, to make gross price.

net price : gross price = 1 : $1\frac{2}{17}$ = 17 : 19.

16. If $\frac{7}{13}$ are added, $\frac{7}{13+7}$ or $\frac{7}{20}$ are subtracted: and $\frac{7}{20}$ of £100 = 35 p.c

56.

1. $\frac{£100 \text{ stock}}{£2000 \text{ stock}} = \frac{£85}{\text{req. sum}}$ *Ans.* £1700.
2. $\frac{£100 \text{ stock}}{£1250 \text{ stock}} = \frac{£93\frac{5}{8}}{£1170 \text{ 6s. 3d.}}$
3. If £$90\frac{1}{2}$ will buy £100 stock, how much will £$1176\frac{1}{2}$ buy? *Ans.* £1300.
4. If £$94\frac{1}{2}$ will bring £3 income, how much will bring £500? *Ans.* £15750.
5. If £93 will give £3, what will £100 give? *Ans.* £3 4s. $6\frac{6}{31}$d.
6. £37 17s. $9\frac{373}{789}$d.
7. The price of the stock is the sum that will produce £$3\frac{1}{2}$ interest, if therefore £999 19s. $11\frac{1}{4}$d. stock produces £44 0s. 6d., what will give £$3\frac{1}{2}$? *Ans.* £$79\frac{1}{2}$.
8. If £1875 produces £75, what must £100 produce? *Ans.* £4.
9. Allowing for brokerage, he buys at $87\frac{5}{8}$ and sells at $89\frac{3}{8}$; he gains £$1\frac{3}{4}$ on every $87\frac{5}{8}$ invested: how much on £100,000? [*Ans.* 1997 2s. $11\frac{185}{701}$d.]
10. £71 8s. $6\frac{6}{7}$d.
11. £177 15s. $6\frac{2}{3}$d.
12. A share, bought for £$89\frac{1}{4}$, makes a profit of £$41\frac{7}{8}$; how much will make a profit of £357 2s. $10\frac{2}{7}$d? *Ans.* £$761\frac{13}{67}$.
13. £450 stock in 3 p. c. produces £13 10s.
    £315 ... 4 p. c. „ £12 12s. Difference = 18s. loss.
14. £57 10s. $3\frac{681}{763}$d.
15. His income from the $3\frac{1}{2}$ per cents. = $\frac{£18700}{£93\frac{1}{2}} \times £3\frac{1}{2}$ = £700.

    The one fifth, or £3740 in the 4 per cents. produces £155 16s. 8d.
    The four-fifths, or £14960 in the 3 per cents. „ £498 13s. 4d.
    Difference of income = £45 10s. loss.

16. The 4 p. c. at 102 give 7s. $10\frac{3}{17}$ more than the 3 p. c. at 85.

		£	s.	d.
17.	First investment gives	228	2	$8\frac{3}{5}\frac{3}{5}\frac{3}{5}$
	Second "	205	18	$4\frac{4}{7}\frac{9}{7}\frac{9}{7}$
	Neglecting fracns, diffe =	22	4	4

18. Now, if £100 − 3 p. c. tax = £97, what sum will bring £100 free of tax?

Ans. $\frac{£10000}{97}$ or £$103\frac{9}{97}$.

∴ if £97 will bring £$3\frac{1}{2}$, how much will bring £$103\frac{9}{97}$.

Ans. £$2857\frac{1}{7}$.

57

1. $4\frac{2}{3}$ months.
2. $2\frac{79}{123}$ yrs.
3. Remr = $\frac{13}{60}$ths; ∴ equated time = $\dfrac{\frac{4}{3}+\frac{3}{2}+\frac{8}{5}+\frac{13}{5}}{\text{whole or } 1}$ months $= 7\frac{1}{30}$ months
4. Equated time = $\dfrac{£20\ (1+2+3+4+5+6+7+8+9+10)}{£200}$ month

$= 5\frac{1}{2}$ months

58.

1. 8925 francs.
2. 1914 frs. $30\frac{1}{4}$ cts.
3. 11283 fr. $3\frac{3}{4}$ cts.
4. £361.
5. £833 19s. $8\frac{2}{5}\frac{5}{11}\frac{6}{8}$d.
6. £70 15s. $5\frac{3}{5}\frac{6}{8}$d.
7. 5813 mrks. 12 sch.
8. 5098 mrks. $9\frac{3}{8}$ sch.
9. 1179 mrks.
10. £74 1s. $5\frac{7}{9}$d.
11. £641 1s. $10\frac{2}{4}\frac{2}{3}\frac{6}{7}$.
12. £258 8s. $10\frac{3}{7}\frac{6}{8}$d.
13. 480 dollars.
14. $2080\frac{3}{8}$ dollars.
15. $6113\frac{3}{4}$ dollars.
16. £208 6s. 8d.
17. £675.
18. $5106\frac{1}{4}\frac{3}{7}$ rupees.
19. £757 16s. 3d.
20. 5687 francs.
21. $1555\frac{5}{9}$ florins.
22. $1973\frac{1}{3}$ mrks.
23. £1 = 24 fr. $34\frac{76}{121}$ cts.
24. $4816\frac{1}{2}$ rees.
25. £1 = 24 fr. $97\frac{1}{7}\frac{1}{2}$ cts.

26. £1 = 240d.
54d. = 1000 rees
800 rees = 5 lire
25 lire = $10\frac{2}{3}$ flor.
$11\frac{3}{4}$ flor. = $13\frac{1}{2}$ mrks.

$$\therefore £1 = \frac{240 \times 1000 \times 5 \times 10\frac{2}{3} \times 13\frac{1}{2}}{54 \times 800 \times 25 \times 11\frac{3}{4}} \text{ mrks.}$$

$= 13$ mrks. $9\frac{4}{4}\frac{1}{7}$ sch.

And there would be a gain of 35 mks. $3\frac{3}{4}\frac{9}{7}$ sch., or about £2 11s. 6d by the direct route.

27. The difference for £1 = ·065 francs,

$$\text{or} = \frac{\cdot065}{25\cdot225}\text{£}1, \text{ since } 25\cdot225 \text{ francs} = \text{£}1.$$

$$= \cdot00257 \text{ \&c. £}$$

∴ difference in £100 = ·257 &c. £, or is 0·26 per cent. nearly.

59. M.

1. 75.

2. $1\frac{1}{2}$ mill. × $1\frac{1}{2}$ lbs. = $2\frac{1}{4}$ mill. lbs.

∴ $\frac{2,250,000}{20 \times 4 \times 8}$ quarters are required per day.

∴ number of acres for 1 week $= \frac{7 \times \text{this No. of quarters}}{4\frac{1}{2} \text{ quarters}} = 5468\frac{3}{4}$.

3. 1,030,301.

4. $\frac{\text{per centage}}{\text{£}100} = \frac{2\text{s. } 2\frac{1}{4}\text{d.}}{4\text{s. } 0\frac{3}{4}\text{d.}}$ ∴ per centage = £$53\frac{11}{13}$.

5. Gain of $\frac{1}{4}$ p.c. on 150* millions = £375,000
Loss of $2\frac{1}{2}$ p. c. on 6 millions = £150,000
Gain . . . . = £225,000

6. 135 ac.; 90 ac.; 45 ac. See Art. 158.

7. $\frac{9}{80}$; ·00225.

8. £1 19s. $7\frac{679}{800}$d.

9. To A, £31 0s. $8\frac{8}{29}$d.
To B, £41 7s. $7\frac{1}{29}$d.
To C, £77 11s. $8\frac{20}{29}$d.

10. £2 17s. 6d.

11. £1500 stock cost 15 times £$88\frac{5}{8}$, or £1329 7s. 6d

12. $\frac{\text{£}100}{8} = \text{£}12\frac{1}{2}$ per cent.

13. £157 10s.

N.

2. 121 yds.

3. ·0000486; 33750.

4. £3 4s. $4\frac{1}{2}$d.

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* The gain must be counted on 150 m. not on 144 m.; for stock is never converted into a lower per cent. stock, unless money is sufficiently plentiful to induce capitalists to take up the stock of those who dissent; so that there will still be 150 m. held.

5. $2\frac{2}{7}$.

6. £1000; £1666 13s. 4d. £2333 6s. 8d.

7. See Arts. 97 and 101.

8. $\frac{7}{9000}$ sh.; $\frac{7}{45000}$ cr.

9. £350. See Ex. V. in Art. 146.

10. £28 2s. 6d.

11. £450.

12. $328\frac{3}{8}$ yds.

13. See Arts. 75 and 76, and Exs. therein.

14. $\dfrac{17}{\frac{2}{3} \text{ of } \frac{7}{8} \text{ of } 3\cdot\dot{3}} = 8\cdot7\dot{4}2857\dot{1}$.

15. $14\frac{8}{9}$ yrs. See Art. 148.

O.

1. £87 10s.

2. 30 yds. 22 ft. 1094 in.

3. 80.

4. 500 francs. See note to Art. 170, Chain Rule.

5. $\dfrac{1}{\frac{3}{4} \times \frac{7}{9} \times 2\frac{2}{11}} = \dfrac{11}{14}$.

6. Each share is worth £75; No. of shares $= \dfrac{£150{,}000}{£3\frac{3}{4}} = 40$

$\therefore$ whole line is worth $40{,}000 \times £75 = £3{,}000{,}000$.

7. $\dfrac{£100}{£17} = 5\frac{15}{17}$ p. c.

8. £117 : £100 = 15s. 9d. : 13s. $5\frac{7}{13}$d. the prime cost.

9. £15600.

10. £21 1s. $0\frac{12}{19}$d.; £23 13s. $8\frac{4}{19}$d.; £25 5s. $3\frac{3}{19}$d. See Ex. V. in

11. $\dfrac{3}{89} < \dfrac{3\frac{1}{2}}{94}$; $\therefore$ the latter investment is the better by

$$10000 \times \left(\frac{3\frac{1}{2}}{94} - \frac{3}{89}\right) £ = £35\ 5s.\ 2\tfrac{3454}{4183}d.$$

12. $\dfrac{2625,\ 13242,\ 1000}{1{,}500{,}000}$

13. 5s. $11\frac{721}{780}$d. See Art. 157.

14. 26775 francs.

15. £357 10s.

P

1. $\frac{1}{8}$; $52\frac{1}{12}$.
2. 590 : 1.
3. $6\frac{8}{13}$ months.
4. See Art. 149, and Ex. II. in 151. *Ans.* 13s.
5. $3\frac{3}{4}$ p. c.
6. $32\frac{2}{89}$ p. c.
7. I receive £3 per an. for £$55\frac{1}{2}$, which is £$5\frac{15}{37}$ p. c., or £5 8s. $1\frac{11}{37}$d.
8. £43 7s. 6d. £2 17s. 10d. per cent.
9. 225.
10. In the 3 per cents, the price $= \frac{100 \times 3}{3\frac{1}{4}}$£ = £92 6s. $1\frac{11}{13}$d.

   „ $3\frac{1}{2}$ „ = £107 13s. $10\frac{2}{13}$d.

   „ 4 „ = £123 1s. $6\frac{6}{13}$d.
11. £617 12s. $11\frac{5}{17}$d.; £308 16s. $5\frac{11}{17}$d.; £123 10s. $7\frac{1}{17}$d.
12. Since the invoice and net prices are as 11 : 8, ∴ $\frac{3}{11}$ths of invoice price must be thrown off: or, per centage of discount $= \frac{3}{11} \times$ £100 = £$27\frac{3}{11}$.

   For Ex., if the invoice price be 88s.

$$\text{Then, discount} = \frac{27\frac{3}{11}}{100} = \frac{300}{11 \times 100} = \frac{3}{11}.$$

$$\therefore \text{ net price} = \tfrac{8}{11} \text{ of } 88\text{s.} = 64\text{s.}$$

$$\text{and net price : invoice price} = 64 : 88 = 8 : 11.$$

13. £415 10s. in Eng^d = £446 13s. 3d. in Amer^a = $1985\frac{1}{6}$ dollars.
14. If £$110\frac{1}{2}$ Amer. = £100 Eng., 3500 dollars at 4s. 6d. will give £712 13s. $4\frac{159}{221}$ Eng.
15. $£1 = \frac{25{\cdot}5 \text{ fr.} \times 55 \text{ flor.} \times 13 \text{ mks.}}{117 \text{ fr.} \times 11 \text{ flor.}} = 14\frac{1}{6}$ mks.

60.

1. 37 sq. ft. 9 sup. pr. 9 sq. in.
2. 372 „ 3 „ 9 „
3. 28 „ 5 „ 3 „
4. 65 „ 3 „ 9 „
5. 81 ft. 2 pr. 7 sec. 0 th. 8 fourths.
6. 196 ft. 9 pr. 4 sec. 6 th. 5 fourths.
7. $\frac{20\frac{1}{2} \text{ ft.} \times 4\frac{1}{4} \text{ ft.}}{9 \text{ sq. ft.}} \times 30\text{s.} =$ £14 10s. 5d.

8. £3 6s $8\frac{88}{108}$d.

9. 3 sq. ft. 42 sq. in.

10. Circumference = 2 length + 2 breadth = 24 ft. 4 in.
Whole surface = height × circumference + 2 length × breadth
= 147 sq. ft. $35\frac{7}{8}$ sq. in.

11. 45 sq. ft. 54 sq. in.

12. £161 8s.

13. 63 sq. ft. 120 sq. in

61.

1. 83 sol. ft. 11 pr. 9 sec. 6 sol. in.
2. 38 „ 9 „ 2 „ 6 „
3. 112 „ 9 „ 7 „ 9 „
4. 6 „ 5 „ or 924 sol. in.
5. 199 „ 5 „ 10 „ 3 „
6. 2304 bricks.
7. £531 14s. $6\frac{3}{8}$d.
8. 29 sq. ft. 6 pr. 9 th. 6 frths.
9. 3827 lbs. 1 oz. 5 dwts. 7·6 grs.
10. Height × area of base = 164 × $277\frac{1}{4}$ cub. in.

$$\therefore \text{ ht.} = \frac{164 \times \frac{1109}{4} \text{ cub. in.}}{2218 \text{ sq. in.}} = 20\tfrac{1}{2} \text{ lin. in.}$$

$$= 1 \text{ ft. } 8\tfrac{1}{2} \text{ in.}$$

11. 44 a. 0 r. 2·712 p.
12. 13 chains.

62.

1.	27.	5.	$5\frac{1}{4}$.	9.	3·25.
2.	105.	6.	$19\frac{2}{7}$.	10.	14·05.
3.	90909.	7.	$35\frac{4}{17}$.	11.	6·0285,
4.	1683.	8.	$110\frac{1}{10}$.	12.	·049.

63.

1. 1·7320, &c.

2. $\dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} = \dfrac{1{\cdot}7320, \&c.}{3} = {\cdot}5773$, &c.

3. $\sqrt{6{\cdot}249} = \sqrt{6{\cdot}25} = 2{\cdot}5.$

4. $\sqrt{15{\cdot}\dot{3}} = \sqrt{15\tfrac{3}{9}} = \sqrt{\dfrac{138}{9}} = \dfrac{11{\cdot}7473, \text{ \&c.}}{3} = 3{\cdot}9157, \text{ \&c.}$

5. Side $= \sqrt{\text{area}} = \sqrt{800 \times 1800} = \sqrt{144000} = 1200.$

6. Sum of areas $= 225 + 625 = 850.$ Side $= \sqrt{850} = 29{\cdot}1547$, &c. ft.

7. Length of diagonal $= \sqrt{(64)^2 + (48)^2} = \sqrt{4096 + 2304} = 80.$

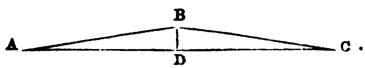
8. Area of field $= 602\tfrac{58}{121}$ sq. per. $= \dfrac{72900}{121}$ sq. per.

$\therefore$ side of field $= \dfrac{270}{11}$ lin. pol. $= \dfrac{270}{11} \times \dfrac{11}{2}$ lin. yds. $= 135$ yds.

9. No. of sq. yds. $= \dfrac{£22\ \ 15s.\ 7\tfrac{1}{2}d.}{7\tfrac{1}{2}d.} = \dfrac{455\tfrac{5}{8}\text{ sh.}}{\tfrac{5}{8}\text{ sh.}} = 729.$

$\therefore$ number of yds. in side $= 27.$

10.



Base $=$ AD $+$ DC

$$AD = \sqrt{AB^2 - BD^2} = \sqrt{(236{\cdot}25)^2 - (2{\cdot}4)^2} = 236{\cdot}237$$

$$DC = \sqrt{BC^2 - BD^2} = \sqrt{(243{\cdot}75)^2 - (2{\cdot}4)^2} = 243{\cdot}738.$$

Whole base $= 479{\cdot}975.$

64.

1. 35.
2. 99.
3. 187.
4. 3201.
5. $3\tfrac{1}{4}$.
6. $11\tfrac{1}{10}$.
7. $13\tfrac{2}{7}$.
8. $75\tfrac{1}{8}$.
9. 4·27.
10. 65·8.
11. 11·01.
12. $\dfrac{{\cdot}074}{{\cdot}4}$ or ·185.
13. 2·46, &c.
14. 5·37, &c.

15. $\sqrt[3]{342{\cdot}\dot{9}} = \sqrt[3]{343} = 7.$

16. $\sqrt[3]{\dfrac{1}{15}} = \sqrt[3]{\dfrac{15^2}{15^3}} = \dfrac{\sqrt[3]{15^2}}{\sqrt[3]{15^3}} = \dfrac{\sqrt[3]{225}}{15}$

$= \dfrac{6{\cdot}082}{15} = {\cdot}406$ nearly.

17. 24.

18. The edge $= \sqrt[3]{72 \times 24 \times 27} = \sqrt[3]{2^3 \times 3^2 \times 2^3 \times 3 \times 3^3}$

$= \sqrt[3]{2^3 \times 2^3 \times 3^3 \times 3^3} = 2 \times 2 \times 3 \times 3 = 36.$

19. $\sqrt[3]{94\text{ y. } 14\text{ ft. } 1088\text{ in.}} = \sqrt[3]{4{,}410{,}944\text{ sol in.}} = 164$ lin. in.

20. The six surfaces $= 6 \times (\text{edge of cube})^2$.

And edge $= \sqrt[3]{15 \text{ sol. ft. } 1080 \text{ sol. in.}} = \sqrt[3]{27000 \text{ sol. in.}} = 30$ lin. in.

$\therefore$ whole six surfaces $= 6 \times (2\frac{1}{2} \text{ ft.})^2$

$$= 6 \times \frac{25}{4} \text{ ft.} = 37\tfrac{1}{2} \text{ sq. ft.}$$

65.

Q.

1. See Arts. 179, 180.
2. See Arts. 177, 181.
3. £9180.
4. Cost £27 18s. Gain per cent $= £40\frac{140}{279}$.
5. £3 18s. 3[illegible]d.
6. $\dfrac{212{\cdot}15 \text{ ft.} \times 12 \text{in.}}{3 \times 1760} = {\cdot}48215\dot{9}\dot{0}$ of an inch.
7. The expressions $= \frac{533}{120}$ and $\frac{45}{120}$; the required ratio $= \frac{289}{244}$.
8. 322 sq. ft. 1 sup. pr. 4 sq. in.; or 322 sq. ft. 16 sq. in.
9. $\dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{3}\sqrt{3}} = \dfrac{1}{3}\sqrt{3}$. The latter; see Art. 194.
10. 80·15, and $\frac{3}{4}$.
11. 43 ft. 7 pr. 6 sec. 8 sol. in.; or 43 sol. ft. 1088 sol. in.
12. £1 7s. 0d. See Art. 151, Ex. 11.
13. £59 14s. $5\frac{1}{3}$d.
14. £1 = 25 fr. $63\frac{13}{20}$ cents.

R.

1. 64, 81.
2. 342 lbs. $10\frac{1}{2}$ oz.
3. 2 ft. 11 in.; 51 sq. ft. 6 sq. in.
4. $4\frac{6}{7}$; 5·46, &c.
5. Surface = (twice length + twice breadth) × ht. + twice length × breadth
   = 174 sq. ft. 88 sq. in.
6. 1st surface $= (15\frac{1}{4} + 10) \times 2 \times 9\frac{1}{2}$ sq. ft.
   2nd surface $= (18\frac{1}{6} + 11\frac{7}{12}) \times 2 \times 12\frac{2}{3}$ sq. ft.
   Cost of 2nd surface = £8 8s. $10\frac{64}{101}$d.
7. $24\frac{1}{4}$d. = 1 rupee.
8. $10\frac{1}{3}$ per cent gain.
9. £59 5s. $2\frac{2}{9}$d.

10. If on £150 there was lost £75$\frac{1}{2}$, on how much would £7550 be lost?
Ans. £15000.

11. 13 a. 1 r. 4$\frac{1}{2}$ p.

12. 9$\frac{1}{2}$ chains. See Art. 193, Ex. 1.

13. 5$\frac{1}{30}$ months.

14. See Art. 203.

8.

1. £6 16s. 3d.

2. 54.

3. $\frac{196}{1815}$.

4. Present worth = £150. $\therefore$ *Ans.* $= \frac{100}{150} \times$ £3 = £2 for half year, or 4 per cent. per an.

5. 5 yrs.

6. 19$\frac{1}{21}$ yrs.

7. £1500.

8. £240 to *A*, £144 to *B*, £216 to *C*.

9. 7265$\frac{5}{8}$ lbs.

10. Aver. ht. = 29·94; daily rise $= \frac{1\cdot58}{12}$.

$$\therefore \text{ required ratio} = \frac{79}{17964}.$$

11. Area = $\frac{1}{2}$ sq. of diagonal $= \frac{1}{2} \times (160)^2 = 12800$ sq. yds.
$= 2$ a. 2 r. 23$\frac{17}{121}$ p.

12. 1 inch of gold weighs $\frac{1260\frac{2}{3}}{1728}$ lbs. (A)

1 lb. of cork $= \frac{1}{15}$ of 1 foot.

$$\therefore \frac{1260\frac{2}{3}}{1728} \text{ lbs. of cork} = \frac{1260\frac{2}{3}}{1728} \times \frac{1}{15} \text{ ft.} = \frac{3782}{3} \times \frac{1}{15} \text{ inches}$$
$$= 84\tfrac{2}{45} \text{ sol. in.}$$

But $\frac{1260\frac{2}{3}}{1728}$ lbs. of cork = 1 inch of gold in weight, from (A)

i.e. weight of 1 inch of gold = that of 84$\frac{2}{45}$ sol. in. of cork.

13. Vol. of cube $= (3\cdot35)^3$ sol. in.

vol. of wire = length × area of section
= length × ·561125 sq. in.

length × ·561125 sq. in. $= (3\cdot35)^3$ sol. in.
or length = 67 in.

14. See Art. 193. Req^d Mult^r = 45.

66.

1. 101,011,001
2. 21.
3. 1,212,221.
4. 334,401.
5. 40262.
6. 4666.
7. 1,010,010,000,100.
8. 31.
9. 220,130.
10. $(21021)_3 = (196)_{10} = (524$
11. 101,115.
12. 8548*e*1.
13.
```
      187t56
       t4789
     135tt16
    1192e80
   1007126
  6t75t0
 1526870
 15t67881016
```
14.
$$456)\ 18763\ (37$$
```
456) 18763 (37
     1480
      3863
      3536
       326
```
$$\frac{326}{456} = \frac{108}{148}.$$

15. 15 ft. 11 in. = 13·*e* written decimally in duodenary scale.
```
7 ft. 9 in. =  7·9
              ee3
             935
             103·43
```
or 123 sq. ft. 51 sq. inches.

Exs. 66. A.

1. $558\frac{6}{43}$
2. £5 17s. $11\frac{5}{42}$d.
3. $5\frac{19}{51}$
4. £47 16s. 3d.
5. £7 7s. $5\frac{5}{8}$d.
6. 118800.
7. £1118 15s.
8. $3\frac{2}{5}$; 1.
9. $6\frac{1}{2}$ hours after B's departure.
10. 9008.
11. 5s. 5d.
12. $\frac{27}{80}$; ·3375.
13. 14 ft. 4 in.
14. 4200*l*.
15. $\frac{13}{20}$; ·65.
16. 334 ft. 714 in.
17. £1012 9s. $9\frac{1}{2}$d.
18. £7 3s. $4\frac{1}{2}$d.
19. £82 3s. 4d.
20. £1 10s.
21. 199 yds. 1 ft. $6\frac{5}{9}$ in.
22. £3012 10s.
23. $\frac{125}{216}$ oz.
24. $12\frac{1}{2}$ p. c.
25. $7\frac{5}{9}$ roods.
26. 6·05.
27. 450; ·024.
28. ·7736 of a day.
29. £43 9s. $4\frac{736}{827}$d.
30. £43 5s.
31. 2.
32. £3 17s. $1\frac{6}{7}$d.
33. £1200.
34. $7\frac{1}{2}$; $\frac{9}{10}$.

35. 5s. $6\frac{2}{3}$d.; ·1241419.
36. £1491 13s. 4d.
37. 24·075; ·00321.
38. 30 cwt. 15 lbs.
39. £1 2s.
40. £4 4s. $6\frac{18}{38}$d.
41. £7 10s.
42. $4\frac{1}{2}$.
43. $\frac{4}{9}$; ·6.
44. £164 15s. 10d.
45. 10 weeks.
46. $133\frac{1}{3}$.
47. $3\frac{3}{4}$ p.c.
48. £5 6s. 3d.; £5 13s. 4d.
49. 24.
50. £6000.
51. £5.
52. 17s. 6d.
53. 340.
54. A's, 2000: B's, 4000; C's, 6000
55. 22.
56. £1 19s. $6\frac{18}{35}$d.
57. 3 years.
58. £475 13s. 9d.
59. £22 5s. 6d.; £165.
60. 216.
61. 44; 31.
62. $26\frac{1}{4}$; $551\frac{1}{4}$.
63. 12600.
64. $7\frac{23}{96}$.
65. See (46); $\frac{7700, 7875, 8000, 8064}{8400}$
66. $\frac{3}{5}$, $\frac{3}{4}+\frac{4}{5}$, $\frac{4}{5}$.
68. $\frac{3\frac{9}{10}}{12}$
69. See Art. (179).
70. See Art. (176).
71. See Art. (179).
73. 606·4 sq. yds.
74. 1·984; 1·401; ·128.
75. $\frac{25}{64}$; $\frac{186}{640}$.
76. 350*l*, 500*l*, 650*l*.
77. $8\frac{1}{4}$.
78. $15\frac{3}{7}$.
79. $22\frac{13}{22}$.
80. 15d.
81. $73\frac{1}{4}$ sq. in.
82. $42\frac{7}{8}$ sol. in.
83. $3\frac{13}{27}$.
84. 36; $1\frac{1}{9}$.
85. £106 9s. 2d.
86. £603 7s. 3d.
87. £1001 19s. $1\frac{1}{4}$d.
88. $4\frac{1}{2}$.
89. 6 years.
90. $2\frac{3}{4}$ p.c.
91. £42 2s. $5\frac{11}{26}$d.
92. $23\frac{9}{17}$ p.c.
93. $3\frac{7}{11}$ p.c.
94. £14 9s. $6\frac{30}{40}$d.
95. $20\frac{5}{6}$ p.c.
96. £9184 5s. 4d.
97. £6544 1s. $4\frac{1}{2}$d.
98. £15.
99. £1250.
100. £250.
101. £422 8s. $2\frac{3}{4}$d.
102. 75.
103. 36lbs.
104. 1s. $4\frac{1}{2}$d.; ·55.
105. $\frac{4}{405}$.
106. 415.
107. £12 2s. 8d.
108. 10·36 miles per hour.
109. $\frac{7}{9}$, the greatest; $\frac{4}{7}$, the least.
110. $62\frac{17}{40}$.

111. $\frac{416}{1213}$.
112. $\frac{24}{34}$.
113. $1\frac{17}{24}$ to each.
114. $37\frac{1}{33}$.
115. 350.
116. $6\frac{1}{4}$ p.c.
117. £1 2s. $3\frac{8}{19}$d.
118. $133\frac{1}{3}$ p.c.
119. 20 p.c.
120. £3285 18s. 9d.
121. £7950.
122. £2500.
123. ·0668402$\dot{7}$.
124. £38965 3s. 3d.
125. 2s. 9d.
126. $2\frac{13}{18}$d.
127. 25 p.c.
128. 9s. $7\frac{1}{2}$d.
129. £4303 15s. $10\frac{1}{2}$d.
130. £9 18s. $10\frac{1}{2}$d.
131. $9\frac{3}{8}$.
132. £26 11s. $0\frac{3}{7}$d.
133. $22\frac{10}{11}$ sh.
134. 995328.
135. £5 13s. 6d.
136. £13 9s. 11·55d.
137. 125.
138. £53 13s. $6\frac{3}{8}$d.
139. $40\frac{10}{21}$.
140. £30 17s. $9\frac{63}{128}$d.
141. $1\frac{19}{88}$.
142. £253 18s. $1\frac{1}{2}$d.
143. £61 12s.
144. 30000.
145. £4 2s. 6d.
146. 6 mths. 10 dys.
147. 16·4012 . . yds.
148. 17s. $4\frac{4}{27}$d.
149. 39 ; 1·5885.
150. 4 chs. 40 links.

67.

1. $\frac{97}{100}$ths of £333 6s. 8d. = £323 6s. 8d.
2. In 120 days; 14 min. to 2, 16 min. past 2.
3. 6s. 8d. £3333 6s. 8d. £3500. £166 13s. 4d.
4. $\frac{25}{48}$d.
5. 1st + 2nd empty $\frac{1}{32}$ in 1 min. (1)

   1st + 3rd „ $\frac{1}{24}$ „ (2)

   2nd + 3rd „ $\frac{1}{16}$ „ (3)

   ∴ sub^{s} (2) from (1) 2nd − 3rd $= \frac{1}{32} - \frac{1}{24}$. (4)

   Also, from (3) 2nd + 3rd $= \frac{1}{16}$.

   Adding, twice 2nd $= \left(\frac{1}{32} + \frac{1}{16} - \frac{1}{24}\right)$

   $= \frac{5}{96}$, or 2nd $= \frac{5}{192}$

   Sub^{s} (4) from (3), twice 3rd $= \left(\frac{1}{16} - \frac{1}{32} + \frac{1}{24}\right)$

   $= \frac{7}{96}$, or 3rd $= \frac{7}{192}$

   And 1st $= \frac{1}{32} - $ 2nd $= \frac{1}{32} - \frac{5}{192} =$ or $\frac{1}{192}$

   ∴ first discharges $\frac{1}{192}$ of 384 gal., or 2 gal.

   Second „ 10 gal. Third discharges 14 gals.

6. $29\frac{13}{32}$ sq. ft., or 29 ft. $58\frac{1}{2}$ in. See **Exs. 61.**

7. £22 0s. 10d.

8. If £100 come to £107 2s. 5d. (neglecting frac^{ns}), what sum will come to £360 10s. *Ans.* £336 10s. 8d.

9. $$\frac{7+\sqrt{6\frac{1}{4}}}{6\frac{1}{3}\times(3+\sqrt[3]{3\frac{3}{8}})} = \frac{7+\frac{5}{2}}{\frac{19}{3}\times\left(3+\frac{3}{2}\right)} = \frac{\frac{19}{2}}{\frac{19}{3}\times\frac{9}{2}} = \frac{1}{3}.$$

10. $$\text{Required number} = \frac{\text{whole vol.}}{\text{vol. of 1 cube}} = \frac{18\times 7\frac{1}{2}}{\left(\frac{3}{4}\right)^3} = 320.$$

11. $291\frac{3}{8}$ lbs.

12. The distance of min. hand from 12 = 12 times the distance of hour hand from 1. And the former dist. = 5 min. + dist. of hour hand from 1 + min. in 60 degr^s (or 10 min.) = 15 min. + dist. of hour hand from 1.

$\therefore$ 12 times dist. of hour hand from 1 = 15 min. + once the dist. of hour hand from 1,

$\therefore$ sub^g once the dist. of hour hand, from the two equal quan^s, we have

11 times dist. of hour hand = 15 min.

$$\therefore \text{this dist.} = \frac{15\text{ min.}}{11} = 1\text{ min. }21\tfrac{9}{11}\text{ sec.}$$

$$\therefore \text{dist. of min. hand from } 12 = 15\text{ min.} + 1\text{ min. }21\tfrac{9}{11}\text{ sec.}$$
$$= 16\text{ min. }21\tfrac{9}{11}\text{ sec.}$$

13. $$\begin{array}{ccccc} \text{lb.} & & \text{dwt.}\ \ \text{grs.} & & \\ 1 & : & 7\ \ 11 & :: & £46\ 12s.\ 6d.\ :\ £1\ 9s.\ 2{\cdot}52675d. \\ \frac{11}{12} & & \frac{924}{1000} & & \end{array}$$

14. Larger circum. = 3·1416 × 45 yds.
Smaller „ = 3·1416 × 40.
Diff^e = 3·1416 × 5 = 15 yds. 2·124 feet.

15. $$\text{The quantity gained in each step} = \frac{49-45}{50}\text{ yds.} = \frac{2}{25}\text{ yds.}$$

$$\therefore \text{if } \frac{2}{25} \text{ yds. are gained in 1 step}$$

$$1 \text{ yd. is gained in } \frac{25}{2} \text{ steps.}$$

$$\text{and 100 yds. is gained in } \frac{100\times 25}{2} \text{ steps, or 1250 steps.}$$

16. 115 yds. £2 3s. $1\frac{1}{2}$d.

17. Rate = $\frac{1}{15}$ of a mile per min.

$$\therefore \text{volume in feet} = 30\times \overset{40}{\cancel{600}} \times \frac{1}{\cancel{15}} \times 5280 = 6{,}336{,}000 \text{ ft.}$$

weight = $176785\frac{5}{7}$ tons.

18. If it gain $3\frac{1}{2}$ min. per day, it will gain $\frac{7\frac{1}{2}}{24}$ of $3\frac{1}{2}$ min., *i. e.* $1\frac{3}{32}$ min. in $7\frac{1}{2}$ hours.

$\therefore$ the clock must at 12 o'clock be put back $1\frac{3}{32}$ minutes, *i.e.* the time shown by it must be $58\frac{29}{32}$ min. past 11.

19. If $\frac{2}{7}$ths are done in 13 days, $\frac{1}{7}$th is done in $\frac{13}{2}$ dys.,

and all in $\frac{91}{2}$ or $45\frac{1}{2}$ days.

also, by both, $\frac{5}{7}$ths are done in 6 days, or $\frac{5}{42}$ in 1 day.

and the first does $\frac{2}{91}$ in 1 day, $\therefore$ the second does $\left(\frac{5}{42} - \frac{2}{91}\right)$ths,

or $\frac{53}{546}$ in 1 day, $\therefore$ the time of doing it is $\frac{546}{53}$ dys., or $10\frac{16}{53}$ dys.

Ans. $45\frac{1}{2}$, $10\frac{16}{53}$.

20. If profit in half year = £63 5s. 2d. on £189 15s. 6d., how much on £100? *Ans.* £$33\frac{1}{3}$, or £$66\frac{2}{3}$ per an.

21. 168 francs, or an average of £6 9s. 5·757d., &c.

22. $\frac{233}{240}$ of his rent = £553 7s. 6d., $\therefore$ his rent = $\frac{240}{233}$ of £553 7s. 6d.

= £570.

23. Length of rows × breadth between rows = whole surface,

$$\therefore \text{length} = \frac{7\frac{1}{2}\text{ ac.}}{\frac{15}{36}\text{ yd.}} = 87120 \text{ yds.}$$

24. 10 days.

25. A rate of 1 mile per minute would give $6\frac{1}{15}$ m., or 6 m. 0 f. $117\frac{1}{3}$ yds. in 6 min. 4 sec., and the distance of 3 m. 4 f. 93 yds. is short of that by 2 m. 4f. $24\frac{1}{3}$ yds., or $\frac{1}{6\frac{1}{15}}$th of this per minute.

Ans. 3 f. $69\frac{26}{91}$ yds.

26 $\frac{115}{1392}$; 1.

27. Wt. of coal = 2000 × 2240 × 16 oz.

Wt. of water of equal bulk = $\frac{2000 \times 2240 \times 16 \text{ oz.}}{1\cdot12}$

and $\therefore$ vol. of this water (in feet) = $\frac{2000 \times 2240 \times 16 \text{ oz.}}{1\cdot12 \times 1000 \text{ oz.}}$ = 64000 ft.

and edge of volume = 40 feet.

28. 250 marks = $250 \times \frac{2}{3}$£ = $\frac{500}{3}$£,

or the pres. worth of the £500 is $\frac{2}{3}$ of £500.

$\therefore$ the present worth of £150 is £100,

or the interest of £100 in 4 yrs. is £50.

$\therefore$ in 1 yr. is $12\frac{1}{2}$ per cent.

29. 2.

30. $64 = 2^5 \times 2$; and $64 \times 16 = 2^5 \times 2^5$ $\therefore$ the required number is 16.

31. Rate of approach to each other $= 17\frac{1}{2}$ miles per hour,

$$\therefore \text{ they meet in } \frac{51\frac{1}{2}}{17\frac{1}{2}} \text{ hours, or in } 2\tfrac{33}{35} \text{ hrs.}$$

$$\text{Dist. from Lond.} = 9\tfrac{1}{2} \times 2\tfrac{33}{35} \text{ m.} = 27\tfrac{67}{70} \text{ m.}$$

$$\text{,, } \quad \text{Camb.} = 8 \times 2\tfrac{33}{35} \text{ m.} = 23\tfrac{19}{35} \text{ m.}$$

32. For credit of 6 months, every £100 must be charged as £102 10s.

$$\therefore \text{ the ratio of price} = \frac{£100}{£102\frac{1}{2}} = \frac{40}{41} \text{ or } 40 : 41.$$

33. 15.

34. Diam. $= \frac{113}{355} \times$ circumf. $= \frac{113}{355} \times 360 \times 69\frac{1}{22}$ m. $= 7912\frac{20}{781}$ miles.

35. $20\frac{5}{7}$ p. c.

36. A does $\frac{1}{30} - \frac{1}{70}$ or $\frac{2}{105}$ per day, $\therefore$ he does it in $52\frac{1}{2}$ days.

A does more than B by $30 \times \left\{\frac{2}{105} - \frac{1}{70}\right\}$ or $\frac{1}{7}$th.

37. The mean daily motion $= \dfrac{360^\circ}{365\frac{29}{120}} = 59$ min. 8·33, &c. seconds.

38. The expr^n (when red. to L. C. D.) $= \dfrac{5(6^2 - 5^2) + 5(6^2 + 5^2)}{\sqrt{6^4 - 5^4}}$

$$= \frac{360}{\sqrt{671}} = \frac{360}{671}\sqrt{671}. \text{ See Art. 194.}$$

39. Since $A + B$ do $\frac{1}{30}$th, and A's work $= \frac{5}{4}$ B's.

$\therefore \frac{9}{4}$ B's $= \frac{1}{30}$th. Hence B does it in $67\frac{1}{2}$ days.

A ,, 54 days.

40. 1·14.

41. Gain per yd. $= \dfrac{£136 \text{ 2s. 6d.}}{4840} = 6\frac{3}{4}$d. $\therefore$ price req^d = 5s. $0\frac{3}{4}$d. per yd.

42. Length $= \dfrac{\text{vol.}}{\text{breadth} \times \text{thickness}} = \dfrac{5 \text{ cub. ft.}}{1 \text{ ft.} \times \frac{1}{6} \text{ ft.}} = 30$ feet.

43. Beginning at the last fr^n, I have

$$\frac{1}{5\frac{2}{3}} = \frac{3}{17} \quad \therefore 3 + \frac{1}{4 + \dfrac{1}{5\frac{2}{3}}} = 3 + \frac{1}{4 + \dfrac{3}{17}} = 3 + \frac{17}{71} = \frac{230}{71}$$

$$\therefore \text{ whole exp}^n = 1 + \frac{2}{\dfrac{230}{71}} = 1\tfrac{71}{115}.$$

$$\text{Again, } \frac{\sqrt{1 + \frac{1}{3}} \div \sqrt{1 - \frac{1}{5}}}{\sqrt{1 + \frac{1}{3}} \times \sqrt{1 - \frac{1}{5}}} = \frac{1}{1 - \frac{1}{5}} = \frac{1}{\frac{4}{5}} = 1\tfrac{1}{4}.$$

44. ·9486, &c

45 1 sq. ft. $86\frac{3}{8}$ sq. in.

46. 5 : 8.

47. Rent + expenses + £625 + $\frac{1}{10}$th of £625 = £820.

$\therefore$ selling price of each beast = £16 8s. Profit = £3 18s.

48. Vol. = 64000 sol. in., edge = 40 in., diag. of face = $40\sqrt{2}$

$= 56{\cdot}56$, &c.

Diag. of box $= 40\sqrt{3} = 69{\cdot}28$, &c

49. £240

50. Lower layer contains $7 \times 6 = 42$; next, 30; next, 20; next, 12; next, 6; last, 2. Total = 112.

51. 1 mile or 1760 yds. = 160 of these units.

$$\therefore 1 \text{ unit} = \frac{1760}{160} \text{ yds.} = 11 \text{ yds. or } 2 \text{ poles.}$$

Also, 12960 min. $= 60 \times 24 \times 9$ min., or 9 days.

20160 min. $= 60 \times 24 \times 14$ min., or 14 days.

Ans. 2 poles, 1 day.

52. $$\frac{\text{Large Area}}{\text{Small Area}} = \left(\frac{24 \text{ in.}}{\frac{1}{2} \text{ in.}}\right)^2 = 2304 : 1.$$

53. $$\frac{\text{L. Diam.}}{\text{S. Diam.}} = \sqrt{\frac{\text{L. Area}}{\text{S. Area}}} = \sqrt{\frac{49}{9}} = \frac{7}{3} = 7 : 3.$$

54. $$\frac{\text{L. Surface}}{\text{S. Surface}} = \left(\frac{\frac{3}{4}}{27}\right)^2 = 1 : 1296.$$

$$\frac{\text{L. Vol.}}{\text{S. Vol.}} = \left(\frac{\frac{3}{4}}{27}\right)^3 = 1 : 46656.$$

55. The 24 hours clock had gone 8 h. 4 min.

$\therefore$ time from sunset was 8 h. 34 min.; or the sunset time will be found by subtracting 8 h. 34 min. from the real time.

The real time is 4 h. 8 min.

$\therefore$ subtracting 8 h. 34 min. from 16 h. 8 m., reckoning the time from the previous noon, we find the time of sunset to be 7 h. 34 m.

56. $$\frac{\text{Circumf.}}{\text{arc}} = \frac{360^\circ}{8{\cdot}58 \text{ sec.}}$$

$$\therefore \text{circum.} = \frac{360 \times 60 \times 60 \text{ sec.}}{8{\cdot}58 \text{ sec.}} \times 4000 \text{ miles.}$$

$$\text{and rad.} = \frac{\frac{1}{2} \text{ cir.}}{3{\cdot}1416}$$

$$\therefore \text{rad.} = \frac{1}{2} \times \frac{360 \times 60 \times 60}{8{\cdot}58} \times \frac{4000}{3{\cdot}1416} \text{ miles} = 96{,}160{,}523{\cdot}96, \text{ \&c. m's.}$$

57. In 8 seconds 1030·4 ft.

In 9 seconds 1304·1 $\quad\therefore$ in the ninth, 273·7 ft.

58. Diff^{ce} of times of oscillation, in seconds,

$$= 3{\cdot}1416\left\{\sqrt{\frac{39{\cdot}25 \text{ in.}}{386{\cdot}4 \text{ in.}}} - \sqrt{\frac{39{\cdot}2 \text{ in.}}{386{\cdot}4 \text{ in.}}}\right\}$$

$$= 3{\cdot}1416 \times {\cdot}0002 \text{ nearly.}$$

$$= {\cdot}00062832 \text{ sec.}$$

Number of seconds lost $= 86400 \times {\cdot}00062832 = 54{\cdot}2868.$

59. If time is unaltered, and G = altered gravity,

$$\text{then } \sqrt{\frac{39{\cdot}2}{32{\cdot}2}} = \sqrt{\frac{39{\cdot}201}{G}},$$

$$G = \frac{39{\cdot}201}{39{\cdot}2} \times 32{\cdot}2 = 32{\cdot}20082, \text{ \&c.}$$

60. From A. D. 325 to A. D. 1848 = 1523 yrs.

The error, if uncorrected, $= 1523 \times {\cdot}007736$ days.

$= 11{\cdot}781928$ days.

The correction strikes out a day from every century which is not divisible by 4, hence strikes out 11 days, and leaves an error of ·781928 days.

Again, error if uncorrected, would in 400 years be 3·0944 days, or when corrected, ·0944 dys.; hence the time when the error would amount to 1 day would be given by this proportion.

$$\frac{{\cdot}0944 \text{ dys.}}{1 \text{ day}} = \frac{400 \text{ yrs.}}{\text{req}^{\text{d}} \text{ time}} \;\therefore\; \text{req}^{\text{d}} \text{ time} = \frac{400 \text{ yrs.}}{{\cdot}0944} = 4237{\cdot}3 \text{ yrs. nearly.}$$

61. 2·5752 sol. in.

62. Vol. of cube = 1 cub. in.

Vol. of 8 spheres $= 8 \times \frac{4}{3} \times 3{\cdot}1416 \times \left(\frac{1}{4}\right)^3 = {\cdot}5236,$

$\therefore$ space unoccupied $= {\cdot}4764$ sol. in.

63. No; for the factor 7 in the den$^{\text{r}}$ is also contained in the num$^{\text{r}}$; and the fraction $= \frac{3}{20} = {\cdot}15.$

64. The required ratio $= \dfrac{6 \times (1\frac{1}{4})^2 \times 3{\cdot}1416}{6 \times (2\frac{1}{2})^2} = 3{\cdot}1416 : 4.$

65. The number $= \dfrac{(15\frac{2}{3} \times 11\frac{1}{2} - 6\frac{1}{4} \times 2\frac{1}{3})144 \times 11}{165\frac{7}{12}} = 1584.$

66. $\dfrac{\text{Required wt.}}{1 \text{ cub. in.}} = \dfrac{\frac{11}{12} \times (5 \text{ dwts. } 3\frac{1}{4} \text{ grs.})}{252{\cdot}458 \times 19{\cdot}362} = \dfrac{1}{43{\cdot}26}$ nearly.

67. The expr$^{\text{n}}$ $= \dfrac{59}{20} \times \dfrac{155}{12} \times \dfrac{10}{161} = 2\frac{1417}{3864}.$

68. From 1 to 3, A and B will have put in $\frac{2}{3}+\frac{1}{4}$ or $\frac{11}{12}$. And at 3, when all are open, the rate of emptying is $1-\frac{1}{3}-\frac{1}{4}$, or $\frac{5}{12}$, per hour; hence time of emptying the $\frac{11}{12}$ will be $2\frac{1}{5}$ hours.

69. Quantity sold out = £10000 stock; income produced = £350. New income = £360. And if £9350 give £360, £4 will be given by £103$\frac{8}{9}$. *Ans.* £103$\frac{8}{9}$.

70. $\frac{1}{12}$ ox $=\frac{3}{7}$ sheep

$\frac{1}{5}$ sheep $=\frac{3}{4}$£.

$\therefore \frac{1}{12}$ of ox $=\dfrac{\frac{3}{7}\times\frac{3}{4}£}{\frac{1}{5}}$; or, 50 oxen are worth £964$\frac{2}{7}$.

71. Area of walk $=3{\cdot}1416\left\{(8\tfrac{1}{2})^2-(4\tfrac{1}{2})^2\right\}=52\times3{\cdot}1416$ feet.

$$\text{price}=\frac{72}{52\times3{\cdot}1416}\text{d.}=\cdot44\text{d. nearly.}$$

The following references to Arts. are designed to suit the Second Edition of the Arithmetic.

ON FRACTIONS AND THE PRINCIPLES OF PROPORTION.

1. Arts. 1, 2, 3.
2. Arts. 10, 11.
3. Arts. 4, 5.
4. Arts. 7, 8

   $1512=2\times2\times2\times3\times3\times3\times7$.
5. Art. 13. G. C. M. 3.
6. Arts. 23, 24.
7. Arts. 28, 29.
8. Art. 30.
9. Arts. 33, 32.
10. See Fig. 3 in Art. 30.
11. In finding L. C. D.

    Art. 39.
12. Arts. 35, 36, 37, 38
13. Reduced to L. C. D.

    $\frac{3}{4}+\frac{5}{6}=\frac{19}{12}=1\frac{7}{12}$.

    $\frac{7}{8}-\frac{3}{5}=\frac{11}{40}$.
14. Art. 46.
15. Art. 53.
16. Art. 53.
17. Art. 58.
18. See last Ex. in Art. 60.
19. Art. 62.
20. Arts. 65, 66. $\frac{4}{5}$; $\frac{5}{4}$
21. Of lesser inequality;

    of equality;

    of greater inequality

22 $\frac{3s.}{7\frac{1}{2}d.} = \frac{3s.}{\frac{5}{8}s.} = 4\frac{4}{5}$.

23. The equality of two ratios. Art. 69.

24. Arts. 74, 73.

25. Arts. 75, 76.

26. (1) Art. 77, Ex. I.
(2) or, Reduce a moidore to the frn of half a guinea.
$\frac{\text{1 moid}}{\text{hf. guin.}} = \frac{27s.}{10\frac{1}{2}s.} = \frac{54}{21} = 2\frac{4}{7}$

27. Art. 84, Ex. IV.

28. Art. 88.

29. Art. 88.

30. In $\frac{15}{7}$ths of an hour; or $2\frac{1}{7}$ h. Art. 90, Ex. V.

ON DECIMALS.

1. Arts. 92—95.
2. Art. 98 (A).
   „ 99 (B).
   „ 101 (C).
3. Art. 92. DEF. $\frac{16}{10000}$, ·0016.
4. Art. 95.
5. Art. 96. 37500.
6. Art. 96. ·0001876.
7. Arts. 97, 101.
8. Art. 101.
9. Art. 102.
10. Arts. 104—110.
    $\frac{343}{999}$, $\frac{32689}{99900}$.
11. Arts. 111, 112.
12. Arts. 113, 114.
13. Art. 115. ·06475.
14. Art. 121, Exs. II. and III.
    For varieties of pointing, see Exs. IV. and V.
15. Arts. 119, 123.
16. 82·58. Art. 118.
17. Art. 117, Ex. III.
    Prod. = 166·6016188.
18. Art. 124, Exs. I. II.
    Art. 126. Ex.
    Construct similar Exs.

ON PRACTICE.

1. Art. 64.
2. 10s. 6d.
3. Art. 130.
4. Art. 130, Ex. I.
   Ans. £288 15s. 3d.
5. Art. 129.
6. Art. 130, Ex. VI.
7. Art. 130, Ex. VII.
8. Art. 130, Ex. VIII.

ON PROPORTION.

1. Four.
2. Art. 132.
3. Art. 135.
4. Art. 135.
5. $4\text{th} = \frac{2\text{nd} \times 3\text{rd}}{1\text{st}}$ Art. 135.
6. $4\text{th} = \frac{2\text{nd}}{1\text{st}} \times 3\text{rd}$

   = abstract number × 3rd.

   ∴ of the same kind as 3rd.
7. **Exs. 46.** 8. The term 10s. 6d. will not appear in the statement.
8. How long a floor 16 ft. broad will equal a floor 24 feet long and 24 ft. broad?
9. In S. Prop. only three terms are employed.

   In C. Prop. we may have 5, 7, 9, &c., or any *odd* number of terms.
10. Art. 140.

ON THE APPLICATIONS OF PROPORTION.

1. Art. 141.
2. Art. 141.
3. Art. 144.
4. Art. 142.
5. Art. 144.
6. (1) If £175 4s. 2d. produce int. £21 0s. 6d. in 3 yrs., what will £100 give?

   Ans. £12, or 4 p.c. per an.

   (2) If £175 0s. 0d. produce £7 8s. 9d. in 1 year, in how many years will it produce £204 15s. 0d. – £175, or £29 15s.

   Ans. 4 years.

   (3) If £100 in 3 yrs. at 5 p.c. will amount to £115, what sum will amount to £155 5s?

   Ans. £135.

   See Art. 146 for such ques^ns^.
7. Art. 148.
8. Art. 149.
9. Art. 151.
10. Art. 151.
11. Art. 151.
12. Art. 152. (G). (H).

    Ex. I. To throw off $\frac{15}{100}$ths, I put on $\frac{15}{85}$ or $\frac{3}{17}$, and price req^d^ = 24s. $8\frac{8}{17}$d.

    Ex. II. $\frac{3}{8}$ths.
13. Art. 153.
14. Art. 154, Exs. II. III.
15. (1) Multiply the whole sum divided, by one of the shares of the capital, and divide by the whole capital. Repeat this process as often as there are divisions required.

    (2) Multiply each capital by its time; consider these pro-

ducts as the respective capitals, and their sum as the whole capital: then proceed as in (1).

6. Art. 158.
7. Art. 157.
8. Art. 159.
9. (1) If £88 will buy £100 stock, how much must be given for £2520 stock?

   (2) If £$96\frac{1}{4}$ produce £3 int^{t}, what will £100 produce?

   (3) If £$104\frac{3}{8}$ will buy £100 stock, how much can be bought for £1450?

20. Arts. 166, 167.
21. Art 170.
22. Arts. 168, 169.
23. Note to Art. 170.
24. Arts. 163—165.
25. Art. 202, Ex. II.

AREA AND VOLUME.

1. Arts. 171, 172.
2. Arts. 171, 172.
3. Art. 177.
4. Art. 180.
5. Arts. 177, 182.
6. Art. 179.
7. Art. 182.
8. 9. Art. 183.
10. Art. 185.
11. Art. 174.

EXTRACTION OF ROOTS.

1. Art. 186.
2. The fifth root of 18, or that number which when multiplied by itself four times will make 18.
3. Art. 187.
4. Art. 187
5. Art. 188.
6. Art. 189.
7. Art. 193. 5. 75.
8. Art. 189.
9. Art. 190.
10. Arts. 191, 192.
11. Art. 188.
12. Art. 193.
13. Arts. 195, 196.
14. Art. 198.
15. $\sqrt{576} = \sqrt{400 + 160 + 16}$
    $= 20 + 4.$

$\sqrt[3]{1728} = \sqrt[3]{1000 + 600 + 120 + 8}$
$= 10 + 2.$

ERRATA.

H. 10 For " thousandths," read " tenths of thousandths."

51. 2. Read £377 7s. 2d.

67. 54 Change the places of L. and S. in both lines.